SPIDERCRANE: MODEL AND PROPERTIES OF A FAST WEIGHT HANDLING EQUIPMENT

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Abstract: A new crane design, labeled "SpiderCrane", is proposed to handle fast load displacements. Its main particularity is the absence of heavy mobile components. The system beeing underactuated, its control is challenging. Hence, as a step towards easier and more efficient control, this paper proposes a dynamic model and investigates the two control-relevant properties of flatness and observability. SpiderCrane is shown to be flat with respect to certain outputs and observable from the motors positions.

Keywords: Crane Control, Modeling, Underactuated Mechanical Systems, Flatness, Observability.

1. INTRODUCTION

The stabilization of loads that are carried by cranes is tedious, and the lack of truly efficient strategies implies a large economical loss due to the additional time involved in the process. In various industries such as construction or naval transport, the crane drivers move the load in a quasi-static way, i.e. by keeping the cable vertical in order not to induce oscillations. To improve the work rate, it is necessary to abandon the quasi-static approach and introduce a control law that can cope with the dynamic couplings. The problem of classical cranes is the large inertia of the boom, which limits the crane dynamics. Hence, this paper proposes a new crane design, labeled SpiderCrane, that is devoid of heavy mobile components. As a result, SpiderCrane can work at considerably higher speeds.

From a control theoretical point of view, cranes, and in particular SpiderCrane, are underactuated mechanical systems, which gives rise to challenging control issues (Gustafsson, 1996). For instance, the control of these systems tend to generate non-asymptotically-stable internal dynamics. This is the case when dynamic inversion techniques are used with poorly chosen outputs. For instance, if all motors are rigidly blocked, the load, together with its cable, will move freely around the blocked position. Hence, a control strategy that inverts the dynamics using the position of the motors as outputs will fail due to the presence of non-asymptotically-stable zero dynamics corresponding to this oscillatory behavior. However, the flatness formalism (Fliess *et al.*, 1999) is ideally suited to handle the motion planning problem: Trajectory generation and input computation are performed without integrating differential equations. Auspiciously, part of the flat outputs corresponds to the three coordinates of the load, the position of which is to be steered.

Although it is straightforward and reliable to measure the cable position with the winching motors, acquiring usefull information about the main cable angles (unactuated coordinates) is much more difficult, especially in certain hostile environments, where bad weather conditions and obscurity renders sophisticated and delicate optical devices obsolete. Thus, it is of great avail to be able to reconstruct the missing positions on the basis of a dynamic model. Hence, the importance of establishing the observability of all variables based on measurements obtained from the winches.

The main point of view adopted in this study, i.e. using the position of the load for feedforward computations and the position of the motors for feedback purposes, is not new in the context of crane control. For instance, a simple output feedback strategy (without observer) was proposed in (Kiss *et al.*, 2000) and developed further in (Fang *et al.*, 2001) for equilibrium stabilization. The general crane modeling approach proposed in (Kiss *et al.*, 1999) provided considerable insight for designing SpiderCrane.

The paper is organized as follows. Section 2 and 3 describe the system and the model, respectively. Section 4 and 5 discuss the control-relevant flatness and observability properties. Simulations are shown in Section 6, and conclusions and future work are addressed in Section 7.

2. SPIDERCRANE DESIGN

SpiderCrane is made of three fixed pylons and a fixed gibbet. A pulley is mounted at the top of each pylon, allowing the sliding of a cable. These three cables are attached to a ring, and by varying their length, the ring can be moved in the surrounding space. The end of the gibbet is above the plane formed by the three pulleys and at the centre of the triangle formed by the pylons. At the end of the gibbet, another pulley is mounted, allowing the passage of the main cable. This cable goes through the centre of the ring and is attached to the load. The positioning of the load in space is done by adjusting both the positioning of the ring and the length of the main cable. All the cables are controlled by means of DC motors equipped with encoders, making it possible to measure the length as well as the speed of the cables.

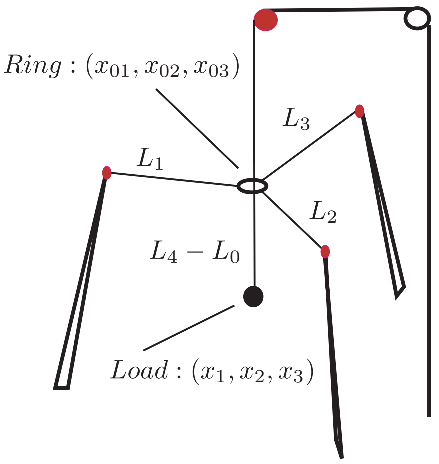

Fig. 1. SpiderCrane

3. MODELING

3.1 Notation

The position of the load of mass m is given by (x_1, x_2, x_3), that of the ring of mass m_0 by (x_{01}, x_{02}, x_{03}). The position of the main motor is (x_{41}, x_{42}, x_{43}) and its equivalent inertia m_4. The positions of the three secondary motors are (x_{11}, x_{12}, x_{13}), (x_{21}, x_{22}, x_{23}) and (x_{31}, x_{32}, x_{33}), respectively, and their equivalent inertias with respect to the cables m_1, m_2 and m_3. The length of the cable connecting the main motor to the load is L_4, with the portion going from the motor to the ring being L_0. The lengths of the cables connecting the secondary motors to the ring are given by L_1, L_2 and L_3, respectively. The four motors are torque-controlled and thus provide directly the forces T_1, T_2, T_3 and T_4.

3.2 Dynamics

Tools of analytical mechanics are used to obtain the dynamic equations of SpiderCrane. First, we define a set q of coordinates, more numerous than the minimal set of generalized coordinates:

$$q = (x_1, x_2, x_3, x_{01}, x_{02}, x_{03}, L_0, L_1, L_2, L_3, L_4)$$

This set of coordinates is constrained by a set of holonomic constraints

$$C_1 = \sum_{i=1}^{3}(x_i - x_{0i})^2 - (L_4 - L_0)^2 = 0 \qquad (1)$$

$$C_j = \sum_{i=1}^{3}(x_{0i} - x_{ji})^2 - L_j^2 = 0 \quad j = 2...4 \quad (2)$$

$$C_5 = \sum_{i=1}^{3}(x_{0i} - x_{4i})^2 - L_0^2 = 0 \qquad (3)$$

which describe the geometric relationship between the position of the crane components and the length of the cables.
The external forces acting in the directions described by q are given by the motors

$$F_{ext} = (0, 0, 0, 0, 0, 0, 0, T_1, T_2, T_3, T_4)$$

Classical Lagrange method cannot be used to obtain the dynamic equations because the set of coordinates is not a set of generalized coordinates. Thus, a Lagrange formalism with constraints is used as in the case of non-holonomic constraints (Greenwood, 1977). It applies directly to SpiderCrane:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \sum_{j=1}^{5} \lambda_j \frac{\partial C_j}{\partial q_i} + F_{ext-i} \qquad i = 1, ..., 11 \qquad (4)$$

where λ_j are the Lagrange multipliers and L is the Lagrangian, i.e. the difference between kinetic and potential energy:

$$L = W_{kin} - W_{pot} \qquad (5)$$

For SpiderCrane, the kinetic energy is given by:

$$W_{kin} = \frac{1}{2}(\sum_{i=1}^{3}(m\dot{x_i}^2 + m_0\dot{x}_{0i}^2) + \sum_{i=1}^{4} m_i\dot{L_i}^2 \quad (6)$$

and the potential energy by:

$$W_{pot} = mgx_3 + m_0gx_{03} \quad (7)$$

Introducing (5), (1), (2) and (3) into (4), gives

$$m\ddot{x_1} = (x_1 - x_{01})\lambda_1, \quad (8)$$
$$m\ddot{x_2} = (x_2 - x_{02})\lambda_2, \quad (9)$$
$$m\ddot{x_3} = (x_3 - x_{03})\lambda_3 - gm, \quad (10)$$
$$m_0\ddot{x_{01}} = (x_{01} - x_1)\lambda_1 + (x_{01} - x_{11})\lambda_2 + (x_{01} - x_{21})\lambda_3 + (x_{01} - x_{31})\lambda_4 + (x_{01} - x_{41})\lambda_5, \quad (11)$$
$$m_0\ddot{x_{02}} = (x_{02} - x_2)\lambda_1 + (x_{02} - x_{12})\lambda_2 + (x_{02} - x_{22})\lambda_3 + (x_{02} - x_{32})\lambda_4 + (x_{02} - x_{42})\lambda_5, \quad (12)$$
$$m_0\ddot{x_{03}} = (x_{03} - x_3)\lambda_1 + (x_{03} - x_{13})\lambda_2 + (x_{03} - x_{23})\lambda_3 + (x_{03} - x_{33})\lambda_4 + (x_{03} - x_{43})\lambda_5 - gm_0, \quad (13)$$
$$0 = (L_4 - L_0)\lambda_1 - L_0\lambda_5, \quad (14)$$
$$m_1\ddot{L_1} = T_1 - L_1\lambda_2 - L_0 \quad (15)$$
$$m_2\ddot{L_2} = T_2 - L_2\lambda_3 - L_0 \quad (16)$$
$$m_3\ddot{L_3} = T_3 - L_3\lambda_4 - L_0 \quad (17)$$
$$m_4\ddot{L_4} = T_4 + (L_0 - L_4)\lambda_1 \quad (18)$$

These equations, together with (1)- (3), result in a set of differential algebraic equations (DAE) describing the process. Standard integration techniques can be used (Gear and Petzold, 1984). Here, however, it is sufficient to express the Lagrange multipliers with the help of the holonomic constraints: differentiating the constraints twice and introducing the dynamic equations, one can solve for the Lagrange multipliers. The constraints remain satisfied throughout the simulation if the initial conditions satisfy them.

4. FLATNESS

It is shown in (Kiss *et al.*, 1999) that any object belonging to the crane class verifies the flatness property. The SpiderCrane belonging to this class, it should be no exception. However the demonstration given therein is somewhat not trivial and examples presented quite succint. Therefore the exposition will be given in full breadth herafter.

Definition 1. A system $\dot{x} = f(x, u)$ with $u \in \mathbb{R}^m$ and $x \in \mathbb{R}^n$ is said to be flat if there exists an output $y \in \mathbb{R}^n$ such that:
- the components of y are independent;
- x and u can be expressed as functions of y and its derivatives up to the r-th order
$x = \mathcal{F}(y, ..., y^{(r-1)}) \quad u = \mathcal{P}(y, ..., y^{(r)}) \quad r \in \mathbb{N}$
with $\mathcal{F}$ and $\mathcal{P}$ satisfying identically $\dot{\mathcal{F}} = f(\mathcal{F}, \mathcal{P})$

In the case of SpiderCrane, one has:

$$x = (x_1, x_2, x_3, x_{01}, x_{02}, x_{03}, L_0, L_1, L_2, L_3, L_4 \;\; \dot{x_1}, \dot{x_2}, \dot{x_3}, \dot{x_{01}}, \dot{x_{02}}, \dot{x_{03}}, \dot{L_0}, \dot{L_1}, \dot{L_2}, \dot{L_3}, \dot{L_4})$$
$$y = (x_1, x_2, x_3, x_{03})$$
$$u = (T_1, T_2, T_3, T_4)$$

Using (8), (9) and (10), x_{01}, x_{02} and λ_1 can be expressed as:

$$x_{01} = x_1 - \frac{m\ddot{x}_1}{\lambda_1} = \mathcal{F}_1(x_1, \ddot{x}_1) \quad (19)$$
$$x_{02} = x_2 - \frac{m\ddot{x}_2}{\lambda_2} = \mathcal{F}_2(x_2, \ddot{x}_2) \quad (20)$$
$$\lambda_1 = \frac{m\ddot{x}_3 + gm}{x_3 - x_{03}} = \mathcal{F}_3(x_3, x_{03}, \ddot{x}_3) \quad (21)$$

Differentiating (19) and (20) gives:

$$\dot{x}_{01} = \mathcal{F}_4(x_1, \dot{x}_1, ..., x_1^{(3)}) \quad (22)$$
$$\dot{x}_{02} = \mathcal{F}_5(x_2, \dot{x}_2, ..., x_3^{(3)}) \quad (23)$$

Solving the constraint equations (1)-(3) for L_j with $j = 0, ..., 4$ and using (19) and (20) leads to

$$L_j = \mathcal{F}_{6+j}(x_1, \ddot{x}_1, x_2, \ddot{x}_2, x_3, \ddot{x}_3, x_{03}) \quad j = 0, ..., 4 \quad (24)$$

Time differentiation of (24) gives:

$$\dot{L}_j = \mathcal{F}_{11+j}\ (x_1, ..., x_1^{(3)}, x_2, ..., x_2^{(3)}, x_3, ..., x_3^{(3)}, x_{03}, \dot{x}_{03}) \quad j = 0, ..., 4. \quad (25)$$

Equations (19)-(25) establish that the states can be expressed as functions of the chosen outputs and their derivatives.
Now, it remains to express the inputs as functions of the outputs and their derivatives and, for this purpose, (22), (23) and (25) need to be differentiated:

$$\ddot{x}_{01} = \mathcal{F}_{16}(x_1, \dot{x}_1, ..., x_1^{(4)}) \quad (26)$$
$$\ddot{x}_{02} = \mathcal{F}_{17}(x_2, \dot{x}_2, ..., x_2^{(4)}) \quad (27)$$
$$\ddot{L}_j = \mathcal{F}_{17+j}(x_1, ..., x_1^{(4)}, x_2, ..., x_2^{(4)}, x_3, ..., x_3^{(4)}, x_{03}, ..., \ddot{x}_{03}) \quad j = 0, ..., 4 \quad (28)$$

Solving (11)-(14) for λ_2, λ_3, λ_4 and λ_5, and using (19), (20), (21), (24), (26) and (27), gives:

$$\lambda_{1+i} = \mathcal{P}_i \ (x_1, ..., x_1^{(4)}, x_2, ..., x_2^{(4)}, x_3, ..., x_3^{(4)}, \\ x_{03}, ..., \ddot{x}_{03}) \qquad i = 1, ..., 4 \qquad (29)$$

Finally, solving (15)-(18) for T_1, T_2, T_3 and T_4, and using (21), (24), (25), (28) and (29), results in:

$$T_i = \mathcal{P}_{4+i} \ (x_1, ..., x_1^{(4)}, x_2, ..., x_2^{(4)}, x_3, ..., x_3^{(4)}, \\ x_{03}, ..., \ddot{x}_{03}) \qquad i = 1, ..., 4 \qquad (30)$$

The condition expressing the inputs as functions of only the outputs and their derivatives is then satisfied. Hence, SpiderCrane is a flat system.

Remark 1. Expressions (30) allow computing in a simple manner the inputs to be applied to SpiderCrane in order to move the load along a prescribed trajectory.

5. OBSERVABILITY

In the nonlinear definition of observability according to Hermann and Krener (1977), a certain output function is given as a function of time $t \to y(t, 0, x(o), u)$. A state x_1 is called indistinguishable from x_2, if $y(t, 0, x_1, u) = y(t, 0, x_2, u)$ for every admissible input u. The system is observable if x_1 beeing indistinguishable from x_2 implies $x_1 = x_2$. This definition has a drawback when one seeks to go beyond the local, inasmuch as a long time could possibly be needed so as to distinguish x_1 from x_2. Additionaly the classical extension of the algebraic test existing for linear system ascertains observability only in a local sense. In the following definition, it is admitted that the output function y is known together with all its derivatives up to a certain fixed order. Then if the state can be determined *instanteneously* based on the knowledge of the aforementioned quantities, the system will be called observable.

Definition 2. A system $\dot{x} = f(x, u)$, $y = h(x)$ is observable if there exists a function $\Im$ such that $x = \Im(y_{mes}, ..., y_{mes}^{(r)}, u, ..., u^{(p)})$ and $\dot{\Im} = f(\Im, u)$, where x are the states, y_{mes} the measured outputs and u the inputs, $r, p \in \mathbb{N}$.

For the following states, outputs and inputs, it will be shown that SpiderCrane is observable according to Definition 2:

$$\begin{aligned} x &= (x_1, x_2, x_3, x_{01}, x_{02}, x_{03}, L_0, L_1, L_2, L_3, L_4 \\ &\quad \dot{x_1}, \dot{x_2}, \dot{x_3}, \dot{x_{01}}, \dot{x_{02}}, \dot{x_{03}}, \dot{L}_0, \dot{L_1}, \dot{L_2}, \dot{L_3}, L_4) \\ y_{mes} &= (L_1, L_2, L_3, L_4) \\ u &= (T_1, T_2, T_3, T_4) \end{aligned}$$

Using the constraints (2), we try to determine the ring position as a function of the lengths L_1, L_2 and L_3. This problem is equivalent to finding the intersection of 3 spheres centered on the secondary pulleys 1, 2 and 3 and with radius L_1, L_2 and L_3. This intersection is given by the two points 1 and 2 in Fig. 2, which is a well-known result in analytical geometry (Gabriel-Marie, 1996).

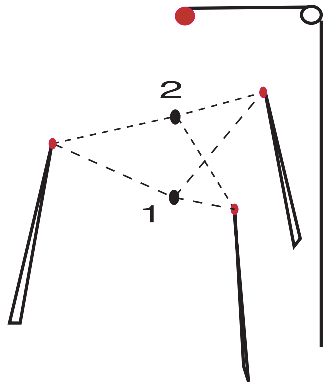

Fig. 2. The constraints give the ring position

Position 2 can be eliminated by inspection since, in this case, the cables need to push the ring load, which is an infeasible scenario. Hence, it is straightforward to express the ring position as a function of L_1, L_2 and L_3:

$$x_{0i} = \Im_1(L_1, L_2, L_3) \quad i = 1, ..., 3 \qquad (31)$$

Combining (3) with (31) gives:

$$L_0 = \Im_4(L_1, L_2, L_3, L_4) \qquad (32)$$

Remark 2. The solution of Constraint (3) provides two values for L_0, including a negative one. Since the length of the cable cannot be negative, only the positive solution will be considered.

Then, simple successive differentiations of (31) and (32) give:

$$\begin{aligned} \dot{x}_{0i} &= \Im_{4+i}(L_1, \dot{L}_1, L_2, \dot{L}_2, L_3, \dot{L}_3) \\ &\quad i = 1, ..., 3 \qquad (33) \\ \dot{L}_0 &= \Im_8(L_1, \dot{L}_1, L_2, \dot{L}_2, L_3, \dot{L}_3, L_4, \dot{L}_4) \\ \ddot{x}_{0i} &= \Im_{8+i}(L_1, ..., \ddot{L}_1, \ddot{L}_2, ..., \ddot{L}_2, L_3, ..., \ddot{L}_3) \\ &\quad i = 1, ..., 3 \qquad (34) \\ \ddot{L}_0 &= \Im_{12}(L_1, ..., \ddot{L}_1, L_2, ..., \ddot{L}_2, L_3, ..., \ddot{L}_3, \\ &\quad L_4, ..., \ddot{L}_4) \qquad (35) \end{aligned}$$

Injecting (32) into (14)-(18), and upon simple algebraic manipulations, the Lagrange multipliers can be expressed as:

$$
\begin{aligned}
\lambda_1 &= \frac{m_4\ddot{L}_4 - T_4}{\Im_4 - L_4} \\
&= \Im_{13}(L_1, L_2, L_3, L_4, \ddot{L}_4, T_4) \qquad (36) \\
\lambda_2 &= -\frac{m_1\ddot{L}_1 - T_1 + \Im_4}{L_1} \\
&= \Im_{14}(L_1, L_2, L_3, L_4, \ddot{L}_1, T_1) \qquad (37) \\
\lambda_3 &= -\frac{m_2\ddot{L}_2 - T_2 + \Im_4}{L_2} \\
&= \Im_{15}(L_1, L_2, L_3, L_4, \ddot{L}_2, T_2) \qquad (38) \\
\lambda_4 &= -\frac{m_3\ddot{L}_3 - T_3 + \Im_4}{L_3} \\
&= \Im_{16}(L_1, L_2, L_3, L_4, \ddot{L}_3, T_3) \qquad (39) \\
\lambda_5 &= \frac{m_4\ddot{L}_4 - T_4}{\Im_4} \\
&= \Im_{17}(L_1, L_2, L_3, L_4, \ddot{L}_4, T_4) \qquad (40)
\end{aligned}
$$

Remark 3. The Lagrange multipliers are not defined when the length of the corresponding cable tends towards zero. Physically, this means that singularities appear when: (i) the load position is identical to the ring position, (ii) the ring is at one of the secondary pulleys, or (iii) the load is at the main pulley.

From (11)-(13) and considering (31)-(34), the load position can be determined:

$$
\begin{aligned}
x_1 &= \frac{-m_0\Im_9 + (\Im_1 - x_{11})\Im_{14} + (\Im_1 - x_{21})\Im_{15}}{\Im_{13}} \\
&\quad + \frac{(\Im_1 - x_{31})\Im_{16} + (\Im_1 - x_{41})\Im_{17}}{\Im_{13}} - \Im_1, \qquad (41) \\
x_2 &= \frac{-m_0\Im_{10} + (\Im_2 - x_{12})\Im_{14} + (\Im_2 - x_{22})\Im_{15}}{\Im_{13}} \\
&\quad + \frac{(\Im_2 - x_{32})\Im_{16} + (\Im_2 - x_{42})\Im_{17}}{\Im_{13}} - \Im_2, \qquad (42) \\
x_3 &= \frac{-m_0\Im_{11} + (\Im_3 - x_{13})\Im_{14} + (\Im_3 - x_{23})\Im_{15}}{\Im_{13}} \\
&\quad + \frac{(\Im_3 - x_{33})\Im_{16} + (\Im_3 - x_{43})\Im_{17}}{\Im_{13}} \\
&\quad - \frac{gm_0}{\Im_{13}} - \Im_3. \qquad (43)
\end{aligned}
$$

Remark 4. Expressions (41)-(43) become singular when $\Im_{13}$ tends towards zero. It follows from (36), i.e. $\Im_{13} = \lambda_1 = \frac{m_4\ddot{L}_4 - T_4}{\Im_4 - L_4}$, that $m_4\ddot{L}_4 - T_4 \neq 0$ is necessary to avoid singularity. In order to interpret this condition physically, we rewrite (18) as $m_4\ddot{L}_4 - T_4 = (L_0 - L_4)\lambda_1$. Since $(L_0 - L_4)\lambda_1$ represents the tension in the main cable, the load position is unspecified when this tension is zero. This occurs either when the mass of the load vanishes or when the inertial force $m_4\ddot{L}_4$ compensates exactly the motor force T_4, a very rare situation.

By inspection of (41)-(43), we can conclude that the load position depends only on the cable lengths, their derivatives and the inputs T_1, T_2, T_3 and T_4:

$$
\begin{aligned}
x_i = \Im_{17+i}(&L_1, ..., \ddot{L}_1, L_2, ..., \ddot{L}_2, L_3, ..., \ddot{L}_3, \\
&L_4, ..., \ddot{L}_4, T_1, T_2, T_3, T_4) \\
&i = 1, ..., 3 \qquad (44)
\end{aligned}
$$

Finally, differentiating (41) and (43) gives:

$$
\begin{aligned}
\dot{x}_i = \Im_{20+i}(&L_1, ..., L_1^{(3)}, L_2, ..., L_2^{(3)}, L_3, ..., L_3^{(3)}, \\
&L_4, ..., L_4^{(3)}, T_1, \dot{T}_1, T_2, \dot{T}_2, T_3, \dot{T}_3, T_4, \dot{T}_4) \\
&i = 1, ..., 3 \qquad (45)
\end{aligned}
$$

Expressions (31)-(34), and (44)-(45) show that x_1, x_2, x_3, x_{01}, x_{02}, x_{03}, L_0, L_1, L_2, L_3, L_4, $\dot{x}_1$, $\dot{x}_2$, $\dot{x}_3$, $\dot{x}_{01}$, $\dot{x}_{02}$, $\dot{x}_{03}$, $\dot{L}_0$, $\dot{L}_1$, $\dot{L}_2$, $\dot{L}_3$, $\dot{L}_4$ can be reconstructed from L_1, L_2, L_3, L_4, their time derivatives, the inputs T_1, T_2, T_3, T_4 and their time derivatives. This confirms observability as per Definition 2.

6. SIMULATION

In order to illustrate the flatness property presented above, the behavior of SpiderCrane is evaluated in simulation for a displacement from an equilibrium point A to an equilibrium point B. Reference trajectories for the flat outputs are chosen constant for x_2, x_3, x_{03} and as a polynomial for x_1 (Fig. 3). To calculate the inputs, the flat outputs and their derivatives up to the 4^{th} order are needed according to (30). Thus, in order to construct the polynomial trajectory, not only the initial and final positions must be specified, but also all derivatives up to 4^{th} order. Hence, 10 conditions are enforced on the trajectory (5 initial conditions and 5 terminal conditions), thus making the minimal polynomial of order 9: $x(0)_{2Ref} = 0.8$, $\dot{x}(0)_{1Ref} = \ddot{x}(0)_{1Ref} = x(0)^{(3)}_{1Ref} = x(0)^{(4)}_{1Ref} = 0$, $x(T_f = 1)_{1Ref} = 0.2$, $\dot{x}(1)_{1Ref} = \ddot{x}(0)_{1Ref} = x(1)^{(3)}_{1Ref} = x(1)^{(4)}_{1Ref} = 0$.

Fig.4.i represents a slow quasi-static displacement that takes 10 sec. Fig.4.ii illustrates the same displacement in a much faster mode (1 sec) using the inputs T_1, T_2, T_3 and T_4 calculated from (30). A comparison of the two figures indicates that it is necessary to use the ring dynamics in a more efficient way to improve the speed of displacement. On the one hand, when a quasi-static displacement is performed, the ring position is almost

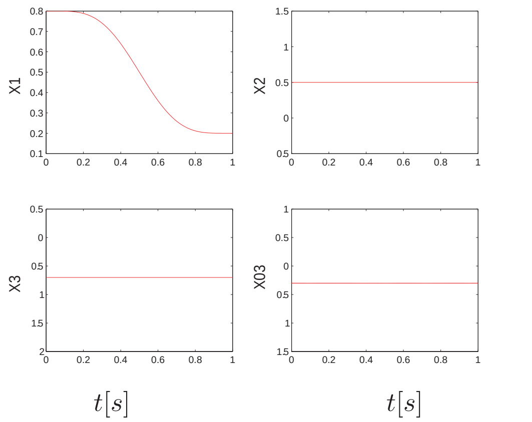

Fig. 3. Reference trajectories for the flat outputs

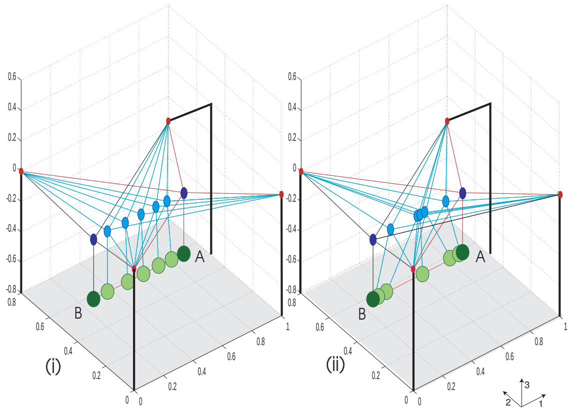

Fig. 4. Displacement from point A to B. (i) In a quasi-static manner (10 sec).(ii) In a highly dynamic way (1 sec)

vertical at the load's position ($x_1 \cong x_{01}, x_2 \cong x_{02}$). Hence the control law is just given by the geometric constraints. On the other hand, for a fast load displacement, the control law needs to manage the strong inherent dynamical couplings. This results in a complex displacement of the ring. For instance, Fig.5 gives the system states and inputs for a fast displacement. The reference trajectories are perfectly tracked by SpiderCrane.

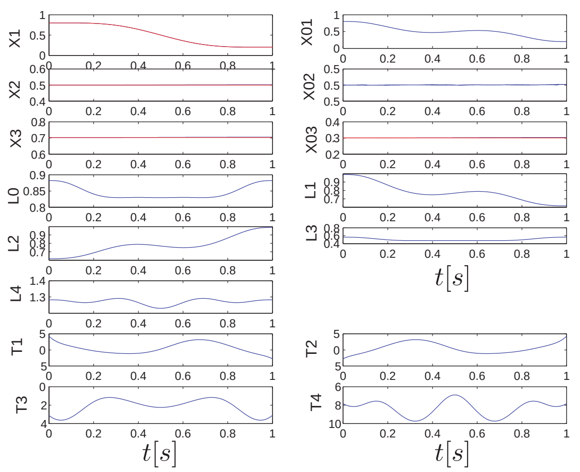

Fig. 5. States and inputs for a fast displacement.

7. CONCLUSION

SpiderCrane is a new crane design allowing fast displacements of the load. Its flatness and observability properties provide several advantages. First, thanks to flatness, motion planning (i.e. trajectory generation and computation of the corresponding inputs) is achieved with ease. Secondly, observability gives the possibility to construct monitoring tools directly from the motor sensors, i.e. without having to rely on a position measurement using, for example, a camera.
Several research alleys can be envisioned from this initial work such as the possibility of rejecting a disturbance (e.g. a strong gale) by synchronizing the load on a reference trajectory. Furthermore, the construction of a laboratory-scale model of SpiderCrane could be useful in ascertaining the research done so far and exploring new paths.

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