ATTAINABILITY AND SUBOPTIMAL MINIMAL TIME CONTROL OF A CLASS OF BIOLOGICAL SEQUENCING BATCH REACTORS

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Abstract: In this paper, geometric tools are used for the attainability study of biological Sequencing Batch Reactors (SBR). It consisted in finding the set of initial conditions from which one can reach a final state target with admissible control. The possible solutions can be determined easily once the set of reachability is fixed. In the second step, optimal time control problem is considered. It consist in finding the switching instant between the different steps of the batch. Pontryagin's Maximum principle is used to solve a part of this problem. We propose asuboptimal solution allowing at most two commutation. A new problem is considered where the criteria is parameterized by the switching concentrations. A numerical solution is then proposed to solve this new minimal time problem.

Keywords: Suboptimal control, Reactor control modeling, Reachability, Numerical optimuzation

1. INTRODUCTION

Nitrogen removal in batch reactor for wastewater treatment is realized in two successive steps: denitrification and nitrification. (?). Usually, the reactors are a priori designed for a known range of concentrations. The reactor volume and the phases duration are fixed. However, if there are significant variations of the influent concentrations, no guarantee on the effluent purification is given and the possibility of carrying out both reactions in the batch is not assured. In this kind of process, the reactions are related and the final concentrations have to reach a terminal target. This leads to a controllability analyses. The possible solutions can be analyzed and the trajectories are optimized. To handle the problem we processed as follow :

- The batch process is modeled using standard laws of biological kinetics modeling. The dynamic behavior is studied and all the dependence between the states variable are derived to reduce the model and to simplify the reachability study.
- With terminal state constraints representing a target set, admissible solutions are analyzed to estimate the necessary number of switching.
- Optimal time control problem is solved to reduce the phases duration and to find the switching instants. Initial guess of optimal trajectory is determined using Pontryagin's maximum principle and numerical algorithme is used to get suboptimal control law.

This work was motivated by practical optimisation problem. It consiste in constructing low cost industriel batch reactor for carbon and nitrogen removal. Reducing the total cycle time is equivalent to increasing the volume treated per day or decreasing the reactor volume. Small and effective reactor can be proposed for industrial use.

2. BIOLOGICAL REACTIONS MODELLING

Let us consider activated sludge bioreactor. To eliminate both carbon and nitrogen two operating modes are necessary.

- aerobic mode: With aeration, two kinds of biomass X_1 and X_2 consume respectively the carbon and the nitrogen and they ensure two independent reactions: carbon removal (reaction 1) and nitrogen removal (reaction 2) where nitrogen is converted to nitrates.

$$S_1 + O_2 \xrightarrow{r1} X_1 \quad (1)$$
$$S_2 + O_2 \xrightarrow{r2} X_2 + S_3 \quad (2)$$

- anoxic mode: with luck of oxygen the denitrification takes place. The X_1 biomass replace the oxygen by the nitrates S_3 to consumes the carbon with respect to the third reaction

$$S_1 + S_2 \xrightarrow{r3} X_1 \quad (3)$$

with respect to these reactions two models are derived. Biological Reactions are modeled using standard laws of biological kinetics (see for instance (?)):

- aerobic model:

$$\dot{X}_1 = r_1(S_1, X_1) \quad (4)$$
$$\dot{X}_2 = r_2(S_2, X_2) \quad (5)$$
$$\dot{S}_1 = -k_{11} r_1(S_1, X_1) \quad (6)$$
$$\dot{S}_2 = -k_{22} r_2(S_2, X_2) \quad (7)$$
$$\dot{S}_3 = k_{32} r_2(S_2, X_2)) \quad (8)$$

- anoxic model:

$$\dot{X}_1 = r_3(S_1, S_3, X_1) \quad (9)$$
$$\dot{X}_2 = 0 \quad (10)$$
$$\dot{S}_1 = -k_{13} r_3(S_1, S_3, X_1) \quad (11)$$
$$\dot{S}_2 = 0 \quad (12)$$
$$\dot{S}_3 = -k_{33} r_3(S_1, S_3, X_1) \quad (13)$$

where $r_j(.)$ is the biomass growth given by $\mu_j\left(S_1, .., S_n\right) X_i$, where μ_j the growth rate modeled with a positif map which vanish if and only if one of the S_i vanishes. X_i is strictly positive.

k_{ij} are the stoichiometric yield coefficients related to the i^{th} substrat of the j^{th} reaction.

We assume that, in aerobic phase, O_2 is controlled and fixed at a constant value.

Particularly for these biological reactions, the yield coefficient k_{11} and k_{13} in reactions (1) and (3) are the same.

3. REACHABILITY ANALYSES

The substrate's concentration, at the end of the batch process, have to be less than the threshold of rejection. The control problem to be solved is to find the switching sequences to ensure the three reactions (1-3) and eliminate the substrates.

Definition 1. Lets $u \in U = \{0, 1\}$ the control variable which corresponds to the switching signal : $u = 1$ for aerobic mode and $u = 0$ for anoxic mode.

By associating the two models in the following matrix form :

$$\begin{bmatrix} \dot{X}_1 \\ \dot{S}_1 \\ \dot{X}_2 \\ \dot{S}_2 \\ \dot{S}_3 \end{bmatrix} = \left[\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -k_{11} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -k_{33} \end{bmatrix} + u \begin{bmatrix} 1 & 0 & -1 \\ -k_{11} & 0 & k_{11} \\ 0 & 1 & 0 \\ 0 & -k_{22} & 0 \\ 0 & k_{32} & k_{33} \end{bmatrix} \right] \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \quad (14)$$

we can derive easily the following relationships :

$$\dot{S}_1 + k_{11} \dot{X}_1 = 0 \quad (15)$$
$$\dot{S}_2 + k_{22} \dot{X}_2 = 0 \quad (16)$$

By integration we get :

$$M_1 = S_1 + k_{11} X_1 = S_1(0) + k_{11} X_1(0) \quad (17)$$
$$M_2 = S_2 + k_{22} X_2 = S_2(0) + k_{22} X_2(0) \quad (18)$$

Where M_1 and M_2 depend only on the initial conditions and remain constant during the batch whatever the control. This is the case of the majority of bioreactors (?)(?), For each reaction, there exists a mass balance conservation between the biomass and substrates. It is invariant for each input control law. Thus one obtains a linear relation between S_1 and X_1 (respectively S_2 and X_2):

$$X_1 = -\frac{S_1 - M_1}{k_{11}} \quad (19)$$
$$X_2 = -\frac{S_2 - M_2}{k_{22}} \quad (20)$$

3.1 Model simplification

In the case of carbon and nitrogen removal the kinetic functions $\mu_i(.)$ are monod kinetics. Growth rates are given by the following form :

$$r_1(S_1, X_1) = \mu_{1max} \frac{S_1}{k_{S1} + S_1} X_1 \quad (21)$$

$$r_2(S_2, X_2) = \mu_{2max} \frac{S_2}{k_{S2} + S_2} X_2 \quad (22)$$

$$r_3(S_1, S_3, X_1) = \mu_{1\max} \rho \frac{S_1}{k_{S1} + S_1} \frac{S_3}{k_{S3} + S_3} X_1 \quad (23)$$

One can replace expressions of X_1 and X_2 in (21) - (23) to obtain the reduced model :

$$\begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \\ \dot{S}_3 \end{bmatrix} = \begin{bmatrix} f(S_1) \\ g(S_2) \\ -\alpha g(S_2) \end{bmatrix} u + \begin{bmatrix} f(S_1)h(S_3) \\ 0 \\ \beta f(S_1)h(S_3) \end{bmatrix} (1-u) \quad (24)$$

Where $\alpha = \frac{k_{32}}{k_{22}}$ and $\beta = \frac{k_{33}}{k_{11}}$ and $\rho < 1$
$f(S_1) = \mu_{1\max}(S_1 - M_1)\frac{S_1}{k_{S1}+S_1}$, $h(S_3) = \rho\frac{S_3}{k_{S3}+S_3}$ and $g(S_2) = \mu_{2\max}(S_2 - M_2)\frac{S_2}{k_{S2}+S_2}$

A compact form of the model is given by :

$$\begin{cases} \dot{Z} = F(Z) + G(Z)u \\ Z_0 = [S_1(0),\ S_2(0),\ S_3(0)] \end{cases} \quad (25)$$

Where $Z^t = [S_1, S_2, S_3]$ is the state vector. F and G are analytic field vectors in $\mathbb{R}^3$ easily identified from equation (24).

3.2 Dynamic behavior

Proposition 2. Let $S_1(t, S_1(0), S_3(0), u)$, $S_2(t, S_2(0), u)$ and $S_3(t, (S_1(0), S_2(0), S_3(0), u)$ be the solutions of the differential equation (24)where u is a constant control.

- $S_1(.)$ and $S_2(.)$ are two decreasing maps and tend towards zero.
- $S_3(t, Z_0, 1)$ decreases and $S_3(t, Z_0, 0)$ increases.

Proof

- $M_1 = S_1 + k_{11}X_1 > S_1$ (res $M_2 = S_2 + k_{22}X_2 > S_2$) so for all $S1$ (res $S2$), $f(S_1) < 0$ (res $g(S_2) < 0$). Since $h(S_3) > 0$ and $u \in \{0, 1\}$ one can deduce that $\frac{dS_1}{dt} < 0\ \forall S_1 > 0$ (res $\frac{dS_2}{dt} < 0\ \forall S_2 > 0$) moreover $\frac{dS_1}{dt}\Big|_{S_1=0} = 0$ (res $\frac{dS_1}{dt}\Big|_{S_1=0} = 0$) so using Lyapunov theorem for asymptotic convergence (**?**) we deduce that $S_1(.)$ and $S_2(.)$ decrease and tend asymptotically toward zero. □
- For the same reason, if $u = 1$, $\frac{dS_3}{dt} > 0$ then $S_3(t, Z_0, 1)$ increases, and if $u = 0$, $\frac{dS_3}{dt} < 0$ then $S_3(t, Z_0, 0)$ decreases. □

Proposition 3. Since u is a piecewise constant mapping, the solution of system (25) verify the following properties :

- $Z(t, Z_0, 1) \subset \Sigma(Z_0)$
- $Z(t, Z_0, 0) \subset \Delta(Z_0)$

where

$$\Sigma(Z_0) = \{Z \in \mathbb{R}^{3+}/S_1 < S_1(0), S_2 < S_2(0), \\ S_3 - S_3(0) = -\alpha(S_2 - S_2(0))\}$$

$$\Delta(Z_0) = \{Z \in \mathbb{R}^3/S_1 < S_1(0), S_2 = S_2(0), \\ S_3 - S_3(0) = \beta(S_3 - S_3(0))\}$$

Proof If $u = 1$, $\dot{S}_3 = -\alpha\dot{S}_2$ then $S_3 - S_3(0) = -\alpha(S_2 - S_2(0))$ moreover $S_1 < S_1(0)$ and $S_2 < S_2(0)$ so $Z(t, Z_0, 1) \in \Sigma(Z_0)$
If $u = 0$, $\dot{S}_2 = 0$ and $\dot{S}_3 = \beta\dot{S}_1$ then $S_2 = S_2(0)$ and $S_3 - S_3(0) = \beta(S_1 - S_1(0))$ moreover $S_2 < S_2(t)$ so $Z(t, Z_0, 0) \in \Delta(Z_0)$ □

Definition 4. (Accessibility and Reachability). Consider the system (25) where $Z(t, Z_0, u)$ is its maximal solution. The set of accessible point at time $T > 0$ is $A^+(Z_0, T) = \cup_{u \in U} Z(t, Z_0, u)$ and the set of accessibility is given as $A^+(Z_0) = \cup_{T>0} A^+(Z_0, T)$ in a similar way, we note $A^-(Z_0, T)$ the set of point from which Z_0 can be reached at time T and $A^-(Z)$ the set of reachable points.

Let now consider a family of the targets C, open sets in $\mathbb{R}^3$, given by : $C(Z_N) = \{Z \in \mathbb{R}^{3+}, Z_N^t = [S_{1N}, S_{2N}, S_{3N}]/S_1 < S_{1N}, S_2 < S_{2N}, S_3 < S_{3N} - \alpha S_2\}$, $S_{iN, i=1..3}$ are the normative rejection norms. The Reachability set $\Omega(Z_N)$ is the set of points from which the target is reached using the dynamics of the system (25). It is given by :
$\Omega(Z_N) = \{A^-(Z_0)/Z(t, Z_0, u) \in C(Z_N)\}$. We define also the sets:
$\Omega_A(Z_N) = \{Z \in \mathbb{R}^{3+}, S_2 < \alpha^{-1}S_{3N}, S_3 < \beta S_1 - \alpha S_2 + S_{3N}\}$
$\Omega_B(Z_N) = \{Z \in \mathbb{R}^{3+}, S_2 > \alpha^{-1}S_{3N}, S_3 < S_1 - \alpha S_2 + \beta(S_3 - S_{3N})\}$
$\Omega_C(Z_N) = \Omega_A(Z_N) \cup \Omega_B(Z_N)$

In a previous work (**?**), we show that from some initial conditions we cannot reach the target and the set of reachability for this problem is given by $\Omega(Z_N) = \Omega_C(Z_N)$. This set is constructed using all the possible concatenations between piecewise trajectories $Z(t, Z_0, 1)$ and $Z(t, Z_0, 0)$ from which the target can be reached. For more detail, refer to (**?**). We prove also that with one anoxic phase the trajectory can reach the target from any point in this reachable set.

In the following, we consider only initial conditions in this reachable set.

4. MINIMAL TIME CONTROL PROBLEM

4.1 Problem and statement

In this section we consider the minimal time problem in which we try to find an optimal sequence of switching between aerobic mode and anoxic mode to minimize the total cycle time. Optimal switching instants have to be determined. In order to solve the problem using the Pontryagyn's maximum principle, we consider the extended problem where the control variable takes values in the closed convex set [0, 1]. The possible solutions are analyzed to check possible bang bang solutions. Our solution is sub-optimal is the sens that we look for at most two commutations

4.2 Pontryagin's Maximum Principle (PMP)

Consider a control-affine system in $\mathbb{R}^3$ of the form:

$$\dot{Z}(t) = F(Z(t)) + G(Z(t))u(t) \tag{26}$$

where F and G are two analytical field vectors in $\mathbb{R}^3$, u a bounded map defined on $\mathbb{R}^+$ and takes value in $U = [u_{\min}, u_{\max}]$. For $Z_0 \in \mathbb{R}^3$, $Z(t, Z_0, u(.))$ is the solution of the differential equation with the initial condition Z_0 at $t = 0$ and control $u(.)$.
Let C be a regular sub manifold in $\mathbb{R}^3$ and $T_Z C$ the tangent space of C at the point Z.
We note $Z^*(t, Z_0, u^*)$ the minimum time trajectory connecting the initial point Z_0 at the target C in the time t^*. The triplet $(Z^*(t), \lambda^*(t), u^*(t))$ verify:

$$\dot{Z}^*(t) = \frac{\partial H^*}{\partial \lambda}\left(Z^*(t), \lambda^*(t), u^*(t)\right) \tag{27}$$

$$\dot{\lambda}^*(t) = -\frac{\partial H^*}{\partial Z}\left(Z^*(t), \lambda^*(t), u^*(t)\right) \tag{28}$$

$$H(Z^*(t), \lambda^*(t), u^*(t)) = \min_{u \in U} H\left(Z^*(t), \lambda^*(t), u(t)\right) \tag{29}$$

with the boundary condition

$$\lambda^*(t^*) \perp T_{(Z^*(t^*))} C, \tag{30}$$

where

$$H\left(Z(t), \lambda(t), u(t)\right) = \lambda^t(t)\left(F(Z(t)) + G(Z(t))u(t)\right) + \lambda_0 \tag{31}$$

The adjoint vector $\lambda(.)$ verify $\lambda(t) \not\equiv 0$ at any time and λ_0 is constant positive or null.

$u^*(.)$ is computed as follow(**?**) :

$$u^*(t) = \begin{cases} u_{\min} & if\ \lambda^t(t)G(Z(t)) > 0 \\ u_s & if\ \lambda^t(t)G(Z(t)) = 0 \\ u_{\max} & if\ \lambda^t(t)G(Z(t)) < 0 \end{cases} \tag{32}$$

with the singular control u_s is computed by solving the following equations at time t :

$$\frac{d}{dt}\left(\lambda^t(t)G(Z(t))\right) = 0 \\ \vdots \\ \frac{d^k}{dt^k}\left(\lambda^t(t)G(Z(t))\right) = 0 \tag{33}$$

k is chosen such that the control variable u appears explicitly in the k^{th} derivative of the switching function

$$\lambda^t(t)G(Z(t)) \tag{34}$$

We first study particular cases without switch and then analyze cases with one anoxic phase.

4.3 Particular case: Solution without switching

Consider again the target $C(Z_N)$ define previously. The trajectory $Z(t, Z_0, u)$ can reaches one of the three sides P_A, P_B or P_C of the target (Cf figure 1). The optimal solutions that reaches $P_A \cup P_B$ verify according to the equation (30) the following transversality conditions : $\lambda_3(tf) = 0$, $\lambda_1(tf) \geq 0$ and $\lambda_2(tf) \geq 0$. The switching function becomes

$$\Phi(t_f) = \lambda_1(t_f)(f(S_1(t_f))(1 - h(S_3(t_f))) + \lambda_2(t_f)g(S_2(t_f)) \tag{35}$$

Since $\lambda(t) \not\equiv 0$, $f(S_1) < 0$, $g(S_2) < 0$ and $0 < h(S_3) < 1$ we have $\Phi(t_f) < 0$ so $u(t_f) = 1$ according to (32).
With $u = 1$ in a non empty interval $[t,\ t_f]$ the adjoint vectors are given by

$$\dot{\lambda}_1(t) = -\lambda_1(t)\left(\frac{\partial f(S_1(t))}{\partial S_1}\right) \tag{36}$$

$$\dot{\lambda}_2(t) = -(\lambda_2 + \alpha\lambda_3(t))\left(\frac{\partial g(S_2(t))}{\partial S_2}\right) \tag{37}$$

$$\dot{\lambda}_3(t) = 0 \tag{38}$$

with the boundary conditions $\lambda_3(tf) = 0$, $\lambda_1(tf) \geq 0$ and $\lambda_2(tf) \geq 0$. The solutions of the differential equations (36-38) verify $\lambda_3(t) = 0$, $\lambda_1(t) \geq 0$ and $\lambda_2(t) \geq 0$, so the switching function is negative and does not change its sign in the interval $[t,\ t_f]$ whatever is $t \in [t,\ t_f]$

Corollary 5. All trajectories $Z(t, Z_0, 1)$ solution of (24) that reach the target $C(Z_N)$ i.e. $Z(t, Z_0, 1) \cap C(Z_N) \neq \phi$ verify the PMP conditions. Thus, it is an optimal trajectory and the optimal control is $u^* = 1$

Proposition 6. The set of initial conditions for which the solution of (24) verify $Z(t, Z_0, 1) \cap C(Z_N) \neq \phi$ is given by :
$\Omega_1(Z_N) = \{Z \in \mathbb{R}^{3+} / S_3 \leq -\alpha S_2 + S_{3N}\}$ refering to the Corollary (5), the optimal control for this set is $u^* = 1$

Proof Lets $Z_0 \in \Omega_1(Z_N)$ so $S_3(0) \leq -\alpha S_2(0) + S_{3N}$. In addition, for $u = 1$, $\dot{S}_3 + \alpha\dot{S}_2 = 0$ so $S_3 - S_3(0) + \alpha(S_2 - S_2(0)) = 0$. When $t \to \infty$ we have $S_1 \to 0$, $S_2 \to 0$ and $S_3^\infty + \alpha S_2^\infty = S_3(0) + \alpha S_2(0) \le S_{3N}$ so $Z^\infty \in C(Z_N)$. □

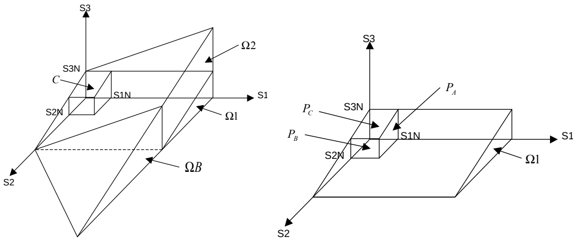

Fig. 1. Reachability set
$\Omega_A(Z_N) = \Omega_1(Z_N) \cup \Omega_2(Z_N)$, $\Omega_C(Z_N) = \Omega_A(Z_N) \cup \Omega_B(Z_N)$

4.4 *General case: solution with one anoxic phase*

The reachability analyses show that for each point in $\Omega(Z_N)$ they exists at most one anoxic phase to reach optimally the target. We tried to solve the optimal time problem with one anoxic phase i.e. with at most two switches aerobic-anoxic-aerobic. To do so we defined the following control :

$$t \in [0\ t_1],\ u = 1 \quad (39)$$
$$t \in [t_1\ t_2],\ u = 0 \quad (40)$$
$$t \in [t_2\ t_f],\ u = 1 \quad (41)$$

to cover all the possible cases, we can have $t_0 = t_1$ and/or $t_1 = t_2$ and /or $t_2 = t_f$. The maximal trajectory is given by: $Z(t \le t_1, Z(t_0), 1) \cup Z(t \le t_2, Z(t_1), 0) \cup Z(t \le t_f, Z(t_2), 1)$

If $Z_0 \in \Omega_1(Z_N)$ we apply corollary (5) so $t_0 = t_1 = t_2$. t_f is given when the trajectory $Z(t, Z(t_2), 1)$ reach the target. If $Z_0 \notin \Omega_1(Z_N)$, t_2 is given when the trajectory $Z(t, Z(t_1), 0)$ reaches $\Omega_1(Z_N)$. If $Z(t, Z(t_1), 0)$ reaches $C(Z_N)$ then $t_f = t_2$. The unknown parameter is t_1 that we parameterize in the following

Proposition 7. The trajectory reach the side P_A of the target $C(Z_N)$. So at $t = t_f$, we have $S_2(t_f) = S_{2N}$.

4.5 *Anoxic phase duration*

In anoxic phase we have $u = 0$. We have then S_2 constant. $S_1(t)$ and $S_3(t)$ are linearly related refereing to the relationships obtained from the first integral $\frac{dS_3}{dt} = \beta \frac{dS_1}{dt}$. In order to parameterize the anoxic phase time, we introduce the following variable : $l = S_1(t_1) - S_1 = \frac{1}{\beta}(S_3(t_1) - S_3)$ so $S_1 = S_1(t_1) - l$ and $S_3 = S_3(t_1) - \beta l$.

Let d be the variation of S_1 in anoxic phase which is proportional to the variation of S_3. It is given by : $d = S_1(t_1) - S_1(t_2) = \frac{1}{\beta}(S_3(t_1) - S_3(t_2))$. The anoxic phase time is that given by :

$$J(S_1^1, S_3^1, d) = \int_0^d \frac{1}{f(S_1^1 - l)} \frac{1}{h(S_3^1 - \beta l)} dl = \int_0^d L(l, S_1^1, S_3^1) dl \quad (42)$$

where $(S_1^1,\ S_2^1,\ S_3^1)$ is the solution of the dynamical system (24) for $u = 1$ at $t = t_1$.

Corollary 8. $L(l, S_1^1, S_3^1)$ is a non-negative and decreasing map. We can deduce that if $d1 < d2$ then $J(S_1^1, S_3^1, d1) < J(S_1^1, S_3^1, d2)$. The minimal value of d to reach the target is given by : $d = \frac{1}{\beta}\left(S_3(0) + \alpha S_2(0) - S_{3N}\right)$. It depend neither on the switching time nor on the dynamic of the system. It depends only on the initial conditions and the target.

Proof By definition $d = \frac{1}{\beta}(S_3(t_1) - S_3(t_2))$. The optimal solution $Z(t, Z(t,1), 0)$ crosses Ω_1 at t_2, then $S_3(t_2) = S_{3N} - \alpha S_2(t_2) = S_{3N} - \alpha S_2(t_1)$ so $d = \frac{1}{\beta}(S_3(t_1) + \alpha S_2(t_1) - S_{3N}$. Moreover t_1 is the final time of the first aerobic phase. In aerobic phase the linear relationships is verified: $S_3(t_1) + \alpha S_2(t_1) = S_3(t_0) + \alpha S_2(t_0)$ because $\frac{dS_3}{dt} = \alpha \frac{dS_2}{dt}$. So $d = \frac{1}{\beta}\left(S_3(0) + \alpha S_2(0) - S_{3N}\right)$. □

4.6 *Reaction time*

In the case when $Z(t, Z(t_2), 1)$ reaches P_B i.e. $S_2(tf) = S_{2N}$ the reaction time is given by :

$$t = \int_{t_0}^{t_1} dt + \int_{t_1}^{t_2} dt + \int_{t_2}^{t_f} dt = \int_{S_2(t_0)}^{S_2(t_1)} \frac{ds}{g(s)} + \int_{t_1}^{t_2} dt + \int_{S_2(t_2)}^{S_2(t_f)} \frac{ds}{g(s)} \quad (43)$$

Since S_2 is constant when $u = 0$ then $S_2(t_1) = S_2(t_2)$. The equation (43) becomes:

$$t = \int_{S_{20}}^{S_{2N}} \frac{ds}{g(s)} + \int_{t_1}^{t_2} dt = cts + \int_{t_1}^{t_2} dt \quad (44)$$

Remark 9. In this case minimizing the total time is equivalent to minimize the anoxic phase time.

5. NUMERICAL OPTIMISATION AND RESULTS

In the following section we propose a numerical algorithm to find the optimal switching concentrations. The next algorithm is used
Algorithm :

I: Solve the dynamical system with $u = 1$ to determine the set of points S_1^1, S_3^1

II: Deduce the subset where anoxic phase can be applied i.e $S_1 > d$ and $S_3 > \frac{d}{\beta}$

III: Using an exploration algorithm find (S_1^{1*}, S_3^{1*}) that minimize $J(S_1^1, S_3^1, d)$. (ex: decent method using the gradient)

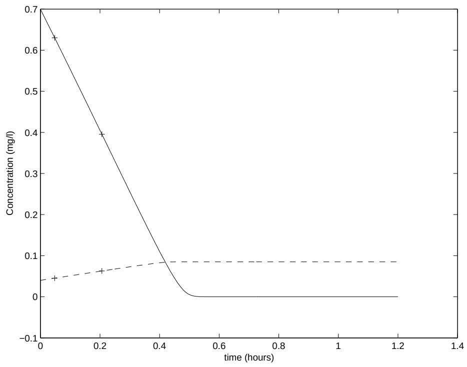

Fig. 2. Set of variations

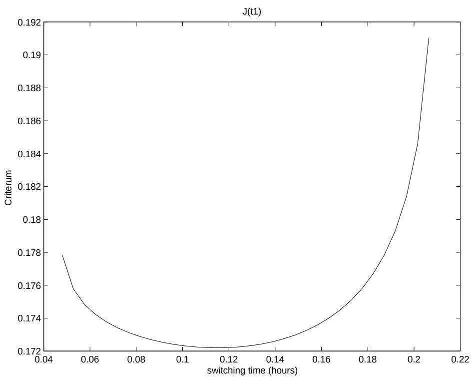

Fig. 3. Criterium to be minimize

6. INTERPRETATION

The case studied in this paper correspond to major cases in practise. The nitrogen removal reaction is very slow compared to the carbon removal. The aerobic time corresponds essentially to the nitrogen removal. Since this last one is not eliminated in anoxic phase, the aerobic phase is time constant. We can reduce the total cycle time by reducing the anoxic phase time. In standard cases the anoxic phase takes place at the beginning of the cycle. The reaction velocity depends on the two substrat concentrations S_1 and S_3. At $t_0 = 0$ the maximum of S_1 is available. In aerobic mode S_1 is eliminated and S_3 is produced. The suboptimal control strategy consists in maximizing the anoxic reaction velocity by increasing S_3 in aerobic phase, (of course, S_1 decreases in this case). The anoxic phase is applied when the maximal possible velocity of the anoxic reaction is reached, so that the total time of this phase is reduced, as it is shown in the latest figures (2 and 3).

7. CONCLUSION

This paper describes a new approche for finding a control law that minimize the total cycle time for carbon and nitrogen removal in SBR process. In this kind of processes, the carbon and nitrogen are treated in aerobic and anoxic conditions. The dynamical behavior of the model is analyzed and it is shown that at most two switches allow to reach the target. In this case, the anoxic phase takes place between two aerobic phases. Altering process sequences can reduce the total time. To find the optimal switching time, a suboptimal time control problem is formulated. The possible solutions and the optimum trajectory are analyzed using Pontryagin's Maximum principle. This leads to find the second switching surface defined by the state variables. Using this information, the problem is translated in a new problem where the criteria is parameterized by the switching concentrations. The new problem is resolved using gradient algorithm to find the first switch. With this approach the total cycle time is optimized.

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