ROBUST NETWORKED PREDICTIVE CONTROL FOR SYSTEMS WITH RANDOM NETWORK DELAY

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Abstract: The robust control of networked predictive control systems with random network delay in the feedback channel is studied in this paper. The stabilisation of systems with constant time-delay is discussed by converting the corresponding Lyapunov inequality to a non-linear inequality. To obtain the maximum domain of uncertainties, the non-linear inequality is evolved as a non-linear optimisation control problem. After the optimisation problem is solved, it yields a controller that can stabilise the system and the domain of uncertainties. Furthermore, for the case of random network induced time-delay, robust stabilisation problem can be formulated as a set of inequalities, which are related to the corresponding constant time-delay, respectively. This result is verified by a numerical example.

Keywords: robustness, predictive control, network, uncertainty, time-delay

1. INTRODUCTION

Using a network in control systems produces many benefits in practical engineering practice: reducing system wiring, ease of system diagnosis and maintenance, and increasing system agility. But it also brings some new: the network-induced delay (sensor-to-controller delay and controller-to-actuator delay), occurrence of packet dropout resulting in control signal break-off, etc. Time-delay is one of the main problems of networked control systems (NCS). With the development of NCS research, some methods have been developed to address this problem. Halevi and Ray (1988) proposed an augmented deterministic discrete-time model method to control a linear plant over a periodic delay network. Luck and Ray (1990, 1994) utilized the deterministic or probabilistic information of an NCS and developed queuing method. Nilsson (1998) proposed the optimal stochastic control method which treats the effects of random network delays in an NCS as a LQG problem. Walsh, et al. (1999a, 1999b) used non-linear and perturbation theory to formulate the network delay effects in an NCS as the vanishing perturbation of a continuous-time system under the assumption that there is no observation noise. Hai Lin, et al. (2003) formulated NCS as discrete-time switched system and proposed a way to study stability and disturbance attenuation issues for a class of NCS under uncertain access delay.

Although much research has been done in networked control systems, most work has ignored a very important feature of networked control systems that communication networks transmit a packet of data at the same time, which is not done in traditional control systems. Just making use of this network feature, Liu et al. (2004) proposed a new networked control scheme—networked predictive control, which can overcome the effects caused by network delay. The paper considered the precise model of NCS with network induced time-delay in the forward channel and didn’t discuss the case of feedback channel time-delay and model uncertainties. This paper addresses these cases by considering NCS with structured uncertainties and network induced time-delay in the feedback channel. The method of networked predictive control is used to handle the network-induced time-delay. Robust stabilization of NCS with constant network induced time-delay is formulated as a constrained non-linear optimisation problem. Both the related control law and boundary of uncertainty can be obtained by solving a non-linear optimisation problem. A NCS with random network induced time-delay can be handled by solving a non-linear optimisation problem that contains a set of non-linear inequalities as constraints corresponding to specific time-delay values. This method shows that in a definite bound, if there exist a controller and an observer which stabilises the augmented system for all constant time-delays, then the control law can robustly stabilise the NCS for random time-delay. In addition, this method is validated by a numerical example.

This paper is organized as follows: Section 2 presents the main results of networked predictive control systems with constant time-delay in the feedback channel; Section 3 discusses robustness analysis of networked predictive control systems with random time-delay in the feedback channel; This method is validated by a numerical example in section 4; Section 5 gives the conclusion.

2. ROBUSTNESS ANALYSIS OF NPCS WITH CONSTANT TIME-DELAY

To overcome the unknown network transmission delay, Liu et al. (2004) proposed a networked predictive control scheme which mainly consists of a control prediction generator and a network delay compensator. The control prediction generator is designed to generate a set of future control predictions. The network delay compensator is used to compensate the unknown random network delay. This networked predictive control system (NPCS) structure is shown in Fig. 1. Only the transmission delay in the feedback channel is considered in this paper.

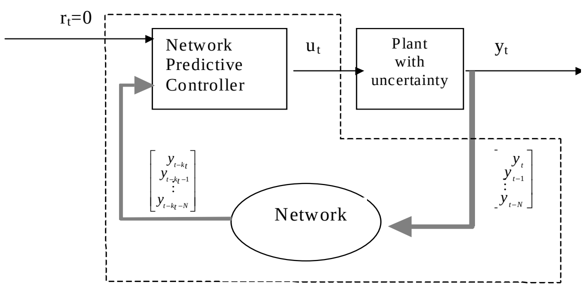

Figure 1. The networked predictive control system

In this section, the case with constant time-delay is discussed. The networked predictive control system with uncertainties can be described as follows

$$\begin{aligned} x_{t+1} &= (A+\Delta A)x_t + (B+\Delta B)u_t \\ y_t &= Cx_t \end{aligned} \qquad (1)$$

where $x_t \in \Re^n$, $y_t \in \Re^l$ and $u_t \in \Re^m$ are the state, output and input vectors of the system, respectively. $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$ and $C \in \Re^{l\times n}$ are the system matrices. The uncertainty items are $\Delta A = H_1 F_t E_1$, $\Delta B = H_2 F_t E_2$ with $H_1 \in \Re^{n\times q}$, $H_2 \in \Re^{n\times q}$, $E_1 \in \Re^{q\times m}$, $E_2 \in \Re^{q\times m}$ and $F_t \in \Re^{q\times q}$ where $F_t^T F_t \le \gamma^2 I$. For the simplicity of stability analysis, the reference input of the system is assumed to be zero.

Assumption1: The pair (A,B) is completely controllable, and the pair (A,C) is completely observable.
Assumption2: The number of consecutive data dropouts must be less than m (m is a non-negative integer). The upper bound of the network induced time-delay is not greater than N .

Remark: Because the control data are transmitted as packages through networks, especially internet, it is reasonable to assume that the network induced time-delays are integer times of sampling period. Assumption 2 indicates that the network will not continuously drop out data packages unlimitedly meanwhile guarantee that the NCS is a closed-loop system. Obviously, $N \ge m$. In addition, for our control scheme, the data dropouts are converted to corresponding time-delay according the used package protocols and sampling period.

Similar to Liu et al (2004), the proposed control scheme is stated as follows. If time-delay is assumed to be i ($i \in \{1,\cdots N\}$), then only the signal of the $t-i$ instant can be used to construct control signal. The following observer and state estimation are designed to predict a series of system states prediction values.

$$\begin{aligned} \hat{x}_{t-i+1|t-i} &= A\hat{x}_{t-i|t-i-1} + Bu_{t-i} + L(y_{t-i} - C\hat{x}_{t-i|t-i-1}) \\ \hat{x}_{t-i+2|t-i} &= A\hat{x}_{t-i+1|t-i} + Bu_{t-i+1} \\ &\vdots \\ \hat{x}_{t|t-i} &= A\hat{x}_{t-1|t-i} + Bu_{t-1} \end{aligned} \qquad (2)$$

where $L \in \Re^{n\times l}$ is the systems observing matrix and $\hat{x}_{t-i+k|t-i} \in \Re^n$ ($k = 1,\cdots i$) and $u_{t-i+k} \in \Re^m$ are the k -step ahead state prediction and the input of the observer the matrix at time $t-i$, respectively. It means that if the designed tolerable time-delay is i , then at any time instant, the predictor generates the state prediction of i -step ahead. The state-feedback controller for the case without network delay is designed by a modern control method, for example, LQG, eigenstructure assignment etc., and for the case with network delay, the controller is of the following form:

$$u_{t-i+k} = K\hat{x}_{t-i+k|t-i}$$

where the state feedback matrix $K\in\Re^{m\times n}$. As $y_{t-i}=Cx_{t-i}$, (2) can be written as

$$\begin{aligned}&\hat{x}_{t-k+i|t-k}=A^{i-1}(A-LC)\hat{x}_{t-k|t-k-1}\\&+\sum_{j=1}^{i}A^{i-j}Bu_{t-k+j-1}+A^{i-1}Cx_{t-i}\end{aligned}\quad(3)$$

for $i=1,2,3,\cdots,k$.

Thus, the output of the networked predictive control at time t is determined by

$$\begin{aligned}&u_t=KA^{i-1}(A-LC)\hat{x}_{t-i|t-i-1}\\&+\sum_{j=1}^{i}KA^{i-j}Bu_{t-i+j-1}+KA^{i-1}LCx_{t-i}\end{aligned}\quad(4)$$

Therefore, the corresponding closed-loop system can be written as

$$\begin{aligned}&x_{t+1}=(A+\Delta A)x_t+(B+\Delta B)[KA^{i-1}(A-LC)\hat{x}_{t-i|t-i-1}\\&+\sum_{j=1}^{i}KA^{i-j}Bu_{t-i+j-1}+KA^{i-1}LCx_{t-i}]\end{aligned}\quad(5)$$

The formula (3), (4), (5) can be described by the following augmented system:

$$\bar{x}_{t+1}=\hat{A}_i\bar{x}_t\quad(6)$$

where $\bar{x}_t=[X_i\ U_i\ \hat{X}_i]^T$ with

$$X_t=[x_t^T\ x_{t-1}^T\ \cdots\ x_{t-i+1}^T\ x_{t-i}^T\ x_{t-i-1}^T\ \cdots\ x_{t-N+1}^T\ x_{t-N}^T],$$

$$U_t=[u_{t-1}^T\ u_{t-2}^T\ \cdots\ u_{t-i+1}^T\ u_{t-i}^T\ u_{t-i-1}^T\ \cdots\ u_{t-N+1}^T\ u_{t-N}^T],$$

$$\hat{X}_t=[\hat{x}_{t|t-1}^T\ \hat{x}_{t-1|t-2}^T\cdots\hat{x}_{t-i+1|t-i}^T\ \hat{x}_{t-i|t-i-1}^T\ \hat{x}_{t-i-1|t-i-2}^T\cdots\hat{x}_{t-N+1|t-N}^T\ \hat{x}_{t-N|t-N-1}^T],$$

and

$$\hat{A}_i=\begin{bmatrix}\tilde{A}_{11}&\tilde{A}_{12}&\tilde{A}_{13}\\\tilde{A}_{21}&\tilde{A}_{22}&\tilde{A}_{23}\\\tilde{A}_{31}&\tilde{A}_{32}&\tilde{A}_{33}\end{bmatrix}\in\Re^{((2n+m)N+2n)\times((2n+m)N+2n)}$$

with

$$\tilde{A}_{11}=\begin{bmatrix}[A+\Delta A\ 0_{n\times(i-1)n}\ \bar{B}KA^{i-1}\ 0_{n\times(N-i)n}]\\[I_{nN\times nN}\quad 0_{nN\times n}]\end{bmatrix}\in\Re^{(N+1)n\times(N+1)n},$$

$$\tilde{A}_{12}=\begin{bmatrix}[\bar{B}KB\ \bar{B}KAB\cdots\bar{B}KA^{i-2}B\ \bar{B}KA^{i-1}B\ 0_{n\times(N-i)m}]\\0_{Nn\times mN}\end{bmatrix}\in\Re^{(N+1)n\times mN}$$

$$\tilde{A}_{13}=\begin{bmatrix}[0_{n\times in}\ \bar{B}KA^{i-1}(A-LC)\ 0_{n\times(N-i)n}]\\0_{Nn\times(N+1)n}\end{bmatrix}\in\Re^{(N+1)n\times(N+1)n}$$

(where $\bar{B}=B+\Delta B$),

$$\tilde{A}_{21}=\begin{bmatrix}[0_{m\times in}\quad KA^{i-1}LC\quad 0_{m\times(N-i)n}]\\0_{m(N-1)\times(N+1)n}\end{bmatrix}\in\Re^{mN\times(N+1)n}$$

$$\tilde{A}_{22}=\begin{bmatrix}[KB\ KAB\ \cdots\ KA^{i-2}B\ KA^{i-1}B\ 0_{m\times(N-i)m}]\\[I_{m(N-1)\times m(N-1)}\quad 0_{m(N-1)\times m}]\end{bmatrix}\in\Re^{mN\times mN},$$

$$\tilde{A}_{23}=\begin{bmatrix}[0_{m\times in}\ KA^{i-1}(A-LC)\ 0_{m\times(N-i)n}]\\0_{m(N-1)\times(N+1)n}\end{bmatrix}\in\Re^{mN\times(N+1)n},$$

$$\tilde{A}_{31}=\begin{bmatrix}[LC\ 0_{n\times(i-1)n}\ BKA^{i-1}LC\ 0_{n\times(N-i)n}]\\0_{Nn\times(N+1)n}\end{bmatrix}\in\Re^{(N+1)n\times(N+1)n},$$

$$\tilde{A}_{32}=\begin{bmatrix}[BKB\ BKAB\ \cdots\ BKA^{i-2}B\ BKA^{i-1}B\ 0_{n\times(N-i)m}]\\0_{Nn\times(N+1)m}\end{bmatrix}\in\Re^{(N+1)n\times Nm},$$

$$\tilde{A}_{33}=\begin{bmatrix}[A-LC\ 0_{n\times(i-1)n}\ BKA^{i-1}(A-LC)\ 0_{n\times(N-i)n}]\\I_{nN\times nN}\quad 0_{nN\times n}\end{bmatrix}\in\Re^{(N+1)n\times(N+1)n}$$

The system described in (6) can be expressed in the form

$$\bar{x}_{t+1}=\left(\bar{A}_i+\bar{H}_i\bar{F}_i\bar{E}_i\right)\bar{x}_t\quad(7)$$

where

$$\bar{A}_i=\begin{bmatrix}\breve{A}_{11}&\breve{A}_{12}&\breve{A}_{13}\\\tilde{A}_{21}&\tilde{A}_{22}&\tilde{A}_{23}\\\tilde{A}_{31}&\tilde{A}_{32}&\tilde{A}_{33}\end{bmatrix}\in\Re^{((2n+m)N+2n)\times((2n+m)N+2n)}$$

with

$$\breve{A}_{11}=\begin{bmatrix}[A\ 0_{n\times(i-1)n}\ BKA^{i-1}\ 0_{n\times(N-i)n}]\\[I_{nN\times nN}\ 0_{nN\times n}]\end{bmatrix}\in\Re^{(N+1)n\times(N+1)n},$$

$$\breve{A}_{12}=\begin{bmatrix}[BKB\ BKAB\cdots BKA^{i-2}B\ BKA^{i-1}B\ 0_{n\times(N-i)m}]\\0_{Nn\times mN}\end{bmatrix}\in\Re^{(N+1)n\times mN},$$

$$\breve{A}_{13}=\begin{bmatrix}[0_{n\times in}\ BKA^{i-1}(A-LC)\ 0_{n\times(N-i)n}]\\0_{Nn\times mn}\end{bmatrix}\in\Re^{(N+1)n\times(N+1)n}$$

and

$$\bar{H}_i=\begin{bmatrix}\tilde{H}_{11}&\tilde{H}_{12}&\tilde{H}_{13}\\0_{(N+1)n\times qN}&0_{mN\times qN}&0_{mN\times(N+1)q}\\0_{(N+1)n\times(N+1)q}&0_{(N+1)n\times(N+1)q}&0_{(N+1)n\times(N+1)q}\end{bmatrix}\in\Re^{((2n+m)N+2n)\times(3qN+2q)}$$

with

$$\tilde{H}_{11}=\begin{bmatrix}[H_1\ 0_{n\times(i-1)q}\ H_2\ 0_{n\times(N-i)q}]\\0_{Nn\times(N+1)q}\end{bmatrix}\in\Re^{(N+1)n\times(N+1)q},$$

$$\tilde{H}_{12}=\begin{bmatrix}[H_2\ H_2\ \cdots\ H_2\ H_2\ 0_{n\times(N-i)q}]\\0_{Nn\times Nq}\end{bmatrix}\in\Re^{(N+1)n\times qN},$$

$$\tilde{H}_{13}=\begin{bmatrix}[0_{n\times in}\ H_2\ 0_{n\times(N-i)n}]\\0_{Nn\times(N+1)q}\end{bmatrix}\in\Re^{(N+1)n\times(N+1)q};$$

$\bar{F}_i=diag\{F_t,F_t,\cdots F_t\}\in\Re^{(3N+2)q\times(3N+2)q}$ and

$$\bar{E}_i=\begin{bmatrix}\tilde{E}_{11}&0_{(N+1)q\times mN}&0_{(N+1)q\times(N+1)n}\\0_{qN\times(N+1)n}&\tilde{E}_{22}&0_{qN\times(N+1)n}\\0_{(N+1)q\times(N+1)n}&0_{(N+1)q\times(N+1)n}&\tilde{E}_{33}\end{bmatrix}\in\Re^{(3qN+2q)\times((2n+m)N+2n)}$$

with

$$\tilde{E}_{11}=\begin{bmatrix}[E_1\ 0_{q\times Nn}]\\0_{(i-1)q\times(N+1)n}\\[0_{q\times in}\ E_2KA^{i-1}\ 0_{q\times(N-i)n}]\\0_{(N-i)q\times(N+1)n}\end{bmatrix}\in\Re^{(N+1)q\times(N+1)n},$$

$$\tilde{E}_{22}=\begin{bmatrix}E_2KB&0&\cdots&0&0&0\cdots0\ 0\\0&E_2KAB&\cdots&0&0&0\cdots0\ 0\\\vdots&\vdots&\ddots&\vdots&\vdots&\vdots\ddots\vdots\ \vdots\\0&0&\cdots&E_2KA^{i-2}B&0&0\cdots0\ 0\\0&0&\cdots&0&E_2KA^{i-1}B&0\cdots0\ 0\\\vdots&\vdots&\ddots&\vdots&\vdots&\vdots\ddots\vdots\ \vdots\\0&0&\cdots&0&0&0\cdots0\ 0\\0&0&\cdots&0&0&0\cdots0\ 0\end{bmatrix}$$

$\in\Re^{qN\times mN}$,

$$\tilde{E}_{33} = \begin{bmatrix} 0_{iq\times(N+1)n} \\ [0_{q\times iq} \;\; E_2KA^{i-1}(A-LC) \;\; 0_{q\times(N-i)n}] \\ 0_{(N-i)q\times(N+1)n} \end{bmatrix} \in \Re^{(N+1)q\times(N+1)n}$$

.

Before the result is presented, the definition of robustly quadratically stable is given below.

Definition 1: System (1) is robustly quadratically stable if there exists a positive definite matrix P such that for any time instant t, Lyapunov function $V(x_t) = x_t^T P x_t$ has the following property:

$$V(x_{t+1}) - V(x_t) = x_{t+1}^T P x_{t+1} - x_t^T P x_t < 0$$

Theorem 1: The corresponding closed-loop system of system (1) with any constant time delay is robustly quadratically stable if and only if there exists a matrices $P = P^T > 0$ and K such that:

$$\Pi_i = \begin{bmatrix} -P^{-1} & \overline{A}_i & \overline{H}_i \\ \overline{A}_i^T & -P + \tau\overline{E}_i^T\overline{E}_i & 0 \\ \overline{H}_i^T & 0 & -\tau\nu I \end{bmatrix} \le 0. \qquad (8)$$

where $\nu = 1/\gamma^2$ and constant $\tau \ge 0$. Furthermore, if (8) holds, then the maximum γ is defined by $1/\sqrt{\nu}$ where ν is determined by the optimisation problem:

min ν

subject to $P = P^T > 0$ and (8).

Before this theorem is proved, two lemmas are given.

Lemma 1 (Petersen and Hollot, 1986): Given $\forall x \in \Re^n$ and $\forall y \in \Re^n$,

$$\max\{(x^T DFEy)^2 : F^T F \le I\} = x^T DD^T x y^T E^T E y.$$

Lemma 2 (Xie and de Souza, 1992): Let X, Y and Z be given symmetric matrices with appropriate dimension such that $X \ge 0, Y < 0$ and $Z \ge 0$. Furthermore, assume that

$$(\eta^T Y\eta)^2 - 4(\eta^T X\eta)(\eta^T Z\eta) > 0$$

for all nonzero $\eta \in \Re^n$. Then there exists a constant $\varepsilon > 0$, such that

$$\varepsilon^2 X + \varepsilon Y + Z \ge 0.$$

Proof: According to definition 1, system (7) is robustly quadratically stable if and only if there exists a positive definite matrix P such that for any time instant t, Lyapunov function $V(\overline{x}_t) = \overline{x}_t^T P \overline{x}_t$ has the following property:

$$\begin{aligned} V(\overline{x}_{t+1}) - V(\overline{x}_t) &= \overline{x}^T{}_{t+1} P \overline{x}_{t+1} - \overline{x}^T{}_t P \overline{x}_t \\ &= \overline{x}_t^T[(\overline{A}_i + \overline{H}_i\overline{F}_i\overline{E}_i)^T P(\overline{A}_i + \overline{H}_i\overline{F}_i\overline{E}_i) - P]\overline{x}_t < 0 \end{aligned} \qquad (9)$$

for $\forall \overline{F}_i \in R^{2q\times 2q}, \overline{F}_i^T\overline{F}_i \le \gamma^2 I$. (9) is equivalent to

$$(\overline{A}_i + \overline{H}_i\overline{F}_i\overline{E}_i)^T P(\overline{A}_i + \overline{H}_i\overline{F}_i\overline{E}_i) - P < 0 \qquad (10)$$

For $P > 0$, apply Schur complement (Zhou, 1996) to formula (10), it gives

$$\begin{bmatrix} -P^{-1} & \overline{A}_i + \overline{H}_i\overline{F}_i\overline{E}_i \\ (\overline{A}_i + \overline{H}_i\overline{F}_i\overline{E}_i)^T & -P \end{bmatrix} < 0 \qquad (11)$$

Now (11) is proved to be right for $P > 0$ if and only if (8) holds.

Necessity: Suppose that (11) has a solution $P > 0$, then

$$\begin{bmatrix} -P^{-1} & \overline{A}_i \\ \overline{A}_i^T & -P \end{bmatrix} + \begin{bmatrix} 0 & \overline{H}_i\overline{F}_i\overline{E}_i \\ \overline{E}_i^T\overline{F}_i^T\overline{H}_i^T & 0 \end{bmatrix} < 0$$

Let

$$Z = \begin{bmatrix} -P^{-1} & \overline{A}_i \\ \overline{A}_i^T & -P \end{bmatrix} \qquad (12)$$

Then, for $\forall \eta_1, \eta_2 \in \Re^{n_s}$, $\eta = [\eta_1^T \; \eta_2^T]^T \ne 0$, it obtains

$$\begin{aligned} \eta^T Z\eta &< -\eta^T \begin{bmatrix} 0 & \overline{H}_i\overline{F}_i\overline{E}_i \\ \overline{E}_i^T\overline{F}_i^T\overline{H}_i^T & 0 \end{bmatrix}\eta \\ &= -(\eta_1^T\overline{H}_i\overline{F}_i\overline{E}_i\eta_2 + \eta_2^T\overline{E}_i^T\overline{F}_i^T\overline{H}_i^T\eta_1) = -2\eta_1^T\overline{H}_i\overline{F}_i\overline{E}_i\eta_2 \end{aligned}$$

Clearly $\eta^T Z\eta < -2\max\{\eta_1^T\overline{H}_i\overline{F}_i\overline{E}_i\eta_2 : \overline{F}_i\overline{F}_i^T \le \gamma^2 I\}$ and $\max\{\eta_1^T\overline{H}_i\overline{F}_i\overline{E}_i\eta_2 : \overline{F}_i\overline{F}_i^T \le \gamma^2 I\} \ge 0$, then

$$(\eta^T Z\eta)^2 > 4\max\{\eta_1^T\overline{H}_i\overline{F}_i\overline{E}_i\eta_2 : \overline{F}_i\overline{F}_i^T \le \gamma^2 I\}^2$$

Therefore, from Lemma 1, it gives

$$\begin{aligned} (\eta^T Z\eta)^2 &\ge 4\gamma^2\eta_1^T\overline{H}_i\overline{H}_i^T\eta_1\eta_2^T\overline{E}_i^T\overline{E}_i\eta_2 \\ &= 4\eta^T\begin{bmatrix} \gamma^2\overline{H}_i\overline{H}_i^T & 0 \\ 0 & 0 \end{bmatrix}\eta\eta^T\begin{bmatrix} 0 & 0 \\ 0 & \overline{E}_i^T\overline{E}_i \end{bmatrix}\eta. \end{aligned}$$

Hence, it follows from Lemma 2 that there exists a constant $\varepsilon > 0$ such that

$$\varepsilon^2\begin{bmatrix} \gamma^2\overline{H}_i\overline{H}_i^T & 0 \\ 0 & 0 \end{bmatrix} + \varepsilon Z + \begin{bmatrix} 0 & 0 \\ 0 & \overline{E}_i^T\overline{E}_i \end{bmatrix} < 0. \qquad (13)$$

Substituting (12) into (13) and applying Schur complement, it gives

$$\begin{bmatrix} -P^{-1} + \varepsilon\gamma^2\overline{H}_i\overline{H}_i^T & \overline{A}_i \\ \overline{A}_i^T & -P + (1/\varepsilon)\overline{E}_i^T\overline{E}_i \end{bmatrix} \le 0 \qquad (14)$$

which is equal to

$$\begin{bmatrix} -P^{-1} & \overline{A}_i & \overline{H}_i \\ \overline{A}_i^T & -P + (1/\varepsilon)\overline{E}_i^T\overline{E}_i & 0 \\ \overline{H}_i^T & 0 & 1/(\varepsilon\gamma^2) \end{bmatrix} \le 0 \qquad (15)$$

Let $\tau = 1/\varepsilon$ and $\nu = 1/\gamma^2$, then Eq. (8) holds.

Sufficiency: Suppose that there exists a constant $\tau > 0$ such that (8) has a solution $P > 0$. Then there exists a constant $\varepsilon > 0$ such that (11) is feasible.

|For $\forall \eta_1, \eta_2 \in \Re^n$, $\eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \ne 0$, and any constant $\varepsilon > 0$, then

$$0 \le \eta^T \begin{bmatrix} \sqrt{\varepsilon}\overline{H}_i\overline{F}_i \\ -\overline{E}_i^T/\sqrt{\varepsilon} \end{bmatrix} \begin{bmatrix} \sqrt{\varepsilon}\overline{F}_i^T\overline{H}_i^T & -\overline{E}_i/\sqrt{\varepsilon} \end{bmatrix}\eta$$
$$= \eta^T \begin{bmatrix} \varepsilon\overline{H}_i\overline{F}_i\overline{F}_i^T\overline{H}_i^T & 0 \\ 0 & 1/\varepsilon\overline{E}_i^T\overline{E}_i \end{bmatrix}\eta - \eta^T \begin{bmatrix} 0 & \overline{H}_i\overline{F}_i\overline{E}_i \\ \overline{E}_i^T\overline{H}_i^T\overline{F}_i^T & 0 \end{bmatrix}\eta$$
$$\le \eta^T \begin{bmatrix} \varepsilon\gamma^2\overline{H}_i\overline{H}_i^T & 0 \\ 0 & 1/\varepsilon\overline{E}_i^T\overline{E}_i \end{bmatrix}\eta - \eta^T \begin{bmatrix} 0 & \overline{H}_i\overline{F}_i\overline{E}_i \\ \overline{E}_i^T\overline{H}_i^T\overline{F}_i^T & 0 \end{bmatrix}\eta$$

Therefore,

$$\begin{bmatrix} \varepsilon\gamma^2\overline{H}_i\overline{H}_i^T & 0 \\ 0 & (1/\varepsilon)\overline{E}_i^T\overline{E}_i \end{bmatrix} \ge \begin{bmatrix} 0 & \overline{H}_i\overline{F}_i\overline{E}_i \\ \overline{E}_i^T\overline{F}_i^T\overline{H}_i^T & 0 \end{bmatrix}.$$

Substituting the above equation into (14) yields

$$0 > \begin{bmatrix} \varepsilon\gamma^2\overline{H}_i\overline{H}_i^T & 0 \\ 0 & (1/\varepsilon)\overline{E}_i^T\overline{E}_i \end{bmatrix} + \begin{bmatrix} -P^{-1} & \overline{A}_i \\ \overline{A}_i^T & -P \end{bmatrix}$$
$$\ge \begin{bmatrix} -P^{-1} & \overline{A}_i + \overline{H}_i\overline{F}_i\overline{E}_i \\ \overline{A}_i^T + \overline{E}_i^T\overline{F}_i^T\overline{H}_i^T & -P \end{bmatrix}$$

Therefore, it shows that (11) holds.

To obtain the maximum value of the uncertainty bound, (15) can be written as a nonlinear optimisation problem described as in the theorem. The minimum value of ν leads to maximum γ.

The proof is completed. ◊

NPCS without time-delay is a special case of constant time-delay. Theorem 1 still works well but the corresponding matrices $\overline{A}_0 \in \Re^{((2n+m)N+2n)\times(2nN+mN+2n)}$, $\overline{H}_0 \in \Re^{((2n+m)N+2n)\times(3N+2)q}$ and $\overline{E}_0 \in \Re^{(3N+2)q\times((2n+m)N+2n)}$ in (7) have the following form:

$$\overline{A}_0 = \begin{bmatrix} [A \quad 0_{n\times(m+n)N} \quad BK \quad 0_{n\times nN}] \\ [I_{nN} \quad 0_{nN\times(nN+mN+2n)}] \\ [0_{m\times(Nn+Nm+n)} \quad K \quad 0_{m\times nN}] \\ [0_{m(N-1)\times(N+1)n} \quad I_{m(N-1)} \quad 0_{m(N-1)\times((N+1)n+m)}] \\ [LC \quad 0_{n\times(m+n)N} \quad A+BK-LC \quad 0_{n\times nN}] \\ [0_{nN\times(mN+nN+n)} \quad I_{nN\times nN} \quad 0_{nN\times n}] \end{bmatrix},$$

$$\overline{H}_0 = \begin{bmatrix} [H_1 \quad 0_{n\times 2qN} \quad H_2 \quad 0_{n\times qN}] \\ 0_{((2n+m)N+n)\times(3N+2)q} \end{bmatrix},$$

$$\overline{E}_0 = \begin{bmatrix} [E_1 \quad 0_{q\times((2n+m)N+n)}] \\ 0_{2Nq\times((2n+m)N+2n)} \\ [0_{q\times(n(N+1)+Nm)} \quad E_2 \quad 0_{q\times nN}] \\ 0_{Nq\times((2n+m)N+2n)} \end{bmatrix}.$$

3. ROBUSTNESS ANALYSIS OF NPCS WITH RANDOM TIME-DELAY

In practical networked control systems, the network induced time-delay size is usually random as network load changes. Based on the assumptions set in section 2, the time-delay is assumed to vary randomly in a set $\{0,1,2,\cdots N\}$. For this case, a similar result is derived as for the case of a constant time-delay.

Theorem 2: The corresponding closed-loop system of system (1) with random time delay (in $\{0,1\cdots,N\}$) is robustly quadratically stable if and only if there exist a common matrix $P = P^T > 0$ and a controller matrix K satisfying all the following inequalities:

$$\Pi_i = \begin{bmatrix} -P^{-1} & \overline{A}_i & \overline{H}_i \\ \overline{A}_i^T & -P + \tau\overline{E}_i^T\overline{E}_i & 0 \\ \overline{H}_i^T & 0 & -\tau\nu I \end{bmatrix} \le 0. \qquad (16)$$

$$for \quad i = 0,1,2,\cdots N$$

where $\nu = 1/\gamma^2$ and constant $\tau \ge 0$. Furthermore, if all the inequalities in (16) holds, then the maximum γ is defined by $1/\nu$; where ν is determined by the optimisation problem:

$$\begin{aligned} &\min \quad \nu \\ &subject \quad to \quad P = P^T > 0 \quad and \quad (16). \end{aligned} \qquad (17)$$

Proof: To robustly stabilise Eq. (1) with random network induced time-delay in the feedback channel, the following cases should be considered:

Case 1: At any time instant t, augmented closed-loop system of (7) is robustly quadratically stable for constant time-delay i.

Case 2: For time instants t to $t+1$, augmented closed-loop system (7) is still robustly quadratically stable when time-delay varies from i to j (Without loss of any generality, it assumes that $i \ne j$).

For the case of constant time-delay, theorem 1 has been proved Case 1, i.e. if the inequalities in (16) are feasible, then the corresponding augmented closed-loop system of (1) is robustly stable at any time instant t for any constant time-delay i $(0 \le i \le N)$.

Therefore, there exists a matrix $P = P^T > 0$ such that the following inequality holds

$$\begin{aligned} &V(\overline{x}_{t+1}) - V(\overline{x}_t) = \overline{x}_{t+1}^T P\overline{x}_{t+1} - \overline{x}_t^T P\overline{x}_t \\ &= \overline{x}_t^T\left((\overline{A}_i + \overline{H}_i\overline{F}_i\overline{E}_i)P(\overline{A}_i + \overline{H}_i\overline{F}_i\overline{E}_i) - P\right)\overline{x}_t < 0 \end{aligned}$$

Assume that at any time instant t, network induced time-delay is i and at time instant $t+1$, it is j (Here $i, j \in [0,\cdots,N]$ are any integer in the interval). Then system (1) is robustly stabilised when network induced time-delay varies from i to j, if there exists a matrix $P = P^T > 0$ such that the following inequality holds:

$$V(\overline{x}_{t+2}) - V(\overline{x}_t) = V(\overline{x}_{t+2}) - V(\overline{x}_{t+1}) + V(\overline{x}_{t+1}) - V(\overline{x}_t) < 0 \qquad (18)$$

For any time instant t

$$\begin{aligned} &V(\overline{x}_{t+2}) - V(\overline{x}_{t+1}) = \overline{x}_{t+2}^T P\overline{x}_{t+2} - \overline{x}_{t+1}^T P\overline{x}_{t+1} \\ &= \overline{x}_{t+1}^T(\overline{A}_j + \overline{H}_j\overline{F}_j\overline{E}_j)P(\overline{A}_j + \overline{H}_j\overline{F}_j\overline{E}_j)\overline{x}_{t+1} - \overline{x}_{t+1}^T P\overline{x}_{t+1} \end{aligned}$$

For $\prod_j \le 0$, $(\overline{A}_j + \overline{H}_j\overline{F}_j\overline{E}_j)P(\overline{A}_j + \overline{H}_j\overline{F}_j\overline{E}_j) - P < 0$.

Therefore

$$V(\overline{x}_{t+2}) - V(\overline{x}_t) < 0 \qquad (19)$$

From (19), it follows that (18) holds. Therefore, it can be concluded that for random network induced time delay, systems (7) is robustly quadratically stabilised if and only if there exist a common matrix $P = P^T > 0$ and matrices K and L satisfying all the inequalities given in (16).

As the proof of Theorem 1, Eq. (16) can be further converted to an optimisation control problem (17) but the corresponding matrix must satisfy all the inequalities in (16). The proof is completed. ◊

4. NUMERICAL EXAMPLES

In this part, a numerical example is constructed to demonstrated the proposed method dealing with the robust control problem of NPCS. An open-loop unstable discrete uncertain system in the form of (1) is described with following matrices:

$$A = \begin{bmatrix} -1.100 & 0.271 & 0.488 \\ -0.482 & 0.100 & 0.240 \\ -0.002 & 0.3681 & 0.707 \end{bmatrix}, B = \begin{bmatrix} 5 & 5 \\ 3 & -2 \\ 5 & 4 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 4 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} 0.11 & -0.10 \\ 0.10 & -0.11 \\ -0.10 & 0.10 \end{bmatrix}, H_2 = \begin{bmatrix} -0.11 & -0.10 \\ 0.13 & 0.11 \\ -0.10 & 0.10 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.15 & -0.15 & 0.15 \\ 0.15 & -0.15 & 0.15 \end{bmatrix}, E_2 = \begin{bmatrix} -0.8 & -0.6 \\ 0.8 & -0.7 \end{bmatrix}.$$

Here, the time-delay is assumed to be no greater than $N = 3$ and assume time-delay varies in the set $\{0,1,2,3\}$ randomly. Solving the combined optimisation problem Π_0, Π_1, Π_2 and Π_3 on the condition of $P = P^T > 0$, the corresponding matrices of the closed-loop system is obtained as follows:

$$K = \begin{bmatrix} -0.1003 & -0.0849 & -0.0333 \\ 0.2347 & 0.0267 & 0.0056 \end{bmatrix},$$

$$L = \begin{bmatrix} -0.2731 & -0.0368 \\ -0.5282 & 0.3776 \\ 0.0367 & 0.0558 \end{bmatrix}$$

and the optimisation specification $\nu = 4.004$. Then the domain of uncertainties is a super circle with a radius $\gamma = 1/\sqrt{\nu} = 0.4998$. This illustrates the validation of the method developed in this paper.

5. CONCLUSIONS

The NCS with structured uncertainties has been studied in this paper. The idea of networked predictive control is used to handle the network induced time-delay of NCS with uncertainties. The definition of robustly quadratically stability is given first, and the problem of robust stabilisation (i.e. the corresponding closed-loop systems is robustly quadratically stable) of NCS with constant network induced time-delay is formulated as a constrained non-linear optimisation problem. The question of finding the related control law and bound of uncertainty is integrated into the non-linear optimisation problem. The problem of NCS with random network induced time-delay is formulated as a non-linear optimisation problem constrained by a set of non-linear inequalities with a common Lyapunov matrix. This method is validated by a numerical example.

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