ONE-POINT FEEDBACK ROBUST CONTROL FOR DISTRIBUTED PARAMETER SYSTEMS

M. Garcia-Sanz, A. Huarte, A. Asenjo

Automatic Control and Computer Science Department, Public University of Navarra
31006 Pamplona, SPAIN. Email: mgsanz@unavarra.es

Abstract: This paper introduces a new technique to design applicable one-point feedback controllers for distributed parameter systems (DPS) with uncertainty. The technique is an extension of the Quantitative Feedback Theory (QFT) to DPS, considering spatial distribution as another parameter of uncertainty. Working on the classical frequency domain, the technique avoids complex double Laplace transforms, partial differential equations, etc., but still represents spatial distributed configurations. The paper extends the classical QFT performance specifications used in lumped systems by introducing a set of inequalities for DPS. An example compares former approaches to the proposed one.

Keywords: Distributed parameter-systems, Robust Control.

1. INTRODUCTION

Traditionally Automatic Control Theory has dealt with ordinary, lumped differential or difference equations to model physical systems to be controlled. In the last century we witnessed the consolidation, successful implementation and validation of diverse control methodologies for lumped model systems. In fact nowadays most industrial control applications have been or were designed using the lumped approach.

However we cannot forget that many physical systems have an intrinsically distributed nature. Large flexible space structures, chemical processes, drainage and sewage water networks, etc. are all examples of systems involving complex DPS control problems. The most significant characteristic of such DPS is the fact that the main variables depend on both spatial and time variations. As a consequence these systems usually have to be modelled by PDEs.

In the last few decades there has been considerable progress in understanding the modelling and stabilisation of distributed parameter systems. One may consult the editorial of the IEEE Trans. Autom. Control, Levis *et al.* (1987), for a summary. Different approaches to the DPS control problem appear in Lions (1988), Chen *et al.*(1987) and in Curtain and Zwart (1995). This last book together with the one by Luo *et al.* (1999) are excellent tutorials of identification and Hinf control of DPS.

Meanwhile, related to the robust control theory QFT (Quantitative Control Theory), Horowitz *et al.* (1989) studied both the frequency domain properties of discrete control loops for uncertain plants and the one-point feedback approach to linear distributed systems also studied under the name of boundary control since early 1990's; Chait *et al.* (1989) showed a Nyquist graphical stability criterion for DPS.

Unfortunately, the reported practical applications of DPS control theories are but a few. In some way this is due to the intrinsic complexity introduced by the PDE methodology. In this context, a new technique is proposed to design applicable one-point feedback controllers for DPS with uncertainty. Working on the classical frequency domain, the technique avoids double Laplace transforms, PDE, etc., but still represents spatial distributed configurations with uncertainty. The new technique is an extension of

QFT to DPS, introducing spatial distribution as another parameter of uncertainty.

The paper begins with the problem formulation in Kelemen *et al.* (1989), stating that the lumped problem is a particular case of the distributed one. Section three introduces an equivalent general homogeneous Pi model for DPS. Section four presents a robust QFT control technique for such systems. Section five applies the new methodology to control an example and compares its results with the ones achieved with former approaches (Kelemen *et al*., 1989). Finally the last section summarises the most relevant ideas of the paper.

2. PROBLEM FORMULATION

Consider a MISO DPS described by linear PDE with constant coefficients and the time variable t > 0, where P_{x2x1} is the transfer function between the input x_1 and the output x_2. The DPS and the feedback loop, shown in Fig. 1, present a general distribution where the sensor, the actuator, the disturbances and the control objective are located at different points x_s, x_a, x_d and x_o respectively.

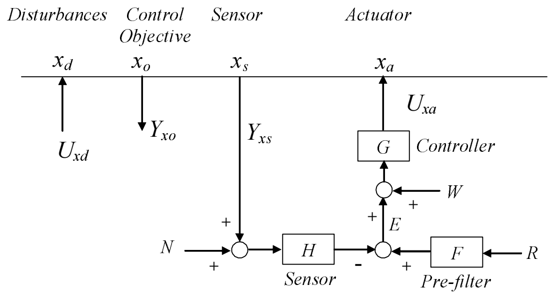


Fig. 1. DPS and Control System

The equations that explain the dynamics of the DPS and the control system are,

$$Y_{xo} = P_{xo\,xd}\;U_{xd} + P_{xo\,xa}\;U_{xa} \qquad (1)$$

$$Y_{xs} = P_{xs\,xd}\;U_{xd} + P_{xs\,xa}\;U_{xa} \qquad (2)$$

$$U_{xa} = G\,\left[F\,R + W - H\left(N + Y_{xs}\right)\right] \qquad (3)$$

where capital letters denote Laplace Transforms (we omit the s variable for simplicity) and *R*, *W*, *N*, U_{xd}, U_{xa} represent the inputs: reference signal, reference disturbances, sensor noise, external disturbances and actuator signals respectively.

Substituting Eq. (2) in (3), and the result in Eq. (1), the variation of the output Y_{xo} due to the inputs is,

$$Y_{xo} = \frac{G\;P_{xo\,xa}}{1+G\;H\,P_{xs\,xa}}\left(F\,R + W\right) + \left[P_{xo\,xd} - \frac{G\,HP_{xo\,xa}P_{xs\,xd}}{1+G\;H\,P_{xs\,xa}}\right]U_{xd} - \frac{G\;H\,P_{xo xa}}{1+G\;H\,P_{xs\,xa}}\;N \qquad (4)$$

where the transfer functions depend on the controllers *G* and *F*, the sensor dynamics *H*, the spatial distribution and the distances between the sensor, the actuator, the disturbances and the control objective through the four transfer functions $P_{xo\ xd}$, $P_{xs\ xa}$, $P_{xs\ xd}$ and $P_{xo\ xa}$.

Remark 1: This model, which represents the distributed parameters system, totally agrees with the lumped model defined in classical automatic control. The case $x_d = x_a$, $x_o = x_s$ ($P_{xo\ xd} = P_{xs\ xa}$; $P_{xs\ xd} = P_{xs\ xa}$; $P_{xo\ xa} = P_{xs\ xa}$) matches the classical lumped system with disturbances at plant input. And the case where $x_d = x_o = x_s \neq x_a$ ($P_{xo\ xd} = 1$; $P_{xs\ xd} = 1$; $P_{xo\ xa} = P_{xs\ xa}$) matches the classical lumped system with disturbances at plant output.

3. GENERAL HOMOGENEOUS DPS MODEL

Generally speaking, the set of differential equations describing the dynamic performance of a distributed physical system can be formulated in terms of *Through Variables* (*I*) and *Across Variables* (*U*) –see for example Takahashi *et al*. (1970), Dorf and Bishop (2000). The *Through Variable I* can represent an electrical current, a mechanical force or torque, a heat flow rate, etc. The *Across Variable V* can represent an electrical voltage difference, a mechanical linear or angular velocity difference, a temperature difference, etc.

When a linear, lumped approximation is used for a distributed element, it can be of algebraic type [$U = I\ R$], or integrated *Through Variable* type [$U = \int C\ i(t)\,dt$] or integrated *Across Variable* type [$U = L\ d\,i(t)/dt$].

In this context, consider the general homogeneous case with different locations of both the outputs and the inputs points ($x_o<x_s$ and $x_d<x_a$). U_{xd} and U_{xa} represent the voltage sources and k_1, k_2 and k_3 represent the distance between the left end and the objective, the objective and the sensor and the sensor and the right end. (See Fig. 2). The “impedance per meter” [*Across Variable* / (*Through Variable* * *Distance*)] is:

$$Z_i = \frac{B_i}{A_i} = \frac{b_{n_i}s^n + \cdots + b_{1_i}s + b_{0_i}}{s^{\alpha_i}\,(\,a_{m_i}s^m + \cdots + a_{1_i}s + a_{0_i}\,)}\ ;i=1,2 \qquad (5)$$

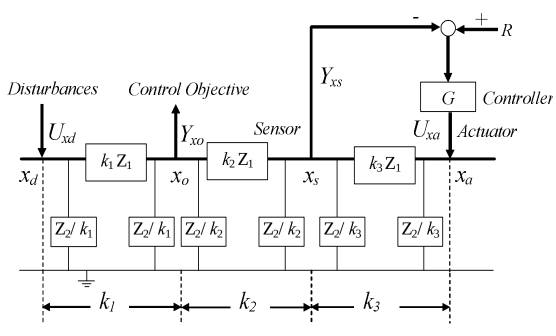


Fig. 2. DPS and Control System. Pi General Case.

The solution of the configuration of Fig. 2 is obtained by applying a well-known electro-technical method - Bruce Carlson (2000)-,

$$\begin{bmatrix} Y_{xo} \\ Y_{xs} \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{k_1}+\frac{1}{k_2}\right)\frac{1}{Z_1}+\frac{k_1+k_2}{Z_2} & -\frac{1}{k_2 Z_1} \\ -\frac{1}{k_2 Z_1} & \left(\frac{1}{k_2}+\frac{1}{k_3}\right)\frac{1}{Z_1}+\frac{k_2+k_3}{Z_2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{U_{xd}}{k_1 Z_1} \\ \frac{U_{xa}}{k_3 Z_1} \end{bmatrix} \tag{6}$$

where, Y_{xo}, Y_{xs} are *Across Variables* and U_{xd}, U_{xa} are the *Through Variables* respectively. From Eq. (6), the four transfer functions of the homogenous Pi system are,

$$P_{xoxd} = \left(\left(\frac{1}{k_2}+\frac{1}{k_3}\right)\frac{1}{Z_1}+(k_2+k_3)\frac{1}{Z_2}\right)\frac{1}{J} \tag{7}$$

$$P_{xsxa} = \left(\frac{1}{k_1 Z_1}+\frac{1}{k_2 Z_1}+\frac{k_1}{Z_2}+\frac{k_2}{Z_2}\right)\frac{1}{J} \tag{8}$$

$$P_{xoxa} = P_{xsxd} = \frac{1}{k_2 Z_1}\frac{1}{J} \tag{9-10}$$

where,

$$J = \frac{-1}{k_2^{\,2} Z_1^{\,2}} + \left(\left(\frac{1}{k_1}+\frac{1}{k_2}\right)\frac{1}{Z_1}+\frac{k_1+k_2}{Z_2}\right)\left(\left(\frac{1}{k_2}+\frac{1}{k_3}\right)\frac{1}{Z_1}+\frac{k_2+k_3}{Z_2}\right)$$

The particular equations describing the effects of the inputs over the control objective can be computed substituting Eqs. (7) through (10) in Eq. (4).

Remark 2: The closed loop transfer functions have the same denominator and depend on the controller G, the impedances per meter Z_1 and Z_2 and the location of the points of the DPS.

Remark 3: Anderson and Parks (1985) present a general study which concludes that the lumped approximation of a DPS is adequate if the number of lumped systems is large enough. In addition, the higher the frequency to be considered in the model, the larger the number of lumped systems. Once the previous conditions are satisfied, the classical control theory (stability, controllability, etc) can be applied to the equivalent lumped model.

Remark 4: In accordance with the Anderson and Parks conditions (1985) the proposed Pi General Case introduced in this paper may include more than one Pi-element between the relevant points of the spatial configuration (x_o-x_d, x_o-x_s or x_s-x_a) depending on the dimension of the original DPS problem.

4. ONE-POINT FEEDBACK ROBUST CONTROL OF DPS

4.1. Lumped QFT

The first technique that introduced a quantitative synthesis and took into account quantitative bounds on the plant uncertainty and quantitative tolerances on the closed-loop system response specifications was presented by Professor Horowitz (1972) in the early seventies: “The Quantitative Feedback Theory” (Houpis *et al.*, 2005).

The QFT method uses the well-known Nichols Chart (NC) to synthesize (loop-shape) the controller law, to reach the desired performance specifications for the whole set of plants with uncertainty. It presents a two-degree of freedom (2DOF) structure, with a loop controller G and a pre-filter F in cascade with the feedback loop, as can be seen in Fig. 3 and Eqs. (11) through (13).

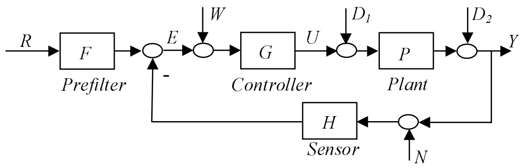


Fig. 3. 2DOF QFT feedback structure.

$$Y = \frac{1}{1+PGH}D_2 + \frac{P}{1+PGH}D_1 + \frac{PG}{1+PGH}(W+FR) - \frac{PGH}{1+PGH}N \tag{11}$$

$$U = \frac{G}{1+PGH}(W+FR) - \frac{GH}{1+PGH}(N+D_2+PD_1) \tag{12}$$

$$E = -\frac{H}{1+PGH}D_2 + \frac{PH}{1+PGH}D_1 + \frac{PGH}{1+PGH}W + \frac{1}{1+PGH}FR - \frac{H}{1+PGH}N \tag{13}$$

The stability and performance specifications are defined in terms of inequalities in the frequency domain. They relate the magnitude of the transfer functions $T_k(j\omega_i)$ of Eqs. (11) through (13) to some expressions $\delta_k(\omega_i)$, so that $\{|T_k(j\omega_i)| \le \delta_k(\omega_i), \forall \omega_i \in \Omega_k, k = 1,2..., i = 1,2...\}$.

The plant model is defined taking into account the parameter uncertainty of the process for every frequency of interest (ω_i), that is to say the plant uncertainty templates: $\Im P(j\omega_i)=\{P(j\omega_i), \omega_i \in \Omega_k\}$. For a nominal plant $P_0 \in \Im P(j\omega_i)$, the QFT methodology converts system specifications and model plant uncertainty to a set of bounds (Horowitz-Sidi Bounds) for every frequency of interest (ω_i), which have to be fulfilled by the nominal open-loop transfer function $L_0 = P_0\, G$. Such a great integration of information in a set of simple curves (the bounds) allows one to design the controller using only a single plant, the nominal plant P_0, and shaping the resulting open loop function L_0. Chait and Yaniv (1993) developed an algorithm to compute the bounds based on quadratic inequalities (see Table 1), simplifying much of the work on traditional manual bound computation.

The ω_i plant template, $\Im P(j\omega_i) = \{P(j\omega_i)\}$, is approximated by a finite set of boundary plants $\{P_r(j\omega_i)$, r = 1,...,m$\}$. Each plant can be expressed in its polar form as $P_r(j\omega_i) = p(\omega_i)\, e^{j\theta(\omega i)} = p\angle\theta$. Likewise the controller polar form is $G(j\omega_i) = g(\omega_i)\, e^{j\phi} = g\angle\phi$. The controller phase ϕ varies from -2π to 0. Therefore, for every frequency ω_i, the feedback specifications $\{|T_k(j\omega_i)| \le \delta_k(\omega_i)$, k = 1,...,5$\}$ in Table

1 –Eqs. (14) through (19)- are translated into quadratic inequalities: $a\,g^2 + b\,g + c \geq 0$, where a, b, c depend on p, θ, ϕ and δ_k. Taking these inequalities into account, it is possible to compute the bounds at the NC –see Chait and Yaniv algorithm (1993)-, and to loop shape the controller afterwards.

Table 1. Lumped systems. H=1. Eqs. (14) to (19)

$$|T_1(j\omega)| = \left|\frac{Y(j\omega)}{R(j\omega)\,F(j\omega)}\right| = \left|\frac{U(j\omega)}{D_1(j\omega)}\right| = \left|\frac{Y(j\omega)}{N(j\omega)}\right| = \left|\frac{P(j\omega)\,G(j\omega)}{1+P(j\omega)\,G(j\omega)}\right| \leq \delta_1(\omega), \quad \omega\in\Omega_1$$

$$|T_2(j\omega)| = \left|\frac{Y(j\omega)}{D_2(j\omega)}\right| = \left|\frac{1}{1+P(j\omega)\,G(j\omega)}\right| \leq \delta_2(\omega), \;\; \omega\in\Omega_2$$

$$|T_3(j\omega)| = \left|\frac{Y(j\omega)}{D_1(j\omega)}\right| = \left|\frac{P(j\omega)}{1+P(j\omega)G(j\omega)}\right| \leq \delta_3(\omega), \;\; \omega\in\Omega_3$$

$$|T_4(j\omega)| = \left|\frac{U(j\omega)}{D_2(j\omega)}\right| = \left|\frac{U(j\omega)}{N(j\omega)}\right| = \left|\frac{U(j\omega)}{R(j\omega)\,F(j\omega)}\right| = \left|\frac{G(j\omega)}{1+P(j\omega)\,G(j\omega)}\right| \leq \delta_4(\omega), \quad \omega\in\Omega_4$$

$$\delta_{5\inf}(\omega) < |T_5(j\omega)| = \left|\frac{Y(j\omega)}{R(j\omega)}\right| = \left|F(j\omega)\,\frac{P(j\omega)\,G(j\omega)}{1+P(j\omega)\,G(j\omega)}\right| \leq \delta_{5\sup}(\omega), \quad \omega\in\Omega_5$$

$$\frac{|G(j\omega)\,P_d(j\omega)|}{|G(j\omega)\,P_e(j\omega)|}\,\frac{|1+G(j\omega)\,P_e(j\omega)|}{|1+G(j\omega)\,P_d(j\omega)|} \leq \delta_5(\omega) = \frac{\delta_{5\sup}(\omega)}{\delta_{5\inf}(\omega)}, \quad \omega\in\Omega_5$$

4.2. Distributed QFT

Consider now the MISO DPS described in Fig.1. Similarly to Eqs. (11) through (13), the expressions that explain the block diagram of Fig. 1 are:

$$Y_{xo} = \frac{GP_{xo\,xa}}{1+GHP_{xs\,xa}}\left(FR+W\right) + \left[P_{xo\,xd} - \frac{GHP_{xo\,xa}P_{xs\,xd}}{1+GHP_{xs\,xa}}\right]U_{xd} - \frac{GHP_{xo\,xa}}{1+GHP_{xs\,xa}}N \qquad (20)$$

$$U_{xa} = \frac{G}{1+G\,HP_{xa\,xs}}\left(F\,R+W\right) - \frac{G\,HP_{xs\,xd}}{1+G\,HP_{xs\,xa}}\,U_{xd} - \frac{G\,H}{1+G\,HP_{xs\,xa}}\,N \qquad (21)$$

$$E = \frac{1}{1+G\,HP_{xs\,xa}}\,F\,R - \frac{G\,HP_{xs\,xa}}{1+G\,HP_{xs\,xa}}\,W - \frac{HP_{xs\,xd}}{1+G\,HP_{xs\,xa}}\,U_{xd} - \frac{H}{1+G\,HP_{xs\,xa}}\,N \qquad (22)$$

$$Y_{xs} = \frac{G\,P_{xs\,xa}}{1+G\;HP_{xs\,xa}}\left(F\,R+W\right) + \frac{P_{xs\,xd}}{1+G\;HP_{xs\,xa}}\,U_{xd} - \frac{G\,HP_{xs\,xa}}{1+G\,HP_{xs\,xa}}\,N \qquad (23)$$

2DOF controller G and F; the sensor dynamics H and the spatial situation of the sensor, the actuator, the disturbances and the control objective

Likewise to Table 1, each feedback distributed problem calculated from Eqs. (20) through (23) turns into a quadratic inequality problem $|T_k(j\omega)| \leq \delta_k(\omega)$, now shown in Table 2.

As in the previous section, the feedback specifications $\{|T_k(j\omega)| \leq \delta_k(\omega),\ k=1,...,8\}$ in Table 2 are translated into quadratic inequalities: $a\,g^2 + b\,g + c \geq 0$. The main difference between classical lumped QFT and the proposed distributed QFT is that now the sets P not only include parametrical and non-parametrical uncertainties but also distributed uncertainty, in the form of the distance between inputs, the distance between outputs and the distance between outputs and the boundaries of the lumped system.

From these new inequalities it is now easy to compute the bounds at the NC as in the previous section, and afterwards to loop shape the controller for the distributed parameter system.

Table 2. DPS. H=1. Eqs. (24) to (32)

$$|T_1(j\omega)| = \left|\frac{Y_o(j\omega)}{R(j\omega)\,F(j\omega)}\right| = \left|\frac{Y_{xo}(j\omega)}{N(j\omega)}\right| = \left|\frac{Y_{xo}(j\omega)}{W(j\omega)}\right| = \left|\frac{G(j\omega)\,P_{xo\,xa}(j\omega)}{1+G(j\omega)\;P_{xs\,xa}(j\omega)}\right| \leq \delta_1(\omega), \omega\in\Omega_1$$

$$|T_2(j\omega)| = \left|\frac{Y_{xo}(j\omega)}{U_{xd}(j\omega)}\right| = \left|P_{xd\,xo}(j\omega) - \frac{G(j\omega)\,P_{xo\,xa}(j\omega)\,P_{xs\,xd}(j\omega)}{1+G(j\omega)\;P_{xs\,xa}(j\omega)}\right| \leq \delta_2(\omega), \;\; \omega\in\Omega_2$$

$$|T_3(j\omega)| = \left|\frac{U_{xa}(j\omega)}{R(j\omega)\,F(j\omega)}\right| = \left|\frac{U_{xa}(j\omega)}{W(j\omega)}\right| = \left|\frac{U_{xa}(j\omega)}{N(j\omega)}\right| = \left|\frac{G(j\omega)}{1+G(j\omega)\;P_{xs\,xa}(j\omega)}\right| \leq \delta_3(\omega), \;\omega\in\Omega_3$$

$$|T_4(j\omega)| = \left|\frac{U_{xa}(j\omega)}{U_{xd}(j\omega)}\right| = \left|\frac{G(j\omega)\,P_{xs\,xd}(j\omega)}{1+G(j\omega)\;P_{xs\,xa}(j\omega)}\right| \leq \delta_4(\omega), \;\omega\in\Omega_4$$

$$\delta_{5\inf}(\omega) \leq |T_5(j\omega)| = \left|\frac{Y_{xo}(j\omega)}{R(j\omega)}\right| = \left|F(j\omega)\;\frac{G(j\omega)\,P_{xo\,xa}(j\omega)}{1+G(j\omega)\;P_{xs\,xa}(j\omega)}\right| \leq \delta_{5\sup}(\omega), \quad \omega\in\Omega_5$$

$$\frac{|G(j\omega)\,P_{xoxad}(j\omega)|}{|G(j\omega)\,P_{xoxae}(j\omega)|}\,\frac{|1+G(j\omega)\,P_{xsxae}(j\omega)|}{|1+G(j\omega)\,P_{xsxad}(j\omega)|} \leq \delta_5(\omega) = \frac{\delta_{5\sup}(\omega)}{\delta_{5\inf}(\omega)}, \;\; \omega\in\Omega_5$$

$$|T_6(j\omega)| = \left|\frac{E(j\omega)}{R(j\omega)F(j\omega)}\right| = \left|\frac{E(j\omega)}{N(j\omega)}\right| = \left|\frac{1}{1+G(j\omega)\;P_{xs\,xa}(j\omega)}\right| \leq \delta_6(\omega), \;\; \omega\in\Omega_6$$

$$|T_7(j\omega)| = \left|\frac{E(j\omega)}{U_{xd}(j\omega)}\right| = \left|\frac{Y_{xs}(j\omega)}{U_{xd}(j\omega)}\right| = \left|\frac{P_{xs\,xd}(j\omega)}{1+G(j\omega)\;P_{xs\,xa}(j\omega)}\right| \leq \delta_7(\omega), \quad \omega\in\Omega_7$$

$$|T_8(j\omega)| = \left|\frac{E(j\omega)}{W(j\omega)}\right| = \left|\frac{Y_{xs}(j\omega)}{R(j\omega)\,F(j\omega)}\right| = \left|\frac{Y_{xs}(j\omega)}{N(j\omega)}\right| = \left|\frac{Y_{xs}(j\omega)}{W(j\omega)}\right| = \left|\frac{G(j\omega)\;P_{xs\,xa}(j\omega)}{1+G(j\omega)\;P_{xs\,xa}(j\omega)}\right| \leq \delta_8(\omega), \;\omega\in\Omega_8$$

4.3 Limitations of the Loop Capabilities

a) The controller G cannot totally control the dynamic of the disturbance effects at output – Eq.(4)- when P_{xoxd} has poles in RHP.

b) Tracking Performance limitation. Due to the parameter uncertainty, the objective signal Y_{xo} may show an offset error. Using a pre-filter F the error can be attenuated -Eq. (33)-.

$$\frac{Y_{xo}}{F\ R} = \frac{G\ P_{xo\,xa}}{1+G\ H\,P_{xs\,xa}} = \frac{G\ P_{xs\,xa}}{1+G\ H\,P_{xs\,xa}}\ \frac{P_{xo\,xa}}{P_{xs\,xa}} \tag{33}$$

5. EXAMPLE

This section explains in more detail the main ideas introduced in the paper using the same heat equation example illustrated by Kelemen *et al.* (1989). Kelemen's PDE model approach to this topic is also compared to the Pi Equivalent approach proposed in this paper.

<u>The Heat Equation Problem</u>: Let us consider a temperature-distributed plant modelled as,

$$\begin{aligned} &\frac{\partial T}{\partial t} - \frac{\partial^2 T}{\partial x^2} = \vartheta(x,t), \quad x \in (0,\pi), \quad t \ge 0 \\ &T(x,0) = 0, \ \ x \in [0,\pi] \\ &T(0,t) = T(\pi,t) = 0 \\ &\vartheta(x,t) = (\delta_{t=0})(\delta_{x=xi}) \end{aligned} \tag{34}$$

where $T(x,t)$ and $\upsilon(x,t)$ are the temperature and input distribution, respectively. It is assumed that the sensor and the actuator point match up, $x_s = x_a = \pi/2$, and a disturbance is applied at x_d. The example shows the design of one feedback loop to meet the desired performance specifications at $x_o = x_d = \pi/4$.

The desired performance specifications are:

(a) Stability bounds. They are defined by the most restrictive expression of the Equations.,

$$\left|\frac{G\ P_{xo\,xa}}{1+G\ P_{xs\,xa}}\right| \le 1.2 \qquad \forall\omega \tag{35}$$

$$\left|\frac{G\ P_{xs\,xa}}{1+G\ P_{xs\,xa}}\right| \le 1.2 \qquad \forall\omega \tag{36}$$

(b) Disturbance Rejection bounds.

$$\left|\frac{Y_{xo}}{U_{xd}}\right| = \left|P_{xo\,xd} - \frac{G\,P_{xo\,xa}\,P_{xs\,xd}}{1+G\,P_{xs\,xa}}\right| \le 0.9\left|P_{xo\,xd}\right|, \ \ \omega < 1.5\text{rad/s} \tag{37}$$

<u>Model calculation:</u> The original paper written by Kelemen *et al.* (1989) shows a irrational transfer Eq.(38) that represents the behaviour of the distributed parameter heat system by using Laplace Transforms of Eq.(34).

$$P_{x\,xi}(s) = \frac{\left(e^{\sqrt{s}(\pi-x)} - e^{-\sqrt{s}(\pi-x)}\right)\left(e^{\sqrt{s}\,x_i} - e^{-\sqrt{s}\,x_i}\right)}{2\sqrt{s}\left(e^{\sqrt{s}\,\pi} - e^{-\sqrt{s}\,\pi}\right)} \tag{38}$$

By taking the equivalent electrical model of this particular heat plant, A=1; r = 1/(kA) =1; c =ρC_pA=1; and solving the resultant system of equations Eq.(6) for the 3, 4 and 8 Pi-elements cases, the respective groups of rational transfer functions P_{xsxa}, P_{xoxa}, P_{xoxd}, P_{xsxd}, are obtained. Fig.4 shows how the equivalent electrical models approximate the irrational Eq.(38).

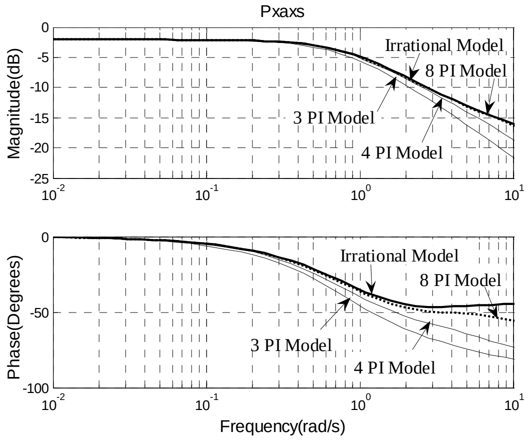


Fig. 4. Bode Diagram of Temperature models. (Similar diagrams represent P_{xsxd}, P_{xoxa}, P_{xoxd})

The 8 Pi-elements equivalent model fits very well the original DPS. According to Anderson and Parks (1985), the approach is valid for the ω, $\omega < 2/(RC) = 2/(\pi/8)^2 = 12.97\,\text{rad/s}$, where R and C are the resistance and capacitance per Pi-element length.

<u>Design Procedure:</u> The templates are calculated from the equivalent transfer functions P_{xsxa}, P_{xoxa}, P_{xoxd}, P_{xsxd} of the 8 Pi elements case. The robust stability and disturbance rejection bounds $B(j\omega)$ are obtained from the quadratic inequalities corresponding to Eq. (24), (31) and (25) of Table 2.

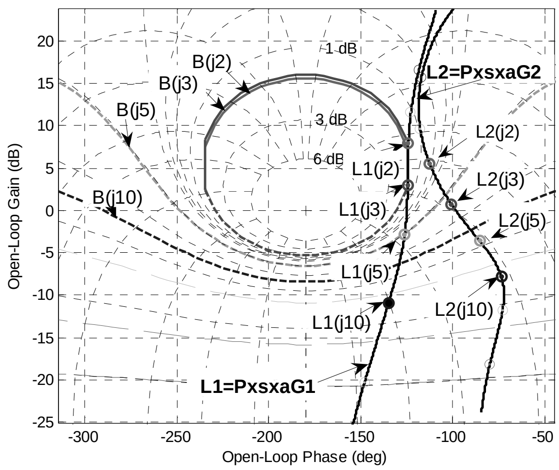


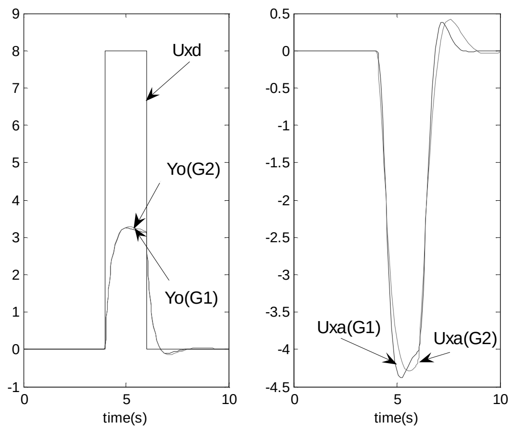
Fig.5. $L_1 = Pxsxa_0\ G_1$, $L_2 = Pxsxa_0\ G_2$ on the NC

The nominal open-loop expression is $L_1 = G_1\ P_{xsxa0}$. By using a standard loop shaping QFT technique, the G_1 controller -Eq.(39)- is designed. Fig. 5 shows on the NC both loop transfer functions (L_1 and L_2), with the proposed controller G_1 and with the Kelemen controller G_2 -Eq.(40)-.

$$G_1(s) = \frac{11.5\left(s/2.9+1\right)}{s\left(s/5+1\right)} \qquad (39)$$

$$G_2(s) = \frac{60\left(s/1.5+1\right)\left(s/2.5+1\right)}{\left(s/0.4+1\right)^2} \qquad (40)$$

Results: Fig. 6 compares the time responses obtained by using both controllers G_1 and G_2. The results show a similar performance of both approaches. However, the proposed methodology is simpler, deals with model uncertainty and is able to work with distributed specifications.



(a) (b)

Fig. 6 (a) Disturbance U_{xd} and output signals Y_{xo} at $x = x_d$ (b) Control signal U_{xa} at $x = x_a$. $F = 2$.

6. CONCLUSIONS

This paper introduced a new technique to design applicable one-point feedback controllers for DPS with uncertainty. The technique is an extension of the QFT to DPS, considering spatial distribution as another parameter of uncertainty. Working on the classical frequency domain, this technique avoids PDEs, etc., but still represents spatial distributed configurations with uncertainty. The paper extended the classical QFT performance specifications used in lumped systems by introducing a set of inequalities for DPS.

To quote Prof. Horowitz (2003) in one of his last papers: *"The door has been opened to the creation of a truly quantitative feedback theory for uncertain distributed systems. There is a tremendous amount of research to be done, both of a pure mathematical nature, and especially of applied computational nature"*.

ACKNOWLEDGEMENT

Authors wish to thank Prof. Isaac Horowitz for his valuable comments, and gratefully appreciate the support given by the Spanish "Ministerio de Ciencia y Tecnología" (MCyT) under grant CICYT DPI'2003-08580-C02-01.

REFERENCES

Anderson, B.D.O and P.C. Parks (1985). Lumped approximation of distributed systems and controllability questions. *IEE Proc.* **132**(3), 89-94.

Bruce Carlson A. (2000). *Circuits*. Brooks: USA.

Chait, Y. and O. Yaniv (1993). Multi-input/single-output computer-aided control design using QFT. *Int. J. Robust & Non-linear Control*, **3**, 47-54.

Chait, Y., C.R.Maccluer and C.J. Radcliffe (1989). A Nyquist Stability Criterion for Distributed Parameter Systems. *IEEE Trans. on Automatic Control*, **34** (1), 90-92.

Chen, G., M.C. Delfour, A.M. Krall and G.Payre (1987). Modelling, Stabilization and Control of Serially Connected Beams. *SIAM J. Contr. Opt.*, **25**, 526-546.

Curtain, R.F and H. Zwart (1995). *An Introduction to Infinite-Dimensional Linear Systems Theory*. Springer-Verlag.

Dorf R.C and R.H. Bishop (2000*).* *Modern Control Systems*. Addison Wesley: USA.

Horowitz, I., Y. Kannai, and M. Kelemen (1989). QFT approach to distributed systems Control and Applications. *Proceedings. ICCON '89. IEEE Int. Conf.*, 516 –519.

Horowitz, I.M and M. Sidi (1972). Synthesis of Feedback Systems with Large Plant Ignorance for Prescribed Time-Domain Tolerances. *International Journal of Control*, **16**(2), 287-309.

Horowitz, I.M. (2003). Some ideas for QFT research*.* *International Journal of Robust and Nonlinear Control*, **13** (7), 599-606.

Houpis, C.H., S.J. Rasmussen and M. Garcia-Sanz (2005). *Quantitative Feedback Theory. Fundamentals and Applications*. 2nd Edition. Marcel Dekker: NY, USA.

Kelemen, M., Y. Kannai and I.M. Horowitz (1989). One-point Feedback Approach to Distributed Linear Systems. *Int. Journal of Control*, **49**(3), 969-980

Levis, A.H., *et al. (*1987). Challenges to Control: A Collective View. *IEEE Trans.Aut.Control,* **32** (4), 275-285.

Lions, J.L. (1988). Exact Controllability, Stabilization and Perturbations for Distributed Parameter Systems. *SIAM Rev*., **30**, 1-68.

Luo, Z.H, B.Z. Guo and O.Morgul (1999), *Stability and Stabilization of Infinite Dimensional Systems with Applications.* Springer-Verlag,

Takahaschi, Y., M. Rabims and D.Auslander (1970). *Control and Dynamic Systems*. Addison Wesley.