AN LMI APPROACH TO ROBUST STABILIZATION OF NETWORKED CONTROL SYSTEMS [1]

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Abstract: The problem of robust stabilization of discrete-time uncertain networked control systems (NCSs) in the presence of packet losses and transmission delays is studied. We model such systems as discrete-time nonlinear systems with delayed input. Sufficient conditions on the stabilization of the NCSs are presented. Stabilizing state feedback controllers can be constructed by using the feasible solution of some linear matrix inequalities (LMIs). One advantage of our method is that the maximum bound on the nonlinearity can be computed by solving a constrained convex optimization problem, another advantage is that the upper bound on the delayed input can be obtained by solving a quasi-convex optimization problem. *Copyright© 2005 IFAC*

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1. INTRODUCTION

Networked control systems (NCSs) is a special class of hybrid systems wherein the control loops are closed through communication networks. NCSs have received increasing attention in recent years because of the popularization and advantages of using network cables in control systems, see, e.g., Ray and Galevi (1988). Industrial applications include automobiles, robotic systems, jacking systems for trains, etc. Because of the distributed structure and the limited bandwidth, the problems of delays and data packet dropout in an NCS are unavoidable and thus the insertion of communication network in the feedback control loop complicates the application of standard results in analysis and design of an NCS (see, e.g., Branicky et al., (2000), Lian et al., (2001) and the references therein).

One issue in NCSs is the unreliable transmission paths because of limited bandwidth and large amount of nodes competing for one network channel, which may result in data packet dropout. The augmented state space method (see, e.g., Zhang, Branicky and Phillips, 2001) is an important method to deal with the problem of data packet dropout. The performance of real-time NCSs with data dropouts is considered by Ling and Lemmon (2002). Azimi-Sadjadi (2003) uses an uncertainty threshold principle to show that under certain conditions there is a rate dropped packets for which an undisturbed NCS is mean square stable. A packet dropping network is modelled as an erasure channel in Hadjicostis and Touri (2002). However, the research mentioned above is concerned primarily with analysis issue rather than control design.

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Time delays typically have negative effects on the systems' stability and performance even if there is no network inserted. So far, various methodologies have been developed to deal with the problem of network delays. Ray and Galevi (1988) have proposed an augmented state vector method to control a linear system over a periodic delay network. Chan and Özgüner (1995) have developed Queueing mechanisms to utilize some deterministic or probabilistic information of NCSs for the control. Nillson (1998) deals with random delays via an optimal stochastic control methodology. With the effects of data packet dropout and transmission delays, Yu et al., (2003) model the continuous-time NCSs as continuous-time linear systems with time-varying input delays and present sufficient conditions on the existence of a stabilizing observer-based dynamic controller.

In distributed NCSs, due to the wide location of sensors whose information length may surpass that of the network packet, multiple-packet transmission is necessary. Zhang, Branicky and Phillips (2001) modelled an NCS in multiple-packet transmission as an asynchronous dynamical system. Their result did not consider the effects of delays and hence their result could only be applied to small scaled NCSs with faster transmission. Lian, Moyne and Tilbury (2001) discussed the modelling and analysis of multi-input and multi-output NCSs with multiple delays. However, they have not consider control design.

Recently, a new approach to dealing with stability and stabilization for linear continuous-time and discrete-time systems under non-linear perturbations has been obtained in terms of linear matrix inequality (LMI) (Siljak and Stipanovic, 2000, Stipanovic and Siljak, 2001), which provides a possibility to reduce the conservativeness in the computations of maximal bounds on non-linear terms. However, when this method is applied to the discrete-time systems, it can only be used for single-input delays and there is structured restrictions on the positive definite matrix. Zuo et al., (2004) has presented a less conservative result with above drawbacks removed and has extended the result to discrete-time systems with constant delays. However, their approach does not work in the case with time-varying input delays.

In this paper, robust stabilization problem is investigated for a class of NCSs with plant uncertainties under the effects of delays and data packet dropout. For simplicity, it is assumed that network communication only occurs between the sensor and the controller through a communication channel with finite bandwidth.

The paper is organized as follows. Section 2 analyzes the effects of delays and data packet dropout on NCSs and models such NCSs as discrete-time non-linear systems with input delays. Section 3 studies the robust stabilization of such NCSs. Section 4 extends the results to NCSs in multiple-packet transmission using modified techniques. Section 5 concludes this paper.

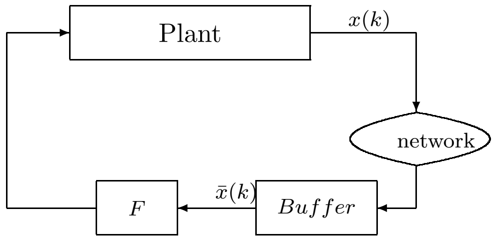

Fig. 1. An NCS controlled via state feedback

Notation: In this paper, $\mathbf{R}^n$ is the set of all n-tuples of real numbers. A^T denotes the transpose of a matrix A. $A > 0 \quad (A < 0)$ means that A is positive definite (negative definite). Z^+ denotes the set of non-negative integers, i.e. $Z^+ = \{0, 1, 2, \cdots\}$.

2. PROBLEM STATEMENT

The state-space model of an NCS shown in Fig. 1 consists of an uncertain discrete-time plant and a discrete-time controller

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + h(k, x(k)) \\ u(k) &= F\bar{x}(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathbf{R}^n, \ u(k) \in \mathbf{R}^m$ are the plant state and input, respectively. F is the state feedback gain matrix to be designed, $A, \ B$ are known real constant matrices with appropriate dimensions. $\bar{x}(k)$ is the output information of the buffer which will be used to construct the controller. $h: \ Z^+ \times \mathbf{R}^n \to \mathbf{R}^n$ is the plant uncertainty. We assume that $h(k, x(k))$ is a nonlinear function in x, satisfying the quadratic constraint condition

$$h^T(k, x(k))h(k, x(k)) \le \alpha^2 x^T(k) H^T H x(k), \tag{2}$$

where $\alpha > 0$ is the bounding parameter on the uncertain function h and H is a constant matrix. Note that constraint (2) is equivalent to

$$\begin{bmatrix} x \\ h \end{bmatrix}^T \begin{bmatrix} -\alpha^2 H^T H & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x \\ h \end{bmatrix} \le 0. \tag{3}$$

For any given H, define the set

$$\begin{aligned} H_\alpha = \{h: \ &\mathbf{R}^{n+1} \to \mathbf{R}^n | h^T(k, x(k))h(k, x(k)) \le \\ &\alpha^2 x^T(k) H^T \times H x(k) \text{ for all } (k, x) \in Z^+ \times \mathbf{R}^n\}. \end{aligned} \tag{4}$$

Because of limited bandwidth and large amount of sensors competing for one communication channel, data packet dropout is unavoidable. When data packet dropout occurs, it might be more advantageous to drop the old packet and transmit a new one than repeated retransmission attempt. The length of the buffer at the controller is equal to one. That is, the controller only uses the new messages transmitted over the network.

We first consider the effect of packet dropout. If $\bar{x}(k)$ has not been updated for $d(k)$ times at step k from the latest updated time, the input of the controller will be given as $\bar{x}(k) = x(k - d(k))$.

Two types of delays are considered: processing delay and communication delay. Usually, the processing delays are constant. Denote the combined sensor/controller processing-communication delay as $t_s(k)$.

Because of network delays, it might happen that more than one sensor messages arrive at the controller at the same steps. Then the most recent message will be used to construct the controller, the rest will be discarded.

Combining the effects of data packet dropout and delays, the input of the controller can be given as

$$\bar{x}(k) = x(k - d(k) - t_s(k)).$$

Define $d(k) + t_s(k) = \tau(k)$. $\tau(k)$ may vary with time step k and it is assumed that

$$\tau(k) \leq \bar{\tau}, \tag{5}$$

where $\bar{\tau}$ is a positive integer.

From the above analysis, the closed-loop system with the effects of packet loss and delays can be given by

$$\begin{aligned} x(k+1) = Ax(k) + BFx(k - \tau(k)) \\ +h(k, x(k)). \end{aligned} \tag{6}$$

Remark 1. $\tau(k)$ is related to data packet dropout and delays. When $d(k) = 0$, it means that $\tau(k) = t_s(k)$ and no packet is dropped or rejected in the transmission. Thus, system (6) can be viewed as a general form of NCS model, where the effects of packet loss and delays are simultaneously considered. Furthermore, system (6) is a discrete-time system with time-varying input delays, so the existing rich theory on delay systems can be applied to studying system (6).

Our major objective is to show how linear constant feedback laws can be formulated to stabilize this type of system and, at the same time, maximize the non-linearity bound which does not violate the stability of the NCSs.

3. ROBUST STABILIZATION

In this section, an LMI approach will be developed to solve the stabilization problem formulated in the previous section.

From the analysis in Section 2, NCS (1) with the effects of packet loss and delays is modelled as time delay system (6) and thus can be written as:

$$\begin{aligned} \Sigma_{c1}: \ x(k+1) &= Ax(k) + BF\sum_{i=1}^{\bar{\tau}} \delta(\tau(k) - i)x(k-i) \\ &\quad +h(k, x(k)), \end{aligned}$$

where

$$\delta(n) = \begin{cases} 1 & \text{if } n = 0, \\ 0 & \text{if } n \neq 0. \end{cases}$$

To establish robust stabilization in the Lyapunov sense, the following Lyapunov function is considered:

$$V(k) = x^T(k)Px(k) + \sum_{i=1}^{\bar{\tau}} \sum_{j=k-i}^{k-1} x(j)^T Qx(j),$$

then the difference of function V along the trajectory of system (6) is given by

$$\begin{aligned} \triangle V(k) &= V(k+1) - V(k) \\ &= x^T(k)[A^T PA + \bar{\tau}Q - P]x(k) \\ &+2x^T(k)A^T PBF \sum_{i=1}^{\bar{\tau}} \delta(\tau(k) - i)x(k-i) + \\ &\sum_{i=1}^{\bar{\tau}}\sum_{j=1}^{\bar{\tau}} \delta(\tau(k) - i)x(k-i)^T F^T B^T PBFx(k-j)\delta(\tau(k) - j) \\ &+2h^T(k, x(k)) \sum_{i=1}^{\bar{\tau}} PBFx(k-i)\delta(\tau(k) - i) \\ &+2x^T(k)A^T Ph(k, x(k)) + h^T(k, x(k))Ph(k, x(k)) \\ &-\sum_{i=1}^{\bar{\tau}} x^T(k-i)Qx(k-i). \end{aligned}$$

Since

$$\sum_{i=1}^{\bar{\tau}} x^T(k-i)Qx(k-i) \geq \sum_{i=1}^{\bar{\tau}} \delta(\tau(k)-i)\delta(\tau(k)-i)x^T(k-i)Qx(k-i),$$

the following inequality holds,

$$\triangle V(k) \leq W(k)^T \Omega W(k),$$

where

$$\Omega = \begin{bmatrix} \Pi & A^T PBF & \cdots & A^T PBF & A^T P \\ * & -\Lambda & \cdots & F^T B^T PBF & F^T B^T P \\ * & * & \ddots & \vdots & \vdots \\ * & * & \cdots & -\Lambda & F^T B^T P \\ * & * & * & * & P \end{bmatrix},$$

$$\Pi = A^T PA + \bar{\tau}Q - P,\ \Lambda = Q - F^T B^T PBF,$$
$$W(k) = \left[x(k)^T |\ x(k-1)^T \delta(\tau(k) - 1)| \cdots |\ x(k-\bar{\tau})^T \delta(\tau(k) - \bar{\tau}) \mid h^T(k)\right]^T.$$

In order to guarantee the stability of NCS (1) with the effects of packet loss and delays, it is required that $\triangle V(k) < 0$, which can be guaranteed by

$$W(k)^T \Omega W(k) < 0. \tag{7}$$

By the well-known S-procedure (Yakubovich, 1977), inequality (7) with constraint (3) can be guaranteed by

$$\left.\begin{array}{c}\begin{bmatrix} \Pi_2 & A^T PBF & \cdots & A^T PBF & A^T P \\ * & -\Lambda & \cdots & F^T B^T PBF & F^T B^T P \\ * & * & \ddots & \vdots & \vdots \\ * & * & \cdots & -\Lambda & F^T B^T P \\ * & * & * & * & P - \mu I \end{bmatrix} < 0, \\ \mu \geq 0, \qquad P > 0, \end{array}\right\} \tag{8}$$

where $\Pi_2 = \Pi + \mu\alpha^2 H^T H$.

It should be noted that inequalities (8) represent non-strict LMI since $\mu \geq 0$. For minimization problem, it is well known (Boyd et al., 1994) that the minimization result under non-strict LMI constraints is equivalent to that under strict constraints. Substitute $\mu > 0$ for $\mu \geq 0$. Then (8) is equivalent to the existence of matrices $\bar{P} =: P/\mu > 0$, $\bar{Q} =: Q/\mu > 0$ such that

$$\begin{bmatrix} \bar{\Pi} & A^T \bar{P}BF & \cdots & A^T \bar{P}BF & A^T \bar{P} \\ * & -\bar{\Lambda} & \cdots & F^T B^T \bar{P}BF & F^T B^T \bar{P} \\ * & * & \ddots & \vdots & \vdots \\ * & * & \cdots & -\bar{\Lambda} & F^T B^T \bar{P} \\ * & * & * & * & \bar{P} - I \end{bmatrix} < 0, \tag{9}$$

where

$$\bar{\Pi} = A^T \bar{P} A + \bar{\tau}\bar{Q} - \bar{P} + \alpha^2 H^T H, \ \bar{\Lambda} = \bar{Q} - F^T B^T \bar{P} BF.$$

Note that (9) can be written as

$$\Phi + \Psi^T W \Psi < 0, \tag{10}$$

where

$$\Phi = diag(\bar{\tau}\bar{Q} - \bar{P},\ -\bar{Q},\ \cdots,\ -\bar{Q},\ -2I),$$

$$\Psi^T = \begin{bmatrix} A^T & H^T & 0 & \cdots & 0 & 0 \\ F^T B^T & 0 & 0 & \cdots & 0 & 0 \\ F^T B^T & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ F^T B^T & 0 & 0 & \cdots & 0 & 0 \\ I & 0 & 0 & \cdots & 0 & I \end{bmatrix}, W = \begin{bmatrix} \bar{P} & 0 & 0 & \cdots & 0 & 0 \\ 0 & \alpha^2 I & 0 & \cdots & 0 & 0 \\ 0 & 0 & \bar{P} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \bar{P} & 0 \\ 0 & 0 & 0 & \cdots & 0 & I \end{bmatrix}.$$

Using Schur complement, (10) is equivalent to

$$\begin{bmatrix} \Phi & \Psi^T \\ \Psi & -W^{-1} \end{bmatrix} < 0. \tag{11}$$

Defining $\gamma := \frac{1}{\alpha^2}$, $X := \bar{P}^{-1}$, $Y := \bar{Q}^{-1}$, $Z := FY^T$, multiplying diag$\{X, Y, \cdots, Y, I, \cdots, I\}$ on both sides of (11), using Schur complements again, we have that

$$\Omega < 0, \tag{12}$$

where

$$\Omega = \begin{bmatrix} -X & 0 & \cdots & 0 & 0 & XA^T & XH^T & 0 & \cdots & 0 & \bar{\tau}X \\ * & -Y & \cdots & 0 & 0 & Z^T B^T & 0 & 0 & \cdots & 0 & 0 \\ * & * & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ * & * & * & -Y & 0 & Z^T B^T & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & * & -2I & I & 0 & 0 & \cdots & I & 0 \\ * & * & * & * & * & -X & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & * & * & * & -\gamma I & 0 & \cdots & 0 & 0 \\ * & * & * & * & * & * & * & \ddots & \vdots & \vdots & \vdots \\ * & * & * & * & * & * & * & * & -X & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & * & -\bar{\tau}Y \end{bmatrix}.$$

To establish robust stabilization in the Lyapunov sense for NCS (1) under constraint (3), the optimization problem is proposed: for given positive integer $\bar{\tau}$

$$\left.\begin{array}{l} \text{minimize } \gamma \\ \text{subject to } X > 0,\ Y > 0,\ Z, \text{ and (12).} \end{array}\right\} \tag{13}$$

Now, for NCS (1) with finite data packet dropout and bounded delays, our main results is presented.

Theorem 1. Consider NCS (1) with data packet dropout and delays. Given a positive integer $\bar{\tau} > 0$, NCS (1) is robustly stabilizable with maximal non-linear bound $\alpha = 1/\sqrt{\gamma}$ if the optimization problem (13) is feasible for data packet dropout and delays satisfy condition (5). Furthermore, the state feedback control law is given by

$$u(k) = ZY^{-1}\bar{x}(k).$$

Remark 2. Theorem 1 provides a sufficient condition on delay-dependent stabilization of discrete-time linear systems with input delays and non-linear uncertainties. For a given γ, the upper bound on the size of the delays can be obtained by solving a quasi-convex optimization problem using the LMI toolbox.

Remark 3. For system (6) without input delays, the stabilization problem has been considered by Stipanovic and Siljak (2001). However their results can not be applied here. Moreover, the drawbacks that their results can only be used for single-input systems and there are some structural restrictions on the Lyapunov matrix and the corresponding variable have been removed. Thus, our result is less conservative and can be applied widely.

Remark 4. For the case when $\tau(k)$ is time-invariant, the controller design problem of system (6) has been investigated in Zuo et al. (2004), whose results can not be applied to the system (6) since $\tau(k)$ here is time-varying.

4. ROBUST STABILIZATION OF NCSS WITH MULTIPLE-PACKET TRANSMISSION

In distributed NCSs, due to the wide location of sensors whose information length may surpass

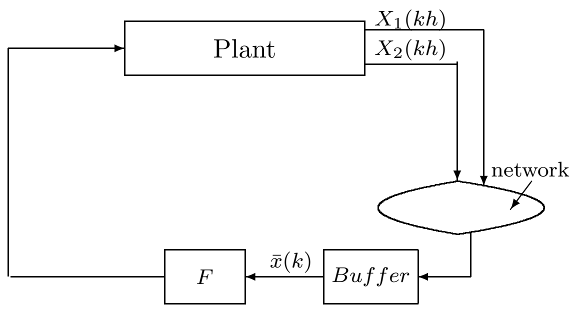

Fig. 2. An NCS with multiple-packet transmission

that of the network packet, multiple-packet transmission is necessary. In multiple-packet transmission, plant output or controller output is split into separate packets. An NCS transmitted in multiple-packet manner with data packet dropout and delays can also be modelled as a non-linear system with multiple-input delays.

Consider the NCS (1) with matrix B being of full column rank. For simplicity, we assume that the plant state is split into two parts $x(k) = \left[X_1^T(k) \;\; X_2^T(k)\right]^T$ with $X_1(k)$ and $X_2(k)$ being transmitted over different network channels, where $X_1(k) = [x_1(k) \;\; \cdots \;\; x_r(k)]^T$ and $X_2(k) = [x_{r+1}(k) \;\; \cdots \;\; x_n(k)]^T$, $r < n$. Fig. 2 illustrates the case where the plant state is transmitted in two packets.

The relation between the plant state, $x(k)$, and the controller input, $\bar{x}(k)$, is studied in the following.

Data packet dropout and delays might happen over each network channel. Then the ith channel of the controller input is given by $\bar{X}_i = X_i(k - \tau_i(k))$ $(i = 1, \cdots, t)$, where $\tau_i(k) \leq \bar{\tau}_i$ reflects the effects of packet loss and delays. Analogous to the single-packet transmission case, we obtain that

$$\bar{x}(k) = \begin{bmatrix} \bar{X}_1(k) \\ \bar{X}_2(k) \end{bmatrix} = \begin{bmatrix} X_1(k - \tau_1(k)) \\ X_2(k - \tau_2(k)) \end{bmatrix} = \sum_{i=1}^{2} C_i x(k - \tau_i(k)),$$

where $C_1 = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$, $C_2 = \begin{bmatrix} 0 & 0 \\ 0 & I_{n-r} \end{bmatrix}$.

When the plant state is transmitted in multiple-packet manner, the closed-loop system with the effects of packet loss and delays can be described as

$$x(k+1) = Ax(k) + BF \sum_{i=1}^{2} C_i x(k - \tau_i(k)) + h(k, x(k)). \tag{14}$$

From the analysis given above, NCS (1) transmitted in two packets with the effects of data packet dropout and delays can be modelled as multiple-delay system (14) and thus can be written as:

$$\Sigma_{c2}: \; x((k+1)) = Ax(k) + BFC_1 \sum_{i=1}^{\bar{\tau}_1} \delta(\tau_1(k) - i) x(k-i) + BFC_2 \sum_{i=1}^{\bar{\tau}_2} \delta(\tau_2(k) - i) x(k-i) + h(k, x(k)),$$

where

$$\delta(n) = \begin{cases} 1 & \text{if } \; n = 0, \\ 0 & \text{if } \; n \neq 0. \end{cases}$$

Define a Lyapunov function $V(k)$ as follows:

$$V(k) = x^T(k) P x(k) + \sum_{\kappa=1}^{2} \sum_{i=1}^{\bar{\tau}_\kappa} \sum_{j=k-i}^{k-1} x(j)^T C_\kappa^T Q_\kappa C_\kappa x(j), \tag{15}$$

where P and Q_κ are positive definite matrices.

Imitating the process of Section 2, in order to guarantee the stability of the closed-loop system, it is required that

$$\begin{bmatrix} \bar{\Pi}_1 & A^T \bar{P} BF & \cdots & A^T \bar{P} BF & A^T \bar{P} BF & \cdots & A^T \bar{P} BF & A^T \bar{P} \\ * & -\bar{\Lambda}_1 & \cdots & F^T B^T \bar{P} BF & F^T B^T \bar{P} BF & \cdots & F^T B^T \bar{P} BF & F^T B^T \bar{P} \\ & & \ddots & \vdots & \vdots & & \vdots & \vdots \\ * & * & & & & \cdots & & \\ * & * & * & -\bar{\Lambda}_1 & F^T B^T \bar{P} BF & \cdots & F^T B^T \bar{P} BF & F^T B^T \bar{P} \\ * & * & * & * & -\bar{\Lambda}_2 & \cdots & F^T B^T \bar{P} BF & F^T B^T \bar{P} \\ * & * & * & * & * & \ddots & \vdots & \vdots \\ * & * & * & * & * & * & -\bar{\Lambda}_2 & F^T B^T \bar{P} \\ * & * & * & * & * & * & * & \bar{P} - I \end{bmatrix} < 0, \tag{16}$$

where

$\bar{\Pi}_1 := A^T \bar{P} A + \bar{\tau}_1 C_1^T \bar{Q} C_1 + \bar{\tau}_2 C_2^T \bar{Q} C_2 - \bar{P} + \alpha^2 H^T H$,
$\bar{\Lambda}_i := \bar{Q}_i - F^T B^T \bar{P} BF$ $(i = 1, 2)$, $\bar{P} := P/\mu$,
$\bar{Q}_i := Q_i/\mu$ $(i = 1, 2)$.

Note that (16) can be written as

$$\Phi + \Psi^T \bar{P} \Psi < 0, \tag{17}$$

where

$$\Phi = \mathrm{diag}(\bar{\tau}_1 C_1^T \bar{Q}_1 C_1 + \bar{\tau}_2 C_2^T \bar{Q}_2 C_2 - \bar{P} + \alpha^2 H^T H, -\bar{Q}_1, \cdots, -\bar{Q}_1, -\bar{Q}_2, \cdots, -\bar{Q}_2, -I),$$

$$\Psi = [A \;\; BF \;\; \cdots \;\; BF \;\; I].$$

It is assumed that

$$\bar{P} B = BM, \tag{18}$$

where M is a matrix with appropriate dimensions. Because B is of full column rank, it follows from (18) that M is also of full rank, and thus invertible.

Define $L = MF$, use Schur complement, it follows from (18) that (17) is equivalent to

$$
\begin{bmatrix}
\Delta & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & A^T\bar{P} & H^T \\
* & -\bar{Q}_1 & \cdots & 0 & 0 & \cdots & 0 & 0 & L^TB^T & 0 \\
* & * & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & -\bar{Q}_1 & 0 & \cdots & 0 & 0 & L^TB^T & 0 \\
* & * & * & * & -\bar{Q}_2 & \cdots & 0 & 0 & L^TB^T & 0 \\
* & * & * & * & * & \ddots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & -\bar{Q}_2 & 0 & L^TB^T & 0 \\
* & * & * & * & * & * & * & -I & \bar{P} & 0 \\
* & * & * & * & * & * & * & * & -\bar{P} & 0 \\
* & * & * & * & * & * & * & * & * & -\gamma I
\end{bmatrix} < 0, \tag{19}
$$

where $\Delta = \bar{\tau}_1 C_1^T \bar{Q}_1 C_1 + \bar{\tau}_2 C_2^T \bar{Q}_2 C_2 - \bar{P}$ and γ is defined as before.

For NCS (1) transmitted in multiple-packet manner, we propose the optimization problem: for given positive integers $\bar{\tau}_i$ (i=1,2)

$$
\left.\begin{array}{l} \text{minimize } \gamma \\ \text{subject to } \bar{P} > 0,\ \bar{Q} > 0,\ L,\ M \text{ (18) and (19).} \end{array}\right\} \tag{20}
$$

The following theorem establishes a sufficient condition on the robust stabilization of the NCS (1) with data packet dropout in multiple-packet transmission.

Theorem 2. Consider NCS (1) transmitted in multiple-packet manner with data packet dropout and delays. Given positive integers $\bar{\tau}_i > 0$ ($i = 1, 2$), NCS (1) is robustly stabilizable with maximal non-linear bound $\alpha = 1/\sqrt{\gamma}$ if the optimization problem (20) is feasible. Furthermore, the state feedback control law is given by

$$
u(k) = M^{-1} L \bar{x}(k).
$$

5. CONCLUSION

This paper has investigated the robust stabilization problem for a class of NCSs with non-linear uncertainties under the effects of delays and data packet dropout. Robust stabilization for the NCSs has been presented. The state feedback controllers can be constructed in terms of LMIs. The admissible upper bound of data packet dropout and transmission delays can be obtained via solving a quasi-convex optimization problem with the efficient LMI toolbox. For NCSs in multiple-packet transmission, similar stabilization results have been established, where single delay has been extended to multiple delays, and state feedback problem has been extended to output feedback problem. The obtained results in this paper can be applied to NCSs without packet loss to save network bandwidth or to find the maximum allowable delay between state update. This is of practical importance in engineering applications.

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