PASSIVITY AND PASSIVITY BASED CONTROLLER DESIGN OF A CLASS OF SWITCHED CONTROL SYSTEMS

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Abstract: Notions of passivity for a class of switched control systems (SCS) are developed first in this article. We then study the following problems: 1) When is an SCS passive? 2) Does passivity imply Lyapunov stability as in the classical passive systems? 3) How to use passivity as a tool to design controllers to stabilize an SCS? For Problem 1), we derive sufficient conditions for passivity. For problem 2), we show that passivity does imply Lyapunov stability when $u = 0$, and the stability result obtained from positive semidefinite storage functions is new in switching systems. For the last problem, we first show that by designing the control law properly other than $u = 0$, stronger stability results can be obtained for a passive SCS. Then, based on this, we solve the stabilization problem of an SCS by making it passive first and then designing the control law for the passified SCS. Examples and simulations are given to support our results.

Keywords: Passivity, switched control system, Lyapunov stability

1. INTRODUCTION

Passivity (Willems, 1972; Sepulchre *et al.*, 1997), besides its physical and intuitive appeal, is a powerful tool for stabilization. The relation between passivity and stability is well established for non-switched systems. Chapter 2 of (Sepulchre *et al.*, 1997) provides a good overview. More recently, passivity is used as a tool for nonlinear feedback design (Kokotovic and Sussman, 1989; Byrnes *et al.*, 1991; Lin, 1996).
For switched control systems, at present, there are very few results available that deal with the subject in hybrid systems (Haddad *et al.*, 2001), (Pogromsky *et al.*, 1998), and (Zefran *et al.*, 2001). Up to now, there remains lack of a systematic study on the following problems:

(1) How to define passivity suitably for an SCS and when is an SCS passive?
(2) Does passivity imply Lyapunov stability as in the classical passive systems?
(3) How to use passivity as a tool to design controllers to stabilize an SCS?

In this paper, we will present a systematic study of the above problems for a class of switched control systems. The rest of paper is arranged as follows: in Section 2, we introduce the model of SCSs. In Section 3, we give notions of passivity for an SCS in the beginning, which are natural extension of notions for classical systems. We then derive some sufficient conditions to check the passivity of an SCS. In Section 4, it is first shown that under some conditions, passivity does imply Lyapunov stability, which extends the results from classical systems to switched control systems. In Section 5, we first show that by taking advantage of

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controller design, stronger stability results can be obtained for passive systems, then we show how to use passivity as a tool to stabilize an SCS. In Section 6, some examples and simulation results are given, and finally certain concluding remarks are made in the last section.

2. SWITCHED CONTROL SYSTEMS

We consider a class of switched control systems (SCS) with M subsystems described as

$$\begin{aligned} \dot{x} &= f_{\sigma(t)}(x) + g_{\sigma(t)}(x)u \qquad x \in R^n \\ y &= h_{\sigma(t)}(x) \qquad\qquad u, y \in R^m \end{aligned} \tag{1}$$

where x, u, and y are the system state, input, and output respectively. $f_i(x)$, $g_i(x)$ and $h_i(x), i \in S = \{1, 2, \cdots, M\}$ are functions from R^n to R^n, from R^n to R^n and R^n to R^m with $f_i(0) = 0$ and $h_i(0) = 0$. Finally $\sigma(t) : [0, \infty) \to S$ is a specific *switching rule*, which is a piecewise constant function of time and/or state. The corresponding system for $\sigma(t) = i \in S$ is called the $i-$th subsystem. In such a case, we also say that the $i-$th subsystem is "active".

Assumptions

A1– For all i, $f_i(x)$ and $g_i(x)$ is locally Lipschitz continuous in x and $u(t)$ is a measurable function of t.

A2– The state does not jump at the switching instants.

When each subsystem is linear, we obtain a *switched linear control system(SLCS)*

$$\begin{aligned} \dot{x} &= A_{\sigma(t)}x + B_{\sigma(t)}u \qquad\qquad x \in R^n \\ y &= C_{\sigma(t)}x \qquad\qquad\qquad\qquad u, y \in R^m \end{aligned} \tag{2}$$

3. PASSIVITY OF SCSS

In this section, we will first introduce two passivity concepts for SCSs which are natural extensions of notions for classical systems. We then derive sufficient conditions to check the passivity of an SCS.

3.1 Notions of passivity for SCSs

We assume that for every $u \in U$, where U is the set of admissible inputs and for any $x^0 \in R^n$,the output $y(\phi(t, x^0, u))$ satisfies $\int_{t_0}^{t_1} \|u^T y\| dt < \infty$ for all $t \geq 0$. Let X be a connected subset of R^n with $0 \in X$.

Definition 1. (Passivity) For system (1) with a specific switching rule $\sigma(t)$, it is said to be passive in $(0, \infty) \times R^n$ if there exists a nonnegative function $S(\sigma(t), x)$ with $S(\sigma(t), 0) = 0$ for any t, such that for all $u \in U$, $x^0 \in X$ and for all $t_1 \leq t_2 \in (0, \infty)$

$$S(\sigma(t_2), x(t_2)) - S(\sigma(t_1), x(t_1)) \leq \int_{t_1}^{t_2} u^T y dt \tag{3}$$

where $x(t) = \phi(t, x^0, u) \in R^n$ for all t is the solution and $S(\sigma(t), x)$ is called storage function.

Definition 2. (Strong Passivity) For system (1), it is said to be strongly passive in $(0, \infty) \times R^n$ if it is passive in $(0, \infty) \times R^n$ for arbitrary switching rule $\sigma(t)$.

3.2 Conditions for passivity

We now give sufficient conditions for an SCS to be passive.

Theorem 1. For an SCS (1) which satisfies Assumptions A1-A2, if all the subsystems are passive in $(0, \infty) \times R^n$ and have a common storage function defined in X, then it is strongly passive in $(0, \infty) \times R^n$.

By using the above theorem and the result in (Hill and Moylan, 1976), we can derive the following result.

Corollary 3. For an SCS given by (1) satisfying Assumptions A1-A2, if there exists a positive semidefinite function $S(x)$ defined on R^n such that for all $i = 1, 2, \cdots, M$

$$\begin{aligned} L_{f_i} S(x) &\leq 0 \\ L_{g_i} S(x) &= h_i^T(x) \end{aligned} \tag{4}$$

Then the SCS is strongly and globally passive with a storage function $S(x)$.

This corollary gives a way to construct the common storage function. It requires to solve a group of partial equations and inequalities, which is generally very difficult. However, for SLCSs, the construction a common storage function is reduced to find a matrix solution of matrix inequalities and equalities.

Corollary 4. For an SLCS given by (2) satisfying Assumptions A1-A2, if there exists a positive definite matrix P such that

$$\begin{aligned} PA_i + A_i^T P &\leq 0 \\ B_i^T P &= C_i, i = 1, 2, \cdots, M \end{aligned} \tag{5}$$

Then the SLCS is strongly passive with a storage function $S(x) = x^T P x$.

Remark 1. The passivity of feedback interconnection of two passive SCSs can be shown using the techniques in (Sepulchre *et al.*, 1997). We do not present the results here because it is obvious.

4. THE RELATION BETWEEN PASSIVITY AND LYAPUNOV STABILITY

For classical systems, the relation between passivity and Lyapunov stability is well established, see (Sepulchre *et al.*, 1997; Byrnes *et al.*, 1991). However, for an SCS and the passivity definition we have, we do not known whether similar relation can be proved. We show an passive SCS's with a positive definite storage function is Lyapunov stable when the input is $u = 0$ in Subsection 4.1. Then, in Subsection 4.2, a passive SCS with positive semidefinite storage functions is also proved to be Lyapunov stable under some conditions when the input is $u = 0$. The following definitions are needed.

Definition 5. $S(\sigma(t), x)$ with $S(\sigma(t), 0) = 0$ is said to be positive definite if $S(\sigma(t), x) > 0$ for any $t \geq 0$ and any $x \neq 0$.

Definition 6. $S(\sigma(t), x)$ with $S(\sigma(t), 0) = 0$ is said to be radially unbounded if the boundness of $S(\sigma(t), x)$ implies the boundness of x.

Definition 7. $S(\sigma(t), x)$ with $S(\sigma(t), 0) = 0$ is said to be positive semidefinite if $S(\sigma(t), x) \geq 0$ for any $t \geq 0$ and any $x \neq 0$.

4.1 A Passive SCS With Positive Definite Storage Function

For a passive SCS with positive definite storage function, we state the following result.

Theorem 2. For an SCS given by (1), assume that its storage function $S(\sigma(t), x)$ is positive definite and with respect to x, then the solution $x = 0$ of the SCS is Lyapunov stable when $u = 0$. If in addition $S(\sigma(t), x)$ is also radially unbounded, then it is globally stable when $u = 0$.

Proof: From the definition of passivity, it is easy to see that along the system trajectory $x(t)$, we have $S(\sigma(t), x(t))$ is decreasing when $u = 0$. With this in hand, the Lyapunov stability can be proved by using the techniques in (Sepulchre *et al.*, 1997).
When $u = 0$, it follows from the passivity that along each the system trajectory $x(t)$, $S(\sigma(t), x(t))$ is bounded. Since $S(\sigma(t), x(t))$ is radially unbounded, the solution $x(t)$ is bounded. This proves the global stability of the solution $x = 0$ when $u = 0$. ¶

The following is an immediate consequence of the above theorem.

Corollary 8. For an SLCS given by (2) satisfying assumptions A1-A2, if it is passive with a storage function $S(x) = x^T P x$, where P is a positive definite matrix, then, the solution $x = 0$ of the SLCS is globally stable.

4.2 A Passive SCS With Positive Semidefinite Storage Function

In this subsection, we will establish the relation between passivity of an SCS with a positive semidefinite storage function and Lyapunov stability. The motivation for this is that passivity of a classical system with positive semidefinite storage function is proved to imply Lyapunov stability under certain conditions. We want to see whether this is still true for switching systems.
To carry out our study, the notion of *conditional stability* is needed, which can be found in (Sepulchre *et al.*, 1997). Also, we need the following definition.

Definition 9. (Zero-state detectability and observability). Consider an SCS given by (1) with $u = 0$, that is $\dot{x} = f_{\sigma(t)}(x)$, $y = h_{\sigma(t)}(x)$, let $Z \subset R^n$ be the largest positively invariant set contained in $\bigcap_{i=1}^{M} \{x \in R^n | y = h_i(x) = 0\}$. We say that the SCS is zero-state detectable (ZSD) if $x = 0$ is asymptotically stable conditionally to Z. If Z=$\{0\}$, we say that the system is zero-state observable (ZSO).

Because an SCS given by (1) is only defined on $[0, \infty)$, to prove our result we need to define the following system starting at $t = 0$ with initial condition x_0 evolving backward in time.

$$\begin{aligned} &\dot{x} = f_{\sigma(t)}(x) + g_{\sigma(t)}(x)u \qquad x \in R^n \\ &y = h_{\sigma(t)}(x) \qquad u, y \in R^m,\ t \leq 0 \end{aligned} \tag{6}$$

where the switching rule $\sigma(t)$ for $t \leq 0$ is defined the same way as $\sigma(t)$ for $t \geq 0$. For example, if $\sigma(t)$ only depends on t, then we define $\sigma(t) = \sigma(-t)$ for all $t < 0$. If $\sigma(t) = \sigma(x(t))$ for $t \geq 0$, then we define $\sigma(t) = \sigma(x(-t_1))$ if $x(t) = x(-t_1)$, where $t, t_1 < 0$.
By using the techniques in the proof of Theorem 2.24 in (Sepulchre *et al.*, 1997), we can prove the following theorem.

Theorem 3. Consider an SCS given by (1) satisfying A1-A2 with $u = 0$ and let $S(\sigma(t), x)$ be a positive semidefinite and continuous with respect to x with $S(\sigma(t), 0) = 0$. Assume that along any trajectory $x(t)$ we have $S(\sigma(t), x)$ is decreasing. Let

Z be the largest positively invariant set contained in $\{x|S(\sigma(t), x) = 0\}$. If $x = 0$ is asymptotically stable conditionally to Z, then the solution $x = 0$ of the SCS with $u = 0$ is stable.

Remark 2. The above result is inspired by Theorem 2.24 in (Sepulchre *et al.*, 1997). We extended the result from classical nonlinear systems to a class of SCSs under weaker conditions. In Theorem 2.24 of (Sepulchre *et al.*, 1997), the function $S(x)$ is required to be C^1, $\dot{S}(x) \leq 0$. In our theorem, $S(\sigma(t), x)$ is only required to be continuous with respect to x, and decreasing along all trajectories. Due to these weaker requirements, our result in the above can be used to analyze the stability of a class of passive SCSs with positive semidefinite storage functions.

By applying Theorem 3, the following result is obtained.

Theorem 4. Consider an SCS given by (1) satisfying A1-A2. Assume that it is passive and the storage function $S(\sigma(t), x)$ is continuous with respect to x and only positive semidefinite. If the SCS is ZSD and $\{x|S(\sigma(t), x) = 0\} \subset \bigcap_{i=1}^{M}\{x \in R^n|y = h_i(x) = 0\}$, then the solution $x = 0$ of the SCS with $u = 0$ is stable.

Proof: Since the SCS is passive, then $S(\sigma(t), x(t))$ is decreasing along all trajectories when $u = 0$. Let Z be the largest positively invariant set contained in $\{x|S(\sigma(t), x) = 0\}$. Because the SCS is ZSD and $\{x|S(\sigma(t), x) = 0\} \subset \bigcap_{i=1}^{M}\{x \in R^n|y = h_i(x) = 0\}$, then the solution $x = 0$ of the SCS with $u = 0$ is asymptotically stable conditionally to Z. Now, because all the conditions in Theorem 3 are satisfied, it follows from Theorem 3 that this theorem is true. ¶

Remark 3. The above stability result is new in switching systems. Although Theorem 2.6 in (Leonessa *et al.*, 2000) could be applied to analyze the stability of switched systems, it requires a semipositive definite Lyapunov $S(\sigma(t), x) \to \infty$ as $\|x\| \to \infty$. We do not need this requirement.

5. PASSIVITY BASED STABILIZATION CONTROLLER DESIGN

In this section, we will show that stronger stabilization results can be achieved by taking advantage of passivity based controller design.
For an SCS with a positive definite storage function, we have the following result.

Theorem 5. For a passive SCS given by (1) satisfying A1-A2, assume that it is ZSD and its storage function $S(\sigma(t), x) = S(x)$ is positive definite and continuous with $S(0) = 0$. Let $\phi(y)$ be a continuous vector function such that $\phi(0) = 0$ and $y^T\phi(y) > 0$ for each nonzero y. If the control law is chosen as

$$u = -\phi(y) \tag{7}$$

Then the solution $x = 0$ of the closed loop SCS is asymptotically stable. If, in addition, $S(x)$ is also radially unbounded, then it is globally asymptotically stable.

Proof: To prove the solution $x = 0$ of the closed loop SCS is asymptotically stable, we need to show it is both stable and attractive. The stability has been proved already in last section, what we need to do is to prove the solution $x = 0$ is attractive.
Because the solution $x = 0$ is stable, for a small $\epsilon_0 > 0$, there exists a positive constant $\delta_0 > 0$ such that all solutions $x(t, x_0)$ are bounded for x_0 satisfying $\|x_0\| < \delta_0$.
For $\|x_0\| < \delta_0$, let $x(t, x_0)$ be the corresponding solution. If we let s_{limit} denote its ω-limit set. Now, we prove that s_{limit} for $x(t, x_0)$ with $\|x_0\| < \delta_0$ is nonempty, compact and invariant. If $\|x_0\| < \delta_0$, then $x(t, x_0)$ is bounded. It follows that s_{limit} is nonempty and bounded. From the definition of s_{limit}, we can easily show that it is also closed, which together with its boundness proves that it is compact. Under the assumptions A1-A2, we can conclude that there exists a unique solution for each initial condition and the solution has the continuity property with respect to initial conditions. Let $\bar{x}$ be a point in s_{limit} and $\bar{x}(t, \bar{x})$ the corresponding solution. By definition, there exists an increasing unbounded sequence $\{t_n\}_0^\infty$ such that $lim_{n\to\infty}x(t_n, x_0) = \bar{x}$. By the continuity property, we get $lim_{n\to\infty}x(t, x(t_n, x_0)) = x(t, \bar{x})$ for all t. By the uniqueness of the solution, we get $\bar{x}(t, \bar{x}) = x(t, \bar{x})$. Again by the uniqueness of the solution, we have $x(t, x(t_n, x_0)) = x(t + t_n, x_0)$ for all t. This implies $x(t, \bar{x})$ belongs to s_{limit}, which implies $\bar{x}(t, \bar{x})$ belongs to s_{limit} for all t. This proves that s_{limit} is invariant.
Because for any solution $x(t, x_0)$ with $\|x_0\| < \delta_0$, $S(x(t))$ is decreasing and nonnegative, we have $lim_{t\to\infty}S(x(t)) = a(x_0) \geq 0$ with $a(x_0)$ a constant.
For any $\bar{x} \in s_{limit}$, we have already proved $\bar{x}(t, \bar{x}) \in s_{limit}$. Therefore, $S(\bar{x}(t, \bar{x})) = a(x_0)$ for all t. Using this and passivity, we get

$$0 = S(\bar{x}(t, \bar{x})) - S(\bar{x}) \leq -\int_0^t y^T(s)\phi(y(s))ds \leq 0 \tag{8}$$

Since $\bar{x}(t, \bar{x})$ is continuous, $y(t)$ is piecewise continuous. It follows from (8) that $y(t) = 0$ for all

$t > 0$. This together with ZSD condition proves that $lim_{t\to\infty}\bar{x}(t,\bar{x}) = 0$ and therefore $a(x_0) = 0$. This implies that $lim_{t\to\infty}S(x(t)) = a(x_0) = 0$, and thus $lim_{t\to\infty}x(t) = 0$.
By far, we have proved $lim_{t\to\infty}x(t, x_0) = 0$ for any $\|x_0\| < \delta_0$, that is, we have proved the solution $x = 0$ is attractive. Therefore the solution is asymptotically stable.
Because $S(x)$ is radially unbounded, the boundness of $S(x(t))$ implies the boundness of all solutions $x(t)$. This proves the solution $x = 0$ is globally asymptotically stable. ¶

Remark 4. Compared with the stability results obtained last section, here an asymptotic stability result is derived. This shows that, for passive systems, we can benefit from the controller design.

The condition $S(\sigma(t), x) = S(x)$ is somewhat restrictive. Although asymptotic stability is not proved for general passive systems, we still have the following result.

Theorem 6. For a passive SCS given by (1) satisfying A1-A2, its storage function $S(\sigma(t), x)$ with $S(\sigma(t), x) = 0$ is positive definite and continuous with respect to x. Let $\phi(y)$ be a continuous vector function such that $\phi(0) = 0$ and $y^T\phi(y) > 0$ for each nonzero y. If the control law is chosen as $u = -\phi(y)$, then the solution $x = 0$ of the closed loop SCS is stable, and we have $lim_{t\to\infty}y(t) = 0$, that is, output regulation is achieved.

Proof: The proof of the stability is already given in Theorem 5. Noticing that along the trajectory $x(t)$ with initial state $x(0)$, according to passivity, we know that $S(t) = S(\sigma(t), x(t))$ is decreasing and bounded and

$$\begin{aligned} S(t) - S(0) &\le -\int_0^t y^T(s)\phi(y(s))ds \\ &\le 0 \end{aligned} \tag{9}$$

Thus, we have

$$0 \le \int_0^\infty y^T(s)\phi(y(s))ds < \infty \tag{10}$$

Note that $y(t)$ is piecewise continuous and $y^T\phi(y) > 0$, it follows from (10) that $lim_{t\to\infty}y(t) = 0$. The theorem is proved. ¶
¿From Theorem 5, we get immediately the following result.

Corollary 10. For an SLCS given by (2) satisfying assumptions A1-A2, assume that it is ZSD. If it is passive with a storage function $S(x) = x^TPx$, where P is a positive definite matrix, and the control law is chosen as $u = -ky$, then, the solution $x = 0$ of the SLCS is globally asymptotically stable.

Theorem 7. Consider an SCS given by (1) satisfying A1-A2. Assume that it is passive with a storage function $S(\sigma(t), x) = S(x)$ which is continuous and only positive semidefinite. Let $\phi(y)$ be any continuous function such that $\phi(0) = 0$ and $y^T\phi(y) > 0$ for each $y \ne 0$. If the SCS is ZSD and $\{x|S(x) = 0\} \subset \bigcap_{i=1}^M \{x \in R^n | h_i(x) = 0\}$, then the solution $x = 0$ of the SCS with $u = -\phi(y)$ is asymptotically stable.

6. EXAMPLES AND SIMULATIONS

In this section, the effects of passivity based controllers are validated through simulations.
Example 1: Let's consider an SCS given below.

$$\begin{aligned} \dot{x} &= f_{\sigma(t)}(x) + g_{\sigma(t)}u \\ y &= x_2 \end{aligned} \tag{11}$$

where $\sigma(t) = 1$ $t \in$ $[2kT, (2k+1)T)$; $\sigma(t) = 2$ $t \in$ $[(2k+1)T, (2k+2)T)$ and $K = 0, 1, 2, ...$ and $T = 0.01$.

$$\begin{aligned} f_1(x) &= \begin{pmatrix} x_1^2x_2 \\ 0 \end{pmatrix}, \quad g_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ f_2(x) &= \begin{pmatrix} -x_1^3 + x_2 \\ x_1 \end{pmatrix}, \quad g_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{aligned} \tag{12}$$

If we use the following control law

$$\begin{aligned} u &= -x_1^3 - \phi(y), \ when\ \sigma(t) = 1 \\ u &= -2x_1 - \phi(y), \ when\ \sigma(t) = 2 \end{aligned} \tag{13}$$

where $\phi(y) = 2y$.
Example 2: Let's consider the following SCS.

$$\begin{aligned} \dot{x} &= f_{\sigma(t)}(x) + g_{\sigma(t)}u \\ y &= x_2 \end{aligned} \tag{14}$$

where $\sigma(t)$ is defined in the same way as in Example 1. , and

$$\begin{aligned} f_1(x) &= \begin{pmatrix} -x_1 + x_2^2 \\ 0 \end{pmatrix}, \quad g_1(x) = \begin{pmatrix} 0 \\ x_1^2 + 1 \end{pmatrix} \\ f_2(x) &= \begin{pmatrix} -3x_1 + 2x_2^2 \\ x_1 \end{pmatrix}, \quad g_2(x) = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \end{aligned} \tag{15}$$

If we use the following control law

$$\begin{aligned} u &= -\frac{\phi(y)}{x_1^2 + 1}, \ when\ \sigma(t) = 1 \\ u &= -\frac{\phi(y)}{2} - \frac{x_1}{2}, \ when\ \sigma(t) = 2 \end{aligned} \tag{16}$$

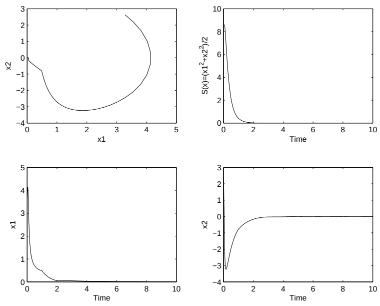

Fig. 1. Simulation results of Example 1

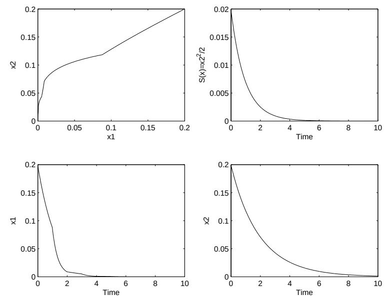

Fig. 2. Simulation results of Example 2

where $\phi(y) = y + y^3$.

The simulation results for Example 1 are presented in Figure 1. While those for Example 2 are given in Figure 2. In each figure, we see that the states of the SCS's tend to zero asymptotically and the storage functions also converge to zero as expected.

7. CONCLUSION

In this article passivity and passivity based controller design was systematically studied for a class of switched control systems. First, necessary conditions and sufficient conditions for an SCS to be passive were established. Second, how to make an SCS passive through choosing the switching rule and/or state feedback was studied, and we gave passification design for general SCSs and some special classes of SCSs. We found that the design would be much easier by allowing the storage function to be positive semidefinite. Third, we proved the relationship between passivity and stability exists in classical systems can be extended to SCSs. As in classical passive systems, under certain conditions, stability and global stability can be achieved even without control for a passive SCS. The stability result obtained for positive semidefinite storage functions is new in switching systems. Fourth, we obtained deeper stability results by designing the control law for passive systems. For SCSs whose subsystems admits common storage functions, asymptotic stability was achieved; while for general SCSs which may have time-varying storage functions, though asymptotic stability was not proved by the authors, output regulation was still obtained, which is often enough in applications. The theoretical results were validated using examples and simulation results.

Since SCSs are only a special class of hybrid systems, how to extend the results reported here to other classes of hybrid systems remains to be investigated.

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