ROBUST FAULT DETECTION FILTER FOR LINEAR STOCHASTIC SYSTEMS

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Abstract
In this paper, we present an unified approach for fault detection and isolation in discrete-time systems affected by noises and faults on measurement and state equations. The proposed robust fault detection filter will be designed under less restrictive conditions compared with classical fault detection filter. After having parameterized the minimum-time left inverse of the system, the degrees of freedom remaining available will be computed to generate an optimal faults estimation. The latter is minimally sensitive to state and measurement noise. An numerical example is given to illustrate the design of the proposed filter.

Keywords: Fault detection filter, left inverse, disturbance rejection, linear system

1. INTRODUCTION

Due to increased complexity, as well as the need for reliability, safety and efficient operation of industrial systems, the robust diagnosis has gained more attention. The robust fault detection and isolation (FDI) based observer has been often studied in a deterministic (Commault, 1999). There exists two main approaches to generate a residual sequence decoupled from unknown input or disturbance : the first is based on eigenstructure assignment (Patton and Chen 1992), (Hsu and Shen 1995) and the second is based on unknown input observers (Wunnenberg and Frank, 1987). There exists also a special form of observer, namely the fault detection filter, first developped by Beard (1971) and Jones (1973), Massoumnia (1986) and by White and Speyer(1987) in the context of eigenstructure assignment. Further improvements were suggested by Park and Rizzoni (1994), Chung and Speyer (1998), Hou and Muller (1994). Recently, Chen and Speyer (2002) have proposed a new robust multiple fault detection filter which is derived by solving an optimization problem in the context where we can not achieve a perfect decoupling. Park and Rizzoni (1994: part2) have extended the FDF in stochastic linear systems. Even if a robust fault diagnosis has been first developed by Nikoukhah (1994), the generated residual sequence is decoupled from unknwon inputs, minimized with respect to the plant and state noises but the isolation of multiple faults is not guaranted. To solve this problem, Keller (1999) has extended the fault isolation filter of Liu and Si (1997) for stochastic systems where the full-order Kalman filter is designed under a particular eigenstructure assignment. Unfortunately, the case where unknown inputs may affect the system produce many false alarms is not treated.

The FDI problem can splitted into two steps. Step1 : Generation of residuals ideally close to zero under no-fault conditions, minimally sensitive to noises or disturbances and maximally sensitive to faults. Step2: Generation of decision rules based on these residuals. The fault reconstruction is not often integrated in the step of residuals generation but is very interesting to simplify the design of decision rules by determining a threshold level directly applied on the faults estimation. Conceptually, the problem of faults reconstruction is very closed to the problem of system inversion. By showing this link in continous-time system, Hou and Patton (1998) have proposed a fault reconstructor designed by means of system inversion which need first and high order derivatives of measurements. In this paper we takes into account both deterministic and stochastic disturbances. The generated residuals will be sensitive to the faults by system inversion and the disturbance will be treated as fault inputs. After having parameterized all the minimum-time left inverses of the system, the remaining design of freedom will be used to minimize the effect of noises on the generated residuals. We note that the obtained filter is designed under less restrictive conditions compared with other approach (Keller, 1999), (Parlangeli,2002).

This paper is organised as follow : section 2 presents the statement of the problem. The section 3 parameterizes the robust fault detection filter. Section 4 uses the remaining design of freedom to minimize the trace of the estimation errors covariance matrix and section 5 gives a numerical example.

2. PROBLEM STATEMENT

Consider the following discrete time linear system

$$x_{k+1} = Ax_k + Bu_k + Md_k + \ell\eta_k + Gw_k \quad (1)$$
$$y_k = Cx_k + Du_k + Nd_k + T\eta_k + Hw_k \quad (2)$$

where $x_k \in \Re^n$ is state vector, $y_k \in \Re^m$ the output vector, $u_k \in \Re^p$ the input vector. $d_k \in \Re^q$ and $\eta_k \in \Re^s$ are respectively the fault vector and unknown input vector. Each component of $d_k \in \Re^q$ represents one possible fault which may occur on measurement and state equations simultaneously. The unitary zero mean white gaussian noise w_k satisfying $E\{w_k w_j^T\} = I\delta_{kj}$ affects measurement and state equations. We assume $rankE = q+s$, $rankH = m$, $q \leq m$, $E = \begin{bmatrix} M & \ell \end{bmatrix}$, $F = \begin{bmatrix} N & T \end{bmatrix}$, $n_k = \begin{bmatrix} d_k & \eta_k \end{bmatrix}$.

$$rank \begin{bmatrix} zI - A & E \\ C & F \end{bmatrix} = n+q+s, \forall z \in C, \mid z \mid \geq 1 \quad (3)$$

$$rank \begin{bmatrix} -e^{jw}I + A & E & G \\ C & F & H \end{bmatrix} = n+m, \ \forall w \in [0, 2\pi] \quad (4)$$

Consider the following residual generator

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + K(y_k - C\hat{x}_k - Du_k) \quad (5)$$
$$r_k = L(y_k - C\hat{x}_k - Du_k) \quad (6)$$

where $\hat{x}_k$ is the state of the filter, $\hat{r}_k$ the output of the filter and where $L \in \Re^{q+s,m}$ and $K \in \Re^{n,m}$ are unknown matrices that we will be designed in order to fulfill fault detection and isolation requirements. The avoid the design of a statistical test on r_k with respect to the very complexe multiple inference decision theory not complete actually, the only solution is compute K and L so that the i^{th} component of d_k and completely from the other and from the disturbances.
From (1) and (5), the state estimation error $e_k = x_k - \hat{x}_k$ propagates as

$$e_{k+1} = (A - KC)e_k + (G - KH)w_k \quad (7)$$
$$+(M - KN)d_k + (\ell - KT)\eta_k \quad (8)$$
$$r_k = L(Ce_k + Hw_k + Nd_k + T\eta_k) \quad (9)$$

We can verify that the filter residual can be expressed

$$r(z) = H_{nr}(z)n(z) + H_{wr}(z)w(z) \quad (10)$$

with

$$H_{nr}(z) = R(z)\left[C(Iz - A)^{-1}E + F\right] \quad (11)$$
$$H_{wr}(z) = R(z)\left[C(Iz - A)^{-1}H + G\right] \quad (12)$$

where

$$R(z) = L\left[I - C[Iz - (A - KC)]^{-1}K\right] \quad (13)$$

Two cases compose our study

* **First case** $rank\left(F\right) = q+s$
  We obtain a perfect reconstructor by computing $\bar{R}(z)$ so that (10) gives
  $$r(z) = n(z) + \bar{R}(z)H_{wy}(z)w(z) \quad (14)$$
  After having parameterized all the solutions of $\bar{R}(z)H_{ny}(z) = I$, ($\bar{R}(z)$ will be the perfect left inverse of $H_{ny}(z) = \left[C(Iz - A)^{-1}E + F\right]$). We specially interest to the second case where $rank(F) < q+s$.

* **Second case** $rank\left(F\right) < q+s$
  The residual r_k will be filtered by a finite impulse reponse (FIR) filter $\hat{\xi}(z)$ so that $\hat{r}(z) = \hat{\xi}(z)r(z)$ is expressed as
  $$\hat{r}(z) = z^{-\alpha}n(z) + \hat{\xi}(z)\hat{R}(z)H_{wy}w(z) \quad (15)$$
  After having parametrized all the solutions of $\hat{\xi}(z)\hat{R}(z)H_{ny}(z) = Iz^{-\alpha}$ ($\hat{\xi}(z)\hat{R}(z)$ is the minimum time left inverse of $H_{ny}(z)$, the degrees of freedom remaining available on K and L will be computed to minimize $\left\|\hat{\xi}(z)\hat{R}(z)H_w(z)\right\|_2$.

3. A ROBUST FAULT DETECTION FILTER

3.1 *A robust perfect fault reconstructor*

The perfect fault reconstructor presented in this section will have no detection space since the maximum predictable space will be the whole state. With $rank(F) = q + s$, the left inverse $R(z) = L\left[I - C[Iz - (A - KC)]^{-1}K\right]$ of $H_{ny}(z)$ satisfying $R(z)H_{ny}(z) = I$ can be parameterized $\bar{K} \in \Re^{n,m-q-s}$ and $\bar{L} \in \Re^{q,m-q-s}$ as

$$K = EF^{+} + \bar{K}\Sigma, \quad L = F^{+} + \bar{L}\Sigma \tag{16}$$

with $\Sigma = \beta(I - FF^{+})$ where β is an arbitrary matrix chosen so that $rank(\Sigma) = m - q - s$.
Under (3), there always exists $\bar{K}$ so that the filter (7;9) of state $e_k = x_k - \hat{x}_k$ represented by

$$\Gamma(\bar{K}, \bar{L}) = \left(\bar{A} - \bar{K}\bar{C},\ 0,\ \begin{bmatrix} F^{+}C + \bar{L}\bar{C} \\ \bar{C} \end{bmatrix},\ \begin{bmatrix} I \\ 0 \end{bmatrix} \right) \tag{17}$$

with $\bar{A} = A - EF^{+}C$ and $\bar{C} = \Sigma C$ is stable.
$\Gamma(\bar{K}, \bar{L})$ is unreachable from the unknown inputs n_k and the state prediction $\hat{x}_k$ of the filter (5;6) is a state prediction of the whole state x_k. Under (3), the H_2 norm of $R(z)H_w(z)$ is minimized with respect to $\bar{K}$ and $\bar{L}$ if and only if

$$\bar{K} = (\bar{A}P\bar{C}^T + \bar{G}\bar{H}^T)(\bar{C}P\bar{C} + \bar{H}\bar{H}^T)^{-1} \tag{18}$$

$$\bar{L} = -F^{+}(CP\bar{C}^T + H\bar{H}^T)(\bar{C}P\bar{C}^T + \bar{H}\bar{H}^T)^{-1} \tag{19}$$

with $\bar{G} = G - EF^{+}H$, $\bar{H} = \Sigma H$ where $P > 0$, solution of

$$\begin{aligned} P = \bar{A}P\bar{A}^T + \bar{G}\bar{G}^T - (\bar{A}P\bar{C}^T + \bar{G}\bar{H}^T) \\ \times(\bar{C}P\bar{C}^T + \bar{H}\bar{H}^T)^{-1}(\bar{A}P\bar{C}^T + \bar{G}\bar{H}^T)^T \end{aligned} \tag{20}$$

is always a stabilizing solution ($\bar{A} - \bar{K}\bar{C}$ stable) under (3) and (4). We have $\left\|\bar{R}(z)H_w(z)\right\|_2^2 = tr(J)$ minimum with

$$\begin{aligned} J = F^{+}[CPC^T + HH^T - (CP\bar{C}^T + H\bar{H}^T) \\ \times(\bar{C}P\bar{C}^T + \bar{H}\bar{H}^T)^{-1}(\bar{C}PC^T + \bar{H}H^T)](F^{+})^T \end{aligned} \tag{21}$$

To show that P is stabilizing solution (poles of $\bar{A} - \bar{K}\bar{C}$ inside the unit circle), (21) can be rewritten as a standard algebraic Ricatti equation

$$P = \tilde{A}P\tilde{A}^T + \tilde{Q} - \tilde{A}P\bar{C}^T(\bar{C}P\bar{C}^T + \bar{H}\bar{H}^T)^{-1}\bar{C}P\tilde{A}^T \tag{22}$$

from $\tilde{A} = \bar{A} - \bar{G}\bar{H}^T(\bar{H}\bar{H}^T)^{-1}\bar{C}$ and $\tilde{Q} = \bar{G}\bar{G}^T - \bar{G}\bar{H}^T(\bar{H}\bar{H}^T)^{-1}\bar{H}\bar{G}^T$. On (22), we can show as in Keller and Darouach (1998) that the detectability of the pair $(\tilde{A}, \bar{C})$ is equivalent to (3) and the existence of no unreachable mode of $(\tilde{A}, \tilde{Q}^{1/2})$ on the unit circle equivalent to (4) leading to an unique stabilizing solution (De Souza et al.,1986)

3.2 *A robust minimum-time fault reconstructor*

Under $rank \begin{bmatrix} -Iz + A & E \\ C & F \end{bmatrix} = n + q + s$ for almost all z, the delay α of $H_{ny}(z)$ is finite and given by the degree of the unitary interactor matrix $\xi(z)$ (a polynomial matrix so that $\xi(z)[\xi(z)]^* = I$) satisfying

$$\hat{H}_{ny}(z) = H_{ny}(z)\xi(z) = C(Iz - A)^{-1}\hat{E}_\alpha + \hat{F}_\alpha \tag{23}$$

with $rank(\hat{F}_\alpha) = q$.
The unitary interactor matrix $\xi(z)$, $\hat{E}_\alpha$ and $\hat{F}_\alpha$ can be computed by the inversion algorithm of Silverman (1969): Assume that the inversion algorithm of Silverman is applied on the transposed system $\Gamma^T = (A^T, C^T, E^T, F^T)$ where $H^T_{ny}(z) = E^T(Iz - A^T)^{-1}C^T + F^T$ and gives $\xi^T(z)$ with $\xi^T(z)[\xi^T(z)]^* = I$ so that $\hat{H}^T_{ny}(z) = \xi^T(z)H^T_{ny}(z) = \hat{E}^T_\alpha(Iz - A^T)^{-1}C^T + \hat{F}^T_\alpha$. By transposing these results, we obtain the unitary interactor matrix $\xi(z) = \xi_0(z)\xi_1(z)\ldots\xi_{\alpha-1}(z)$ satisfying (23) where

$$rank \begin{bmatrix} -Iz + A & \hat{E}_\alpha \\ C & \hat{F}_\alpha \end{bmatrix} = rank \begin{bmatrix} -Iz + A & E \\ C & F \end{bmatrix} \tag{24}$$

(Kobayashi and Nakamizo, 1982)

***Theorem** 1.* From the finite impulse reponse (FIR) filter $\hat{\xi}(z) = z^{-\alpha}\xi(z)$ (which is proper under $\alpha = max\{\rho_i\}$ with ρ_i the column-degrees of the polynomial matrix $\xi(z)$), the minimum-time left inverse $\hat{\xi}(z)\hat{R}(z)$ of $H_{ny}(z)$ (with $\hat{R}(z) = L\left[I - C[Iz - (A - KC)]^{-1}K\right]$) satisfying

$$\hat{\xi}(z)\hat{R}(z)H_{ny}(z) = Iz^{-\alpha} \tag{25}$$

can be parameterized from $\hat{K} \in \Re^{n,m-(q+s)}$ and $\hat{L} \in \Re^{q,m-(q+s)}$ as

$$K = \hat{E}_\alpha\hat{F}^{+}_\alpha + \hat{K}\hat{\Sigma} \ \text{ and } \ L = \hat{F}^{+}_\alpha + \hat{L}\hat{\Sigma} \tag{26}$$

with $\hat{\Sigma} = \hat{\beta}(I - \hat{F}_\alpha\hat{F}^{+}_\alpha)$ where $\hat{\beta}$ is an arbitrary matrix chosen so that $rank(\hat{\Sigma}) = m - q - s$. Under (3), there always exists $\hat{K}$ so that the filter (7;9) described by

$$\Gamma(\hat{K}, \hat{L}) = \left(\hat{A} - \hat{K}\hat{C},\ \hat{E},\ \begin{bmatrix} \hat{F}^{+}_\alpha C + \hat{L}\hat{C} \\ \hat{C} \end{bmatrix},\ \begin{bmatrix} \hat{F}^{+}_\alpha F \\ 0 \end{bmatrix} \right) \tag{27}$$

with $\hat{E} = E - \hat{E}_\alpha\hat{F}^{+}_\alpha F$, with $\hat{A} = A - \hat{E}_\alpha\hat{F}^{+}_\alpha C$, $\hat{C} = \hat{\Sigma}C$ is stable. The state $e_k = x_k - \hat{x}_k$ of $\Gamma(\hat{K}, \hat{L})$ is reachable from the unknown inputs n_k and the state prediction $\hat{x}_k$ of the filter is not a state prediction of the whole state x_k.

***Demonstration** 1.* under (3), the system $\Gamma = (A, E, C, F)$ has a finite delay α. So, $\hat{H}_{nr}(z) = C(Iz - A)^{-1}\hat{E}_\alpha + \hat{F}_\alpha$ has no delay and $\hat{R}(z)\hat{H}_{ny}(z) = I$ can be parameterized as in paragraph 3.1 leading to (26) where $\hat{\xi}(z) = z^{-\alpha}\xi(z)$ is a causal

FIR filter since α is the degree of $\xi(z)$. The relation $\hat{R}(z)\hat{H}_{ny}(z) = I$ can be equivalently rewritten $\hat{R}(z)\hat{H}_{ny}(z) = \xi^*(z)$ where $\hat{\xi}(z)\hat{R}(z)H_{ny}(z) = z^{-\alpha}\xi(z)\xi^*(z)$ leads to (25) since $\xi(z)\xi^*(z) = I$. The filter (7;9) with $w_k = 0$ is then rewritten

$$\Gamma(\hat{K},\hat{L}) = \left(\hat{A} - \hat{K}\hat{C},\ \hat{E} - \widehat{K}\widehat{\Sigma}F,\ \begin{bmatrix} \hat{F}_\alpha^+ C + \hat{L}\hat{C} \\ \hat{C} \end{bmatrix},\ \begin{bmatrix} \hat{F}_\alpha^+ F \\ \widehat{\Sigma}F \end{bmatrix} \right) \tag{28}$$

where $\widehat{\Sigma}F = 0$ holds from the Silverman's algorithm (Under $\widehat{\Sigma}F = 0$, (28) gives (27)). However, compared to the results of section 3.1, the state $e_k = x_k - \hat{x}_k$ of $\Gamma(\widehat{K},\widehat{L})$ is now reachable from the faults since $\widehat{F} \neq 0$ and the state prediction $\widehat{z}_{k+1}$ is not the unbiased minimum variance prediction of the whole state x_{k+1}. From the results of section 3.1, $(\widehat{A},\widehat{C})$ is detectable and the unobservable modes of $(\widehat{A},\widehat{C})$ are the invariant zeros $\Gamma = (A,E,C,F)$. So, (3) under (24) is the unique existence of $\widehat{K}$ such that $\widehat{A} - \widehat{K}\widehat{C}$ is stable. Under $\widehat{A} - \widehat{K}\widehat{C}$ stable, the minimum-time left inverse $\widehat{\xi}(z)\widehat{R}(z)$ of $H_{ny}(z)$ is always stable.

***Theorem** 2.* Let $\hat{G} = G - \hat{E}_\alpha\hat{F}_\alpha^+ H$ et $\hat{H} = \hat{\Sigma}H$. The norm H_2 of the transfer $\hat{\xi}(z)\widehat{R}(z)H_w(z)$ is minimized under (26) with respect to $\hat{K}$ and $\hat{L}$ if and only if

$$\hat{K} = (\hat{A}P\hat{C}^T + \hat{G}\hat{H}^T)(\hat{C}P\hat{C}^T + \hat{H}\hat{H}^T)^{-1} \tag{29}$$
$$\hat{L} = \hat{F}_\alpha^+(CP\hat{C} + H\hat{H}^T)(\hat{C}P\hat{C}^T + \hat{H}\hat{H}^T)^{-1} \tag{30}$$

where P solution of

$$\begin{aligned} P = \hat{A}P\hat{A}^T + \hat{G}\hat{G}^T - (\hat{A}P\hat{C}^T + \hat{G}\hat{H}^T) \\ (\hat{C}P\hat{C}^T + \hat{H}\hat{H}^T)^{-1}(\hat{A}P\hat{C}^T + \hat{G}\hat{H}^T)^T \end{aligned} \tag{31}$$

is a stabilizing solution under (3) and (4). We have $\left\|\hat{\xi}(z)\widehat{R}(z)H_w(z)\right\|_2^2 = tr(J)$.

***Demonstration** 2.* $\left\|\hat{\xi}(z)\widehat{R}(z)H_w(z)\right\|_2^2$

$$\begin{aligned} &= \tfrac{1}{2\pi}\int_{-\pi}^{\pi} tr\left\{[\hat{\xi}(e^{j\theta})\widehat{R}(e^{j\theta})H_w(e^{j\theta})]\ [\hat{\xi}(e^{j\theta})\widehat{R}(e^{j\theta})H_w(e^{j\theta})]^*\right\} \\ &= \tfrac{1}{2\pi}\int_{-\pi}^{\pi} tr\left\{\hat{\xi}(e^{j\theta})\hat{\xi}^*(e^{j\theta})[\widehat{R}(e^{j\theta})H_w(e^{j\theta})]\ [\widehat{R}(e^{j\theta})H_w(e^{j\theta})]^*\right\} \end{aligned} \tag{32}$$

$= \left\|R(z)H_w(z)\right\|_2^2$.

So the minimization of $\left\|\hat{\xi}(z)\widehat{R}(z)H_w(z)\right\|_2^2$ with respect to $\widehat{K}$ and $\widehat{L}$ is then equivalent to the minimization of $\left\|R(z)H_w(z)\right\|_2^2$ with respect to $\bar{K}$ and $\bar{L}$ studied in the last paragraph.

To show that (31) has a stabilizing solution (poles of $\hat{A} - \hat{K}\hat{C}$ inside the unit circle) under (3) and (4), rewrite (31) as a standard algebraic Riccati equation

$$P = \widehat{\widehat{A}}P\widehat{\widehat{A}}^T + \widehat{\widehat{Q}} - \widehat{\widehat{A}}P\hat{C}^T(\hat{C}P\hat{C}^T + \hat{H}\hat{H}^T)^{-1}\hat{C}P\widehat{\widehat{A}}^T \tag{33}$$

where $\widehat{\widehat{A}} = \hat{A} - \hat{G}\hat{H}^T(\hat{H}\hat{H}^T)^{-1}\hat{C}$, $\widehat{\widehat{Q}} = \hat{G}\hat{G}^T - \hat{G}\hat{H}^T(\hat{H}\hat{H})^{-1}\hat{H}\hat{G}^T$. Via relation (24), we can show as in paragraph 3.1 that the detectability of the pair $(\widehat{\widehat{A}},\hat{C})$ is equivalent to (3) and the existence of no unreachable mode of $(\widehat{\widehat{A}},\widehat{\widehat{Q}}^{1/2})$ on the unit circle equivalent to (4). These completes the proof.

Remark1
Assume that the Silverman's inversion algorithm applied on $\Gamma = (A, E = M, C, 0)$ produces the following results $\Gamma = (A, \hat{E}_\alpha, C, \hat{F}_\alpha)$, $\hat{F}_\alpha = \left[\, CA^{\bar{\rho}_1 - 1}e_1\ ...\ CA^{\bar{\rho}_q - 1}e_q \,\right]$, with $\rho_i = min\{\because CA^{t-1}e_i \neq 0, t = 1,2,..\}$ at the final step $\alpha = max\{\bar{\rho}_i\}$. Then $rank\hat{F}_\alpha = q$ is the output separability condition often considered as the existence condition of fault detection filter (Chung and Speyer, 1998; Keller, 1999). In this case, we can verify that the order $\bar{\rho}_i$-order detection spaces $\bar{\Omega}_i = [e_i\ Ae_i\ \ldots\ A^{\bar{\rho}_i - 1}e_i]$, where $\bar{\Omega}_i$ is the detection space associated to the i^{eme} component of d_k, are solutions of $(\hat{A} - \hat{K}\hat{C})\bar{\Omega}_i \subseteq \bar{\Omega}_i$ and $e_i \subseteq \bar{\Omega}_i$ where $C\bar{\Omega}_i \bigcap (\sum_{j\neq i} C\bar{\Omega}_j) = \emptyset$ is clearly satisfied under $rank\hat{F}_\alpha = q$. From $\bar{\mu} = \sum_{j=1}^{q}\bar{\rho}_i$ where $\bar{\mu}$ is the order of $\bar{\Omega}$, we conclude that $\bar{\Omega} = \bar{\Omega}_1 \oplus ..\bar{\Omega}_i.. \oplus \bar{\Omega}_q$ is the direct sum of these subspaces, a result equivalent to the existence of the diagonal interactor matrix $\bar{\xi}(z) = diag\left[z_1^{\bar{\rho}}...z_q^{\bar{\rho}}\right]$ for the transfer of $\Gamma = (A,E,C)$ more restrictive than the existence condition (3) of an interactor matrix diagonal or not.

Remark2
In the case of a diagonal interaction matrix, the detectability indice ρ_i corresponds to time delay between the occurence of the fault d^i and it's first effect on the outputs. In the case of any interaction matrix, the generalization of the last reasoning can be acheived only with the term α corresponding to the degree of the polynomial matrix $\xi(z)$ representing the time delay allowing to faults to have a different repercussion on the outputs. In the stochastic case, the application of an global test necessite to delay all the faults by α, even if there exists one or many faults which have a repercussion more quick than α.

4. NUMERICAL EXAMPLE

This tutorial example describes the different steps for the design of the fault reconstructor in the context of remak1. Consider the following discrete-time system $\Gamma = (A, E = M, C)$ described by

$$A = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 1 & \lambda_2 & 0 & 0 \\ 0 & 1 & \lambda_3 & 0 \\ 0 & 0 & 1 & \lambda_4 \end{bmatrix},\quad C = \begin{bmatrix} 1\ 0\ 0\ 0 \\ 0\ 0\ 1\ 0 \\ 0\ 0\ 0\ 1 \end{bmatrix}, \tag{34}$$

$$E = \left[\, e_1\ \ e_2 \,\right] = \begin{bmatrix} 1\ 1 \\ 0\ 1 \\ 0\ 0 \\ 0\ 0 \end{bmatrix} \quad \text{et} \quad d_k = \begin{bmatrix} d_k^1 \\ d_k^2 \end{bmatrix} \tag{35}$$

where the output separability condition $rank[CA^{\bar{\rho}_1-1}e_1 \;\; CA^{\bar{\rho}_2-1}e_2] = 2$ with $\bar{\rho}_i = min\{t : CA^{t-1}e_i \neq 0, \; t = 1,2..\} = 1$ for $i = 1, 2$ is not satisfied since $rang[CA^{\bar{\rho}_1-1}e_1 \;\; CA^{\bar{\rho}_2-1}e_2] = rangCE = 1 < 2$. Applied on $\Gamma = (A, E, C)$, the inversion algorithm of Silverman produces the following results:

The first step. $\xi_0(z) = Iz$ leads to $\Gamma_1 = (A, E_1, C, F_1)$ with $E_1 = AE = \begin{bmatrix} \lambda_1 & \lambda_1 \\ 1 & 1+\lambda_2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $F_1 = CE = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$.

We have $q_1 = rankF_1 = 1 < 2$

The second step. Compute the orthogonal matrix $S_1 = \begin{bmatrix} 0.5 & -\frac{\sqrt{3}}{2} \\ 0.5 & \frac{\sqrt{3}}{2} \end{bmatrix}$ so that $F_1S_1 = [\,\tilde{F}_1 \; 0\,]$ with $\tilde{F}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $E_1S_1 = [\,\tilde{E}_1 \; \bar{E}_2\,] = \begin{bmatrix} \lambda_1 & 0 \\ 1+0.5\lambda_2 & \lambda_2\frac{\sqrt{3}}{2} \\ 0.5 & \frac{\sqrt{3}}{2} \\ 0 & 0 \end{bmatrix}$ where $rank\tilde{F}_1 = 1$. From $\xi_1(z) = S_1 \begin{bmatrix} 1 & 0 \\ 0 & z \end{bmatrix}$, we obtain $\Gamma_2 = (A, \hat{E}_2, C, \hat{F}_2)$ with $\hat{E}_2 = [\,\tilde{E}_1 \; A\bar{E}_2\,] = \begin{bmatrix} \lambda_1 & 0 \\ 1+0.5\lambda_2 & \frac{\sqrt{3}}{2}\lambda_2^2 \\ 0.5 & \frac{\sqrt{3}}{2}(\lambda_2+\lambda_3) \\ 0 & \frac{\sqrt{3}}{2} \end{bmatrix}$ and $\hat{F}_2 = [\,\tilde{F}_1 \; C\bar{E}_2\,] = \begin{bmatrix} 1 & 0 \\ 0 & \frac{\sqrt{3}}{2} \\ 0 & 0 \end{bmatrix}$ where $rang\hat{F}_2 = 2$ is the stopped condition of the recursive algorithm.

From $\Gamma_2(A, \hat{E}_2, C, \hat{F}_2)$, the fault reconstructor $\hat{z}_{k+1} = \hat{A}\hat{z}_k + \hat{E}_2\hat{F}_2^{+}y_k + \hat{K}\gamma_k$, $\gamma_k = \hat{\Sigma}y_k - \hat{C}\hat{z}_k$ and $r_k = (\hat{F}_2^{+} + \hat{L}\hat{\Sigma})(y_k - C\hat{z}_k)$, parameterizing all the minimum-time inverses the system $\Gamma = (A, E, C)$ from the degrees of freedom $\hat{K} \in \Re^{4,1}$ and $\hat{L} \in \Re^{2,1}$, is obtained by the computing of the left inverse $\hat{F}_2$ given by $\hat{F}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{bmatrix}$. This leads to $\hat{\Sigma} = \hat{\beta}(I - \hat{F}_2\hat{F}_2^{+}) = \begin{bmatrix} 0 & 0 & \tau \end{bmatrix}$ with $\hat{\beta} = \begin{bmatrix} 0 & 0 & \tau \end{bmatrix}$ where the arbitrary scalar τ must be chosen so that $rang\hat{\Sigma} = 1$ and where

$$\hat{A} = A - \hat{E}_2\hat{F}_2^{+}C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.5\lambda_2 & \lambda_2 & -\lambda_2^2 & 0 \\ -0.5 & 1 & -\lambda_2 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{bmatrix} \tag{36}$$

and $\hat{C} = \hat{\Sigma}C = [\,0 \; 0 \; 0 \; \tau\,]$.

The unitary interactor matrix is given by $\xi(z) = \xi_0(z)\xi_1(z) = \begin{bmatrix} 0.5z & -\frac{\sqrt{3}}{2}z^2 \\ 0.5z & \frac{\sqrt{3}}{2}z^2 \end{bmatrix}$ and its degree $\alpha = 2$ is the delay of $\Gamma = (A, E, C)$.The FIR filter $\hat{\xi}(z) = z^{-2}\xi(z)$ is then implemented as $\begin{bmatrix} \hat{n}_k^1 \\ \hat{n}_k^2 \end{bmatrix} = \sum_{j=0}^{2} W_j r_{k-j}$ with $W_0 = \begin{bmatrix} 0 & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} \end{bmatrix}$ and $W_1 = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 0 \end{bmatrix}$.

After the filter's optimization via the computation of $\hat{K}$ and $\hat{L}$ which do not depend of the choice of τ since the pair $(A^{max}, C^{max}) = (\lambda_4, \tau)$ is observable $\forall\tau \neq 0$ ensuring $rank\hat{\Sigma} = 1$, the output of $\hat{\xi}(z)$ gives a minimum variance estimates of the fault $\begin{bmatrix} d_k^1 \\ d_k^2 \end{bmatrix}$ tel que $E(\begin{bmatrix} \hat{r}_k^1 \\ \hat{r}_k^2 \end{bmatrix}) = \begin{bmatrix} d_{k-2}^1 \\ d_{k-2}^2 \end{bmatrix}$ is reached after the convergence of the filter.

This example gives the state transformation matrix $T = \begin{bmatrix} X \\ Q \end{bmatrix}$ with $X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$ so that $XQ^T = 0$. So, the pair $(A^{min}, E^{min}) = \left(\begin{bmatrix} 0 & 0 & 0 \\ -0.5\lambda_2 & \lambda_2 & -\lambda_2^2 \\ -0.5 & 1 & -\lambda_2 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \right)$ is clearly of dimension $\mu = 3$ recovered by the relation $\mu = \rho_1 + \rho_2$ where $\rho_1 = 1$ and $\rho_2 = 2$ are the degrees of the first and second column of $\xi(z)$, respectively . With $C^{min} = \hat{F}_2^{+}C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{3} \end{bmatrix}$, the system $\Gamma^{min} = (A^{min}, E^{min}, C^{min})$ can be equivalently rewritten from $S = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & \lambda_2 \\ 0 & 0 & 1 \end{bmatrix}$ as $\Gamma^{min} = (S^{-1}A^{min}S, S^{-1}E^{min}, C^{min}S) = \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{3} \end{bmatrix} \right)$ where the pair $(S^{-1}A^{min}S, C^{min}S) = \left(\begin{bmatrix} F_1 & 0 \\ 0 & F_2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{3} \end{bmatrix} \right)$ has a deadbeat observable companion form with $F_1 = [0]$ et $F_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ tho nilpotent matrices of index of nilpotency $\rho_1 = 1$ and $\rho_2 = 2$ respectively. With $\alpha = max\{\rho_1, \rho_2\} = 2$, we have $\begin{bmatrix} 0 & 0 & 0 \\ -0.5\lambda_2 & \lambda_2 & -\lambda_2^2 \\ -0.5 & 1 & -\lambda_2 \end{bmatrix}^2 = 0$ illustrating the deadbeat property of detection space Ω.

5. CONCLUSION

This paper has presented an unified approach for robust fault detection and isolation in discrete time systems affected by additive noises and faults on measurement and state equations simultaneously. The fault reconstructor produces a minimum-time estimation of faults and an optimal reduced state prediction of the maximum predictable subspace of the state. Not often studied in the field in FDI, the maximum reduced part of the state which can optimally predicted (or estimated in continous-time systems) by a fault detection filter is of great importance for the integration of the obtained FDI scheme in a FTC system which possesses the ability to accomodate system component failures automatically. From a state feedback based observer, this problem is currently under consideration by the authors where the minimum-time faults estimation and the maximum reduced state prediction are both involved in the reconfigurable control law.

References

Beard, R.V. (1971). *Failure accommodation in linear systems through self-reorganisation.* Ph.D dissertation,MA.

Chen, R.H., Speyer, J.L. (2002). Robust multiple fault detection filter. *International Journal of Robust and Nonlinear Control*, 12:675–696.

Chen, R.H., Mingori, D.L. and Speyer, J.L. (2003). Optimal stochastic fault detection filter. *Automatica*, 39:377–390.

Chung, W.H and Speyer, J.L. (1998). A game theoritic fault detection filter. *IEEE Trans. Automatic Control*, AC-43:933–935.

Commault, C. (1999). On the disturbed fault detection and isolation problem. *Systems Control Letters* , 38:73–78.

De souza, C.E and Gevers, M.R. and Goodwin, G.C (1986). Riccati equations in optimal filtering of nonstabilizable systems having singular state transition matrices. *IEEE Trans. Automatic Control*, AC-31:831–838.

Erme, E and Silverman, L.M. (1976). Minimal dynamic inverse for linear systems with arbitrary initial states. *IEEE Trans. Automatic Control*, AC-21:766–770.

Frank P.M. (1990). Fault diagnosis in dynamic systems using analytical and knowledge based redundancy-a survey and some new results. *Automatica*, 26:459–474.

Jamouli, H., Keller, J.Y., and Sauter, D. (2003). Fault isolation filter with unknown inputs in stochastic systems. *IFAC Safeprocess'03, Washington*, 531–536.

Jones, H.L. (1973). *Failure detection in linear systems.* Ph.D dissertation,MA. USA.

Hsu,P.L and Shen, L.C. (1995). A robust diagnostic approach for uncertain system: an example for the jet engine sensor systems. *SPIE's Conference on Sensing, Actuating, and Control*, 2494:48–59.

Hou,M. and Patton, R.J. (1998). Input observability and input reconstruction. *Automatica*, 34:789–794.

Hou,M. and Muller, P.C. (1994). Fault detection and isolation observers. *International Journal Control*, 60:827–846.

Keller, J.Y. (1999). Fault isolation filter for linear systems. *Automatica*, 35:1701–1706.

Kobayashi, N. and T. Nakamizo (1982). An observer design for systems with unknown inputs. In *International Journal Control*, 35:605–619.

Kwakernaak, H (1976). Asymptotic root loci multivariable linear regulators and linear optimal filters. In *IEEE Trans. Automatic Control*, AC-17:79–86.

Li-Cheng, sh. and Pau-Lo, Hs (1998). Robust design of fault isolation observers. In *Automatica*, 34(1):1421–1429.

Liu, B., Si, J. (1997). Fault isolation filter design for time invariant systems. *IEEE Trans. Automatic Control*, AC-21:775–778.

Massoumnia, M. (1986). A geometric approach to the synthesis of failure detection filters. *IEEE Transactions on Automatic Control*, AC-31:839–846.

Nagpal, K., Helmick, R., Sims, C. (1987). Reduced-order estimation: part1.Filtering. *International Journal of Control*, 45:1867–1888.

Nikoukhah, R (1994). Innovations generation in the presence of unknown inputs : application to robust failure detection. *Automatica*, 30:1851–1867.

Park, J., Rizzoni, G. (1994a). A new interpretation of the fault detection filter: part1. *International Journal of Control*, 60:767–787.

Park, J., Rizzoni, G. (1994b). A new interpretation of the fault detection filter: part2. *International Journal of Control*, 60:1033–1055.

Patton, R.J. (1997). The 1997 situation (survey). *IFAC SAFEPROCESS'97*, Hull, UK, 2:26–28.

Patton, R.J. and Chen, J (1991). A review of parity space approaches for fault diagnosis. *IFAC SAFEPROCESS conference*, Baden-Baden, Germany, 65–81.

Parlangeli, G. and Valcher, M.E (2002). Disturbed fault detection and isolation problems for linear state models in a noisy environment. *IFAC 2002*, Barcelone, Espagne.

Rausch, H.E. (1995). Autonomous control reconfiguration. *IEEE Control Systems Magazine*, 15(6):37–49.

Silverman, L.M. (1969). Inversion of multivariable linear systems. *IEEE Transaction on Automatic Control*, 14:270–276.

White, J.E., Speyer, J.L. (1987). Detection filters design: Spectral theory and algorithms. *IEEE Transaction on Automatic Control*, 32:593–603

Wonham, W.M. (1985). *Linear Multivariable Control: a geometric approach.* Springer,Berlin.

Wolowich, W.A. and Falb, P.L (1976). Invariants and Canonical forms under dynamic compensation. SIAM. Journal Control Optim,14, 996-1008

Wunnenberg, J. and Frank, P.M (1987). Sensor fault using detection via robust observers. System fault diagnostics, Reliability and related Knowledge-Based Approaches,1, 147-160