DISCRETE-TIME H_∞ MODEL MATCHING PROBLEM IN TWO DEGREES OF FREEDOM CONTROL STRUCTURE

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Abstract: In this study, the model-matching problem (MMP) in two degrees of freedom (2DOF) control structure is considered for discrete-time system in the sense of the H_∞ optimality criterion. The problem is solved in Linear Matrix Inequality (LMI) framework using the results on the standard H_∞ OCP and the recent results given by Gören (2003) on the MMP in 2DOF control structure for continuous-time systems.

Keywords: Model Matching Problem; Linear Matrix Inequalities; H_∞ Optimal Control; 2DOF Control Structure; Discrete-Time Systems.

1. INTRODUCTION

The standard H_∞ MMP (Francis & Doyle 1987) is defined as to find a controller transfer matrix $\boldsymbol{R(s)}$ which is stable and proper rational matrix, i.e., $\boldsymbol{R(s)\in RH_\infty}$, that further minimizes the H_∞ norm of $\boldsymbol{T_m(s)-T_1(s)R(s)T_2(s)}$, where the stable and proper rational matrices $\boldsymbol{T_m(s)}$ and $\boldsymbol{[T_1(s),\ T_2(s)]}$ are the model and the system transfer matrices, respectively. The H_∞ norm of a transfer matrix is defined as the maximum value over all frequencies of its largest singular value. This means the performance of the system described by $\boldsymbol{T_1(s)R(s)T_2(s)}$ approximates the desired performance as given in $\boldsymbol{T_m(s)}$, in the sense of the following criterion,

$$\gamma_{opt} = \inf_{R(s)\in\Re H_\infty} \left\{ \left\| T_m(s) - T_1(s)R(s)T_2(s) \right\|_\infty \right\} \quad (1)$$

While this problem which is also known as the bilateral H_∞ MMP, the unilateral H_∞ MMP is defined as to find a controller transfer matrix $\boldsymbol{R(s)\in RH_\infty}$ that minimizes the H_∞ norm of $\boldsymbol{T_m(s)-T(s)R(s)}$, where the stable and proper rational matrices $\boldsymbol{T_m(s)}$ and $\boldsymbol{T(s)}$ are the model and the system transfer matrices respectively. In the literature, there are several results on the standard H_∞ MMP. Two of them are based on Nevanlinna-Pick Problem (NPP) (Doyle, Francis & Tannenbaum, 1992) and Nehari Problem (NP) (Francis, 1987 and Francis & Doyle, 1987). In these studies, the H_∞ MMP has been reduced to the one of these problems and then using the results on the solution of NPP or NP, first the value γ_{opt} defined in (1) is found and finally the controller transfer matrix $\boldsymbol{R(s)}$ is obtained in the form of stable and proper rational matrix. Some studies concerning H_∞ MMP are considered in the concept of the standard H_∞ Optimal Control Problem (OCP). A complete state space solution to the standard H_∞ OCP is given by Doyle, Glover, Khargonekar&Francis (DGKF, 1989). The relationships between model matching problem and DGKF solution for generalized plant setting has been investigated by Green, Glover, Limebbeer and Doyle (1990) via J-spectral factorisation theory. A state space solution of the unilateral H_∞ MMP is given by Hung (1989), and this solution is based on canonical spectral factorisations and solutions of the Algebraic Riccati Equations (ARE). Gahinet & Apkarian (1994) re-derived the solution of the standard H_∞ OCP given by DGKF in the framework of LMI. In Gören & Akın (2002), an LMI-based solution of the unilateral H_∞ MMP is presented; also a solution of a multi-objective H_∞ control problem is obtained using the results given in Gahinet & Apkarian (1994). In all these studies on the standard H_∞ MMP, the controller structures that could be used in feedback configuration have not been considered in the formulation of the problem. However, one can say that the controller $\boldsymbol{R(s)}$ with property of stable and causal rational matrix, which is found in the form of a pre-compensator as a solution of the unilateral H_∞ MMP, can generally be established by dynamic state feedback (Kucera, 1992 and Gören & Akın, 2002).

To consider the control structures for the system to be controlled in the formulation of the problem, the H_∞ MMP is defined as find a controller minimizing the H_∞ norm of $T_m(s)$-$T_{cl}(s)$ in the specific control structure, such that the rational matrices $T_m(s)$ and $T_{cl}(s)$ are the model and the closed loop system transfer matrices respectively. In Gören (2003), the continuous-time H_∞ MMP in the 2DOF control structure is considered and the LMI-based solvability conditions of the problem are presented using the results given in Apkarian & Gahinet 1994, and Gören & Akın 2002.

In recent years, there has been a rapid increase in the use of industrial digital controllers. With the complete continuous-time synthesis theory, a natural approach to discrete-time synthesis problem is to transform the continuous-time controller into discrete-time via the bilinear transformation. Although this procedure will lead to controller formulas for the discrete-time synthesis problem, it has a number of disadvantages. Especially, in practical implementation, choosing the sampling rate gives rise to difficulties when the continuous controller has both high and low speed modes. In addition, there are several theoretical advantages to be gained from a solution in "natural coordinates" (Green&Limebeer, 1995). Moreover, in industrial control situations wherever a digital computer is used to monitor and to control a system, the discrete-time framework is usually a very natural one in which to give a system model description.

In this study, the method developed in Gören (2003) for continuous-time systems will be transferred to discrete-time context and will lead to qualitatively similar results. The following notation will be used throughout the paper: ***Ker M*** and ***Im M*** denote the null space and range of the linear operator associated with M respectively and N^* for the transpose conjugate of N matrix. Finally, $P>0$ denotes that P matrix is positive definite.

2. PROBLEM FORMULATION

Consider a realization (A, B, C, D) of $T(z)$, namely the discrete-time model of the system to be controlled, and (F, G, H, J) of $T_m(z)$, namely model system, so the state space equations of these systems are given as follows,

$$T(z); \quad \begin{aligned} x(t+1) &= A\,x(t) + B\,u(t) \\ y_s(t) &= C\,x(t) + D\,u(t) \end{aligned} \tag{2}$$

$$T_m(z); \quad \begin{aligned} q(t+1) &= F\,q(t) + G\,w(t) \\ y_m(t) &= H\,q(t) + J\,w(t) \end{aligned} \tag{3}$$

Also consider the dynamic 2DOF control structure with output feedback, where the control input $u(t)$ is generated by the reference input $w(t)$ and the system output $y_s(t)$ such that,

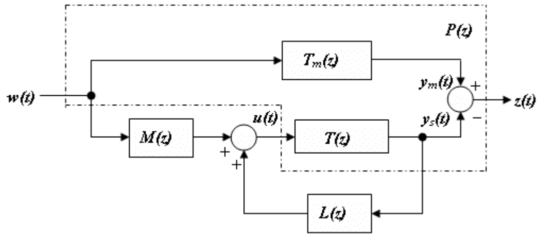

Figure 1

$$U(z) = L(z)Y_s(z) + M(z)W(z) \tag{4a}$$

in these equations $x(t) \in R^n$, $q \in R^{n_m}$, $u(t) \in R^m$, $w(t) \in R^{m_r}$, $y_s(t) \in R^p$, $y_m(t) \in R^p$, $t \in I$. These are illustrated in Figure 1.

At this point, H_∞ MMP in 2DOF control structure defined in Gören (2003), has to be modified for the discrete-time systems and this is done in definition 1.

Definition 1: The H_∞ MMP in 2DOF control structure is to find the controller transfer matrices $M(z), L(z) \in RH_\infty$ that minimizes the H_∞ norm of the transfer matrix $T_{zw}(z)$ defined as,

$$\begin{aligned} T_{zw}(z) &= T_m(z) - \left[I - T(z)L(z)\right]^{-1} T(z)M(z) = \\ &= T_m(z) - T(z)\left[I - L(z)T(z)\right]^{-1} M(z) \end{aligned}$$

such that the proper rational matrices $T_m(z)=H(zI-F)^{-1}+J$ and $T(z)=C(zI-A)^{-1}B+D$ are the model and the discrete-time system transfer matrices the respectively.□

Note that H_∞ norm of a stable discrete-time system is defined as,

$$\|G(z)\|_\infty = \sup_{|z|\geq 1} \bar{\sigma}\,(G(z))$$

3. THE SOLUTION of THE DISCRETE-TIME H_∞ MMP in 2DOF CONTROL STRUCTURE

In this section, the LMI based solvability conditions of the H_∞ MMP in 2DOF control structure is derived. For this purpose, consider a plant $P(z)$ described by,

$$\begin{bmatrix} x(t+1) \\ q(t+1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} \begin{bmatrix} x(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} w(t) + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) \tag{5}$$

$$\begin{aligned} z(t) &= y_m(t) - y_s(t) = \\ &\begin{bmatrix} -C & H \end{bmatrix} \begin{bmatrix} x(t) \\ q(t) \end{bmatrix} + J\,w(t) - D\,u(t) \end{aligned} \tag{6}$$

$$y(t) = \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} w(t) + \begin{bmatrix} D \\ 0 \end{bmatrix} u(t) \tag{7}$$

and a controller $K(z)$ defined as,

$$K(z) = \begin{bmatrix} L(z) & M(z) \end{bmatrix} \tag{8a}$$

As exploited in Figure 2, the closed loop transfer matrix $T_{zw}(z)$ is obtained according to Equation (9a),

$$
\begin{aligned}
T_{zw}(z) &= T_m(z) \\
&- T(z)K(z)\left(\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} - \begin{bmatrix} T(z) \\ 0 \end{bmatrix} K(z)\right)^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix} \\
&= T_m(z) - \left[I - T(z)L(z)\right]^{-1} T(z)M(z)
\end{aligned}
\tag{9a}
$$

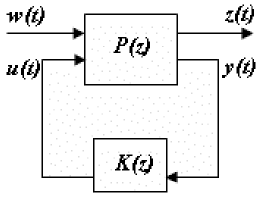

Figure 2

Note that $P(z)$ in Figure 2 is described as follows,

$$
P(z) = \begin{bmatrix} J & -D \\ 0 & D \\ I & 0 \end{bmatrix} + \begin{bmatrix} -C & H \\ C & 0 \\ 0 & 0 \end{bmatrix}
\left[\begin{pmatrix} zI & 0 \\ 0 & zI \end{pmatrix} - \begin{pmatrix} A & 0 \\ 0 & F \end{pmatrix}\right]^{-1} \begin{bmatrix} 0 & B \\ G & 0 \end{bmatrix}
\tag{10a}
$$

To use the results on standard H_∞ OCP while solving H_∞ MMP in 2DOF control structure, we assume $D=0$, in other words, the system to be controlled has to be strictly proper, thus the generalized plant will be well-posed. As it is known, the solution of the standard discrete- time H_∞ OCP gives all admissible controllers $K(z)$ for $P(z)$ shown in Figure 2, such that $\|T_{zw}(z)\|_\infty$ is minimum. The following Preposition provides the existence conditions of internally stabilizing controllers for the plant defined by (10a).

Preposition 1: A necessary and sufficient condition for the existence of internally stabilizing $K(s)$ for Figure 2 and the plant $P(s)$ given in (10) is that $\left(\begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix}, \begin{bmatrix} B \\ 0 \end{bmatrix}, \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix}\right)$ is stabilizable, namely the characteristic polynomial of F is strictly Schur and (A, B, C) is stabilizable.

Proof: Zhou, K, J. C. Doyle &K. Glover (1995). □

Throughout the paper, we assume that (A, B, C) is stabilizable, i.e., there exists a constant matrix K such that $A\text{-}BK$ is strictly Schur, and (A, C) is detectable i.e., there exists a constant matrix L such that $A\text{-}LC$ is strictly Schur. The synthesis theorem for discrete-time H_∞ OCP in formulation of LMIs given by Gahinet & Apkarian (1994) can be written for the discrete-time H_∞ MMP in 2DOF control structure as in the following Lemma.

Lemma 1 A controller $K(z)=[L(z)\ M(z)]$ with order $n_K \geq \dim A + \dim F$ which holds $\|T_{zw}(z)\|_\infty < \gamma$, exists for the plant described by (5-7) and closed-loop system is internally stable for H_∞ Optimal Control Problem if and only if there exist symmetric matrices $X>0$ and $Y>0$ such that

$$
\begin{bmatrix} N_o & 0 \\ 0 & I \end{bmatrix}^*
\begin{bmatrix}
\begin{bmatrix} A^* & 0 \\ 0 & F^* \end{bmatrix} X \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} - X & \begin{bmatrix} A^* & 0 \\ 0 & F^* \end{bmatrix} X \begin{bmatrix} 0 \\ G \end{bmatrix} & \begin{bmatrix} -C^* \\ H^* \end{bmatrix} \\
\begin{bmatrix} 0 & G^* \end{bmatrix} X \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} & -\gamma I + \begin{bmatrix} 0 & G^* \end{bmatrix} X \begin{bmatrix} 0 \\ G \end{bmatrix} & J^* \\
\begin{bmatrix} -C & H \end{bmatrix} & J & -\gamma I
\end{bmatrix}
\begin{bmatrix} N_0 & 0 \\ 0 & I \end{bmatrix} < 0
\tag{11}
$$

$$
\begin{bmatrix} N_c & 0 \\ 0 & I \end{bmatrix}^*
\begin{bmatrix}
\begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} Y \begin{bmatrix} A^* & 0 \\ 0 & F^* \end{bmatrix} & \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} Y \begin{bmatrix} -C^* \\ H^* \end{bmatrix} & \begin{bmatrix} 0 \\ G \end{bmatrix} \\
\begin{bmatrix} -C & H \end{bmatrix} Y \begin{bmatrix} A^* & 0 \\ 0 & F^* \end{bmatrix} & -\gamma I + \begin{bmatrix} -C & H \end{bmatrix} Y \begin{bmatrix} -C^* \\ H^* \end{bmatrix} & J \\
\begin{bmatrix} 0 & G^* \end{bmatrix} & J^* & -\gamma I
\end{bmatrix}
\begin{bmatrix} N_c & 0 \\ 0 & I \end{bmatrix} < 0
\tag{12}
$$

$$
\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0
\tag{13}
$$

where N_o and N_c are full rank matrices whose images satisfy

$$
Im\, N_o = Ker \begin{bmatrix} C & 0 & 0 \\ 0 & 0 & I_{m_1} \end{bmatrix}
\tag{14}
$$

$$
Im\, N_c = Ker \begin{bmatrix} B^* & 0 & 0 \end{bmatrix}
\tag{15}
$$

Proof: The claims of the Lemma are the same as those of the synthesis theorem for discrete-time H_∞ OCP in the framework of LMIs presented by Gahinet & Apkarian (1994), they are only rewritten for the system $P(z)$ given in (5-7).□

In order to obtain some specific results for discrete-time H_∞ MMP, further work has to be carried on Lemma 1. To full fill this aim, the following Lemmas are given.

Lemma 2 Suppose A and Q are square matrices and $Q>0$. Then A is strictly Schur if and only if there exists the unique solution,

$$
X = \sum_{t=0}^{\infty} (A^*)^t Q A^t > 0
$$

for the Lyapunov equation $A^* X A - X + Q = 0$.

Proof: Zhou, K, J. C. Doyle &K. Glover (1995). □

Lemma 3 The block matrix,

$$
\begin{bmatrix} P & M \\ M^* & N \end{bmatrix} < 0
$$

if and only if $N<0$ and $P-MN^{-1}M^*<0$. In the sequel, $P-MN^{-1}M^*$ will be referred to as the Schur Complement of N.

Proof: See Dullerud & Paganini 2000.□

Lemma 4 Suppose A, C, X and Y are square matrices and $\gamma \in R$. If the matrix A is strictly Schur, then for every pair of $\gamma>0$ and $Y>0$, there always exists a matrix $X>0$ such that holds the following inequalities,

$$A^*XA - X + \frac{1}{\gamma}C^*C < 0 \tag{16}$$

$$X - Y^{-1} \geq 0 \tag{17}$$

Moreover, some matrices satisfying (16) and (17) are generated by the following explicit relation,

$$X = \varepsilon\, P_0 + \frac{1}{\gamma} L_0 \tag{18}$$

in which, $\varepsilon \in R^+$ and it satisfies the following inequality,

$$\varepsilon \geq \lambda_{max}\left[\, (P^{-1})^*(Y^{-1} - \frac{1}{\gamma}\, L_0\,)\,P^{-1}\,\right] \tag{19}$$

where L_0 is Observabilty Gramian of (A,C) and is given by equation (20)

$$L_0 = \sum_{t=0}^{\infty}(A^*)^t C^* C A^t > 0 \tag{20}$$

and, $P_0>0$ is a solution of the equation $A^*P_0A - P_0 + Q = 0$ for $Q>0$, and P is a non-singular matrix with satisfying $P_0 = P^*P$.

Proof: Since A is strictly Schur, there always exist a matrix $X>0$ for every $\gamma>0$. Consider the following Lyapunov equation with $Q>0$, and $\varepsilon \in R^+$,

$$A^*XA - X + \frac{1}{\gamma}C^*C + \varepsilon Q = 0$$

The unique solution of the equation can be found from Lemma 2 as in the following form,

$$X = \sum_{t=0}^{\infty}(A^*)^t(\frac{1}{\gamma}C^*C + \varepsilon Q)A^t$$

$$= \frac{1}{\gamma}L_0 + \sum_{t=0}^{\infty}(A^*)^t \varepsilon Q A^t > 0$$

where, L_0 is Observabilty Gramian of (A,C) as in (20). To complete the proof, it will be sufficient to show that there exists a matrix $X>0$ satisfying $X \geq Y^{-1}$, namely (17). In that respect, lets define the matrix $P_0 = \sum_{t=0}^{\infty}(A^*)^t Q A^t$ then from Lemma 2 $P_0>0$ is a solution of the equation $A^*P_0A - P_0 + Q = 0$. Since the matrix P_0 if and only if $P_0 = P^*P$ and P is nonsingular, so there always exists some $\varepsilon \in R^+$ satisfying,

$$\varepsilon \geq \lambda_{max}\left[\, (P^{-1})^*(Y^{-1} - \frac{1}{\gamma}\, L_0\,)\,P^{-1}\,\right]$$

such that, $X = \varepsilon\, P_0 + \frac{1}{\gamma}L_0 \geq Y^{-1}$.□

Lemma 5 Suppose (A, C) is detectable and $Im\ N = Ker\ C$, there exist some $X>0$ such that the following inequality holds,

$$N^*\left(A^*XA - X\right)N < 0 \tag{21}$$

Furthermore, these matrices $X>0$ satisfying (21) can be generated by the following relation,

$$X = \varepsilon X_0\,,\ X_0 = \sum_{t=0}^{\infty}(A_1^{\,*})^t Q A_1^{\,t} > 0 \tag{22}$$

where $A_1 = A-LC$ and strictly Schur and $\varepsilon \in R^+$.

Proof: Since (A, C) is detectable then there always exist the matrices L with compatible dimensions and $X>0$, such that $(A-LC)$ is strictly Schur and thus the following inequality holds,

$$(A - LC)^* X (A - LC) - X < 0 \tag{23}$$

due to Lemma 2. Since $(A-LC)N = A\,N$ and so $N^*(A-LC)^* = N^*A^*$ and the inequality (21) is obtained by pre- and post-multiplying (23) with N^* and N respectively, then the proof is completed using Lemma 2.□

It will be useful to give the following corollary as a straightforward result of last two Lemmas, to provide an easy proof of a theorem on the H∞ MMP in 2DOF control structure, which will be given later.

Corollary 1: Suppose F is strictly Schur and (A, C) is detectable and $Im\ N = Ker\ C$, then for every pair of $\gamma>0$ and $Y>0$, there always exists a matrix $X>0$ such that hold the following inequalities,

$$\begin{bmatrix} N^* & 0 & 0 \\ 0 & I_{n_1} & 0 \\ 0 & 0 & I_{p_1} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} A^* & 0 \\ 0 & F^* \end{bmatrix} X \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} - X & \begin{bmatrix} -C^* \\ H^* \end{bmatrix} \\ \begin{bmatrix} -C & H \end{bmatrix} & -\gamma I \end{bmatrix} \begin{bmatrix} N & 0 & 0 \\ 0 & I_{n_1} & 0 \\ 0 & 0 & I_{p_1} \end{bmatrix} < 0 \tag{24}$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0 \tag{25}$$

The matrices $X>0$ which satisfy (24) and (25) can be generated by the following relations,

$$X = \begin{bmatrix} X_1 & 0 \\ 0 & X_3 \end{bmatrix},\ X_1 = \varepsilon X_0, \quad X_3 = \varepsilon\, P_0 + \frac{1}{\gamma}L_0 \tag{26}$$

in which $\varepsilon \in R^+$ and the following inequality holds,

$$\varepsilon \geq \lambda_{max}\left[\, (P^{-1})^*(Y^{-1} - \frac{1}{\gamma}\, L_0\,)\,P^{-1}\,\right],$$

and $X_0 = \sum_{t=0}^{\infty}(A_1^*)^t Q A_1^t > 0$ such that $A_1=A-LC$ and is strictly Schur, and L_0 is Observabilty Gramian of (F, H) as $L_0 = \sum_{t=0}^{\infty}(F^*)^t Q F^t \geq 0$ and $P_0>0$ is a solution of the equation $F^* P_0 F - P_0 + Q = 0$ for $Q>0$, and P is a non-singular matrix with satisfying $P_0 = P^* P$.

Proof: Let the matrix X be block diagonal with appropriate dimensions as $X = \begin{bmatrix} X_1 & 0 \\ 0 & X_3 \end{bmatrix}$, then the LMI (24) can be written as $\begin{bmatrix} \Psi_1 & 0 \\ 0 & \Psi_3 \end{bmatrix} < 0$ where,

$$\Psi_1 = N^*\left(A^* X_1 A - X_1\right)N < 0, \quad \Psi_3 = \begin{bmatrix} F^* X_3 F - X_3 & H^* \\ H & -\gamma I \end{bmatrix} < 0 \tag{27}$$

so the proof is completed by applying Lemma 3 to (27) and using Lemma 4 and Lemma 5.□

The following theorem can be presented on LMI based solution of the H_∞ MMP in 2DOF control structures as a reduced version of Lemma 1.

Theorem 1 A controller $K(z)=[L(z)\ M(z)]$ with order $n_K \geq \dim A + \dim F$, which the transfer matrices $T_{zw}(z)$ given in (9) hold $\|T_{zw}(z)\|_\infty < \gamma$, exists for the plant described by (5-7) and the closed-loop systems are internally stable, i.e., there exists a solution of the discrete- time H_∞ MMP in 2DOF control structure for the system and model given by (2) and (3) respectively, if and only if there exists a symmetric matrix $Y>0$; such that the following inequality holds,

$$\begin{bmatrix} N_c & 0 \\ 0 & I \end{bmatrix}^* \begin{bmatrix} \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} Y \begin{bmatrix} A^* & 0 \\ 0 & F^* \end{bmatrix} & \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} Y \begin{bmatrix} -C^* \\ H^* \end{bmatrix} & \begin{bmatrix} 0 \\ G \end{bmatrix} \\ \begin{bmatrix} -C & H \end{bmatrix} Y \begin{bmatrix} A^* & 0 \\ 0 & F^* \end{bmatrix} & -\gamma I + \begin{bmatrix} -C & H \end{bmatrix} Y \begin{bmatrix} -C^* \\ H^* \end{bmatrix} & J \\ \begin{bmatrix} 0 & G^* \end{bmatrix} & J^* & -\gamma I \end{bmatrix} \begin{bmatrix} N_c & 0 \\ 0 & I \end{bmatrix} < 0 \tag{28}$$

where N_c is a full rank matrix with,

$$Im\, N_c = Ker \begin{bmatrix} B^* & 0 & 0 \end{bmatrix}$$

and (A, B, C) and (F, G, H, J) are the discrete state space description of the $T(z)$ and $T_m(z) \in RH_\infty$ respectively, such that (A, B) is stabilizable and (A, C) is detectable .

Proof: It is easily seen that the claim of the Theorem is the same as the condition (12) of Lemma 1. To complete the proof, it will be sufficient to show that the conditions (11) and (13) are already satisfied. For this purpose, the condition (11) in Lemma 1 can be rewritten as follows,

$$\begin{bmatrix} N^* & 0 & 0 & 0 \\ 0 & I_{n_1} & 0 & I_{p_1} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} A^* & 0 \\ 0 & F^* \end{bmatrix} X \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} - X & \begin{bmatrix} A^* & 0 \\ 0 & F^* \end{bmatrix} X \begin{bmatrix} 0 \\ G \end{bmatrix} & \begin{bmatrix} -C^* \\ H^* \end{bmatrix} \\ \begin{bmatrix} 0 & G^* \end{bmatrix} X \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} & -\gamma I + \begin{bmatrix} 0 & G^* \end{bmatrix} X \begin{bmatrix} 0 \\ G \end{bmatrix} & J^* \\ \begin{bmatrix} -C & H \end{bmatrix} & J & -\gamma I \end{bmatrix} \begin{bmatrix} N & 0 \\ I_{n_1} & 0 \\ 0 & 0 \\ 0 & I_{p_1} \end{bmatrix} < 0 \tag{29}$$

since $Im\, N_0 = Ker \begin{bmatrix} C & 0 & 0 \\ 0 & 0 & I_{m_1} \end{bmatrix}$ so $N_0 = \begin{bmatrix} N & 0 \\ 0 & I_{n_1} \\ 0 & 0 \end{bmatrix}$ such that $Im\ N=Ker\ C$. Furthermore, the inequality (29) can also be written as follows,

$$\begin{bmatrix} N^* & 0 & 0 \\ 0 & I_{n_1} & I_{p_1} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} A^* & 0 \\ 0 & F^* \end{bmatrix} X \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} - X & \begin{bmatrix} -C^* \\ H^* \end{bmatrix} \\ \begin{bmatrix} -C & H \end{bmatrix} & -\gamma I \end{bmatrix} \begin{bmatrix} N & 0 \\ 0 & I_{n_1} \\ 0 & I_{p_1} \end{bmatrix} < 0 \tag{30}$$

Since (A, C) is detectable and F is strictly Schur, it can easily be seen that there always exist some $X>0$ with satisfying the inequalities (30) and (13) by using Conclusion 1. This means that the conditions (11) and (13) given in Lemma 1, are already satisfied, hence the proof is completed. □

In order to construct the controllers $L(z)$ and $M(z)$, it is useful to give a brief procedure; suppose the matrix $Y>0$ and the minimum value of $\gamma_{opt} \in R^+$ are found as a solution of (28) by using LMI toolbox (Gahinet, Nemirovski & Chilali, 1994). Then a matrix $X>0$ is found by using (26), such that the inequalities given in (30) and (13) hold. Finally, the controller transfer matrix $K(z)$, which minimizes the $\|T_{zw}(z)\|_\infty$ given in (9a) is obtained as,

$$K(z) = D_k + C_k (zI - A_k)^{-1} B_k \tag{31}$$

using the matrices X and Y, via the controller reconstruction procedure given by Gahinet & Apkarian (1994). Thus the transfer matrices of the feedback and the feed-forward controllers $L(z)$ and $M(z)$ respectively, i.e., the solution of the H_∞ MMP in 2DOF control structure for the system and model given by (2) and (3) respectively, are found from the definition $K(z)=[L(z)\ \ M(z)]$.

4. CONCLUSION

In this paper, we have studied on the discrete time H_∞ MMP in the 2DOF control structure. The method developed in Gören (2003) for continuous-time systems will be transferred to discrete-time context and will lead to qualitatively similar results. The LMI-based solution of the problem by using this control structure has been presented with including some relations with the solution of OPC.

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