ADAPTIVE BACKSTEPPING CONTROL OF SYSTEMS WITH UNCERTAIN NONSMOOTH ACTUATOR NONLINEARITY

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Abstract: In this paper, we present a new scheme to design adaptive controllers for uncertain systems containing nonsmooth nonlinearities in the actuator device. The control design is achieved by introducing certain well defined sign functions and Neural Networks approximations and by using the backstepping technique. For the design and implementation of the controller, no knowledge is assumed on the unknown system parameters and nonlinearity. It is shown that the proposed controller not only can guarantee global stability, but also transient performance.*Copyright©2005 IFAC*

Keywords: Adaptive control, backstepping, neural networks, global stability

1. INTRODUCTION

Adaptive control is becoming popular in many fields of engineering and science as concepts of adaptive systems are becoming more attractive in developing advanced applications. It faces many important challenges, especially in nontraditional applications, such as nonsmooth nonlinearities. Several adaptive control schemes have recently been proposed, see for examples (Tao and Kokotovic, 1995*a*), (Pare and How, 1998), (Cho and Bai, 1998), (Ahmad and Khorrami, 1999) and (Su *et al.*, 2000). In these papers, an adaptive inverse technique was constructed to deal with continuous-time model reference adaptive control. An adaptive inverse cascaded with the plant was employed to cancel the effects of nonlinearity. Sometimes, it is not easy to get the inverse of the nonlinearity, so this scheme cannot be employed. The compensation scheme is considered in (Tao and Kokotovic, 1995*a*) for hysteresis, (Tao and Kokotovic, 1995*b*) for backlash, (Lewis *et al.*, 1999) for dead-zone, (Tang *et al.*, 2003) for actuator failure. All the known approaches in dealing with compensation assume that the uncertain parameters in the system and nonsmooth nonlinearities must be inside some known compact sets. Dead-zone pre-compensation using Neural Networks (NNs) have been used extensively in feedback control systems (Selmic and Lewis, 2000). The NN has two layers or weights consisting a NN estimator and a NN compensator. In the above mentioned approaches, the uncertain NN weights must be within a known compact set. Thus, the disturbance-like term from NN approximation will be bounded with known bounds. This assumption will make control design simpler. Also, the developed scheme cannot achieve good transient performance.

This paper will address the control of nonlinear systems with unknown parameters and nonsmooth nonlinearities in the actuator. The existence of such nonsmooth nonlinearities imposes a great challenge for the controller development. The nonsmooth nonlinearity is not required to be symmetric. To address such a challenge, NNs will be adopted to model the plant and the controller

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is constructed based on NNs. The NNs, used to approximate the nonlinearities in the plant, is adjusted by an adaptive law based on the backstepping approach. To compensate for the effect from the NN approximation, we design a new sign function, which is continuous and differentiable, and employs it in the recursive backstepping technique. The estimators are used to handle such terms. Owing to the approximation of nonsmooth nonlinearity, the new function and the backstepping technique, a priori knowledge on system parameters and nonlinearities is no longer needed. Besides showing global stability of the system, transient performance in terms of L_2 norm of the tracking error is derived to be an explicit function of design parameters and thus the proposed scheme allows designers to achieve the closed-loop behavior by tuning design parameters in an explicit way.
This paper is organized as follows: Section 2 states the problem of this paper and assumptions on the nonlinear systems. Section 3 introduces the approximation of nonlinearity using NNs. Sections 4 presents adaptive control design based on the backstepping technique and analyzes stability and performance. Simulation results are presented in Section 5. Finally, Section 6 concludes the paper.

2. PROBLEM STATEMENT

Consider a single-input single-output nonlinear system of the form

$$\dot{x}_i(t) = x_{i+1}(t) + \theta^T \phi_i(\bar{x}_i(t)) \qquad i = 1, \ldots, n-1$$

$$\dot{x}_n(t) = \phi_0(x(t)) + \theta^T \phi_n(x(t)) + \omega \qquad (1)$$

$$\omega = N(u), \quad y(t) = x_1(t) \qquad (2)$$

where $\bar{x}_i(t) = [x_1(t), \ldots, x_i(t)]^T, x(t) = [x_1(t), \ldots, x_n(t)]^T \in R^n$, $\omega \in R$ and $y(t)$ are state variables, system input and output respectively, $\theta \in R^r$ is unknown constant parameter vector and $\phi_i \in R^r, i = 1, \ldots, n$ are known nonlinear functions. The nonlinear system is assumed to be preceded by the actuating device $\omega = N(u)$, ω being the actuator output not available for control and u being the actuator input. Moreover, the actuator is assumed to be contained a nonsmooth nonlinearity, such as hysteresis or dead-zone nonlinearity. It should be noted that more general classes of nonlinear systems can be transformed into this structure (Tao and Kokotovic, 1996; Tao and Lewis, 2001).
The control objective is to design an adaptive control law for $u(t)$ in (2), such that the plant output $y(t)$ follows the desired reference signal $y_r(t)$.
Assumption 1. The desired trajectory $y_r(t)$ and its $(n-1)$th order derivatives are known and bounded.
The control objectives are to design backstepping adaptive control laws such that
- The closed loop system is globally stable in the sense that all the signals in the loop are uniformly ultimately bounded;
- The tracking error $y(t) - y_r(t)$ is adjustable during the transient period by an explicit choice of design parameters and $\lim_{t\to\infty} |y(t) - y_r(t)| \le \delta_1$ for an arbitrary specified bound δ_1.

3. FUNCTION APPROXIMATION USING NEURAL NETWORKS

In this section, we present NN approximation of a piecewise continuous function. For the neural networks, the theoretical ability to uniformly approximate functions with a specified degree of accuracy has been demonstrated in (Tao and Lewis, 2001; Cybenko, 1989).

3.1 NN Approximation of Continuous Functions

Any function can be approximated by a two-layer NN mapping with appropriate weights (Cybenko, 1989) on a compact set. In other words, any function $f(x) \in C(S)$, with S a compact subset of R^n, there exists

$$f(x) = \sum_{k=0}^{L} w_k \sigma_k(m_k^T x + n_k) + \epsilon(x)$$

$$= W^T \sigma(M, x, N) + \epsilon(x) \qquad (3)$$

where $W = [w_0, \ldots, w_L]^T$, $M = [m_0, \ldots, m_L]^T$, $N = [n_0, \ldots, n_L]^T$, $\sigma(\cdot) = [\sigma_0(\cdot), \ldots, \sigma_L(\cdot)]^T$, $\epsilon(x)$ is the NN approximation error, which is bounded by $\| \epsilon(x) \| < \epsilon_N$. Moreover, for any ϵ_N, one can find a NN such that $\| \epsilon(x) \| < \epsilon_N$, for all $x \in S$. The weights m_k in the first layer are selected randomly and will not be tuned. The weights w_k in the second layer are tunable. The function $\sigma(\cdot)$ could be any continuous sigmoid function (Cybenko, 1989). Here, we choose $\sigma(\cdot)$ as

$$\sigma(t) = \frac{1}{1 + exp^{-t}} \qquad (4)$$

This result shows that any continuous function can be approximated arbitrarily well using a linear combination of sigmoidal functions. This is well known as the NN universal approximation property.

3.2 Compensation of Nonlinearity

In this section, an NN precompensator for a general model is given. It is not required to be symmetric. The generality of the method and applicability to a broad range of nonlinear functions make this approach a potentially useful tool for compensation of backlash, hysteresis, and other nonlinearities.

For any unknown nonlinear function $N(u)$, we have the following assumption.

Assumption 2: The function $N(u)$ is invertible and continuous.

By assumption, there exists $N^{-1}(v)$, such that

$$N(N^{-1}(v)) = v \tag{5}$$

The function $N^{-1}(v)$ can be expressed in equivalent form as

$$N^{-1}(v) = v + \omega_{NN}(v) \tag{6}$$

where $\omega_{NN}(v) = N^{-1}(v) - v$. Equation (6) can be viewed as a direct feedforward term plus a correction term.

Based on the NN approximation property, one can approximate the nonlinear function by

$$N(u) = W^T \sigma(M, u, N) + \epsilon(u) \tag{7}$$

Also, we can design an NN for the approximation of the modified inverse function given in (6) by

$$\omega_{NN}(v) = W_0^T \sigma(M_0, v, N_0) + \epsilon_0(v) \tag{8}$$

In these equations $\epsilon(u), \epsilon_0(v)$ are the NN reconstruction error and W, W_0 are ideal target weights. The reconstruction error is bounded by $\| \epsilon \| < \epsilon_N(u)$ and $\| \epsilon_0 \| < \epsilon_{N0}(v)$. The weights in the first layer M, M_0, N, N_0 in both (7) and (8) are fixed.

Define $\hat{W}, \hat{W}_0$ as the estimates of the ideal NN weights, which are given by updating laws. Furthermore, define the weight estimation errors as

$$\tilde{W} = W - \hat{W}, \quad \tilde{W}_0 = W_0 - \hat{W}_0 \tag{9}$$

and estimations of the nonlinearity and modified inverse function as

$$\hat{N}(u) = \hat{W}^T \sigma(M, u, N) \tag{10}$$

$$\hat{\omega}_{NN}(v) = \hat{W}_0^T \sigma(M_0, v, N_0) \tag{11}$$

The expressions (10) and (11) represent, respectively, an NN approximation of the nonlinearity $N(u)$ and of the modified inverse (6). Note that

$$u = v + \hat{\omega}_{NN}(v) \tag{12}$$

Note that we use two NNs. The first NN is used as an estimator of nonlinearity, while the second is used as a compensator. The structure of the NN precompensator and estimator are shown in Figure 1.

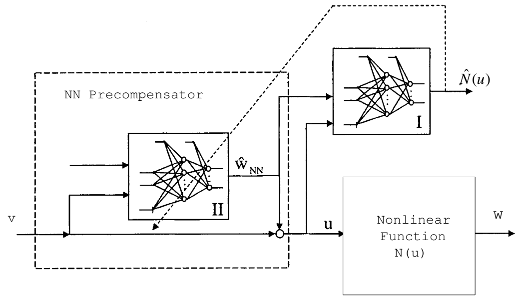

Fig. 1. NN Compensation Scheme

Theorem 1. Given the NN approximation function (11), (12) and the NN observer (10), the approximation of $N(u)$ is given by

$$\begin{aligned} N\,(u) &= v + \tilde{W}^T \sigma(M, u, N) M^T \hat{\omega}_{NN} + d(t) \\ &- \hat{W}^T \sigma(M, u, N) M^T \tilde{W}_0^T \sigma(M_0, v, N_0) \end{aligned} \tag{13}$$

where the modelling mismatch term $d(t)$ is given by

$$\begin{aligned} d(t) &= -\tilde{W}^T \sigma(M, u, N) M^T W_0^T \cdot \\ &\quad \sigma(M_0, v, N_0) - b(t) + \epsilon(u) \quad (14) \\ b(t) &= W^T \sigma\big[M, v + \hat{W}_0^T \sigma(M_0, v, N_0), N\big] \\ &\cdot M^T \, \epsilon_0\,(v) + W^T R_1(\tilde{W}_0, v) + \epsilon(v + \omega_{NN}) \quad (15) \end{aligned}$$

Proof: See (Selmic and Lewis, 2000).

In (13), the first term has known factors multiplying $\tilde{W}$, the second term has known factors multiplying $\tilde{W}_0$, and a suitable bound term can be found for $d(t)$.

The Theorem 1 shows the effectiveness of the proposed NN structure. It shows that the estimates $\hat{W}, \hat{W}_0$ approach the actual neural network parameters W, W_0, and the NN effectively provides a pre-inverse for the nonlinearity. It is shown in Section 4 in deriving the NN update laws for $\hat{W}, \hat{W}_0$ that closed-loop stability can be guaranteed.

Remark 1. Note that the form of (13) is crucial in controller design in deriving the adaptive laws that guarantee closed-loop stability. Moreover, the residual term is bounded by a constant vector multiplied by a known function vector as in (16). Thus, adaptive control techniques can be applied to deal with this residual term. A similar technique has also been used in (Selmic and Lewis, 2000; Tao and Lewis, 2001; Su *et al.*, 2003), where the approximator was constructed by neural networks or fuzzy logic. In general, this neural network scheme or fuzzy logic scheme could be used for any continuous invertible functions. Therefor, it is a powerful result to deal with general nonlinearities in motion control systems.

Firstly, we define $\|\|$ as any suitable vector norm. Given $A = [a_{ij}]$, the Frobenius norm is defined by $\| A \|^2 = tr(A^T A) = \sum_{i,j} a_{ij}^2$, with $tr()$ denoting the trace.
The following result gives the upper bound of the norm $d(t)$. This is an important result to be used in the stability proof.

Lemma 1. The norm of the modelling mismatching term $d(t)$ in (13) is bounded by

$$\| d(t) \| \leq \beta^T Y(t) \tag{16}$$

where $\beta \in R^{4\times 1}$ is an unknown constant vector, being composed of optimal weight matrices and some bounded constants, and $Y(t) = [1, \| \hat{W} \|, \| \hat{W}_0 \|^2, \| \hat{W}_0 \|]^T$ is a known function vector.

Remark 2. Note that β is not assumed to be known. So, the residual term $d(t)$ is bounded by an unknown parameter vector with a known function vector as in (16). Unlike the normal NN approximation using the restricted assumption, the residual term is bounded by a known bound. All these uncertain parameter vectors will be estimated by the proposed adaptive update laws.

4. DESIGN OF ADAPTIVE CONTROLLERS

Before presenting the adaptive control design using the backstepping technique to achieve the desired control objectives, the following change of coordinates is necessary.

$$z_1 = y - y_r \tag{17}$$
$$z_i = x_i - y_r^{(i-1)} - \alpha_{i-1}, \qquad i = 2, 3, \ldots, n \tag{18}$$

where α_{i-1} is the virtual control at the *ith step* and will be determined in later discussions. We define functions $sg_i(z_i)$ and $\eta_i(z_i)$ as in (Zhou *et al.*, 2004).

$$sg_i(z_i) = \begin{cases} \dfrac{z_i}{|z_i|} & |z_i| \geq \delta_i \\ \dfrac{z_i}{(\delta_i^2 - z_i^2)^{n-i+2} + |z_i|} & |z_i| < \delta_i \end{cases} \tag{19}$$

$$\eta_i(z_i) = \begin{cases} 1 & |z_i| \geq \delta_i \\ 0 & |z_i| < \delta_i \end{cases} \tag{20}$$

where $\delta_i (i = 1, \ldots, n)$ is a positive design parameter. This ensures that the resulting functions are differentiable. We now illustrate the backstepping design procedures with details given for the last step.

• *Step i* $(i = 1, \ldots, n-1)$: For $z_i = x_i - \alpha_{i-1} - y_r^{(i-1)}$ and $V_i = V_{i-1} + \frac{1}{n-i+2}(|z_i| - \delta_i)^{n-i+2}\eta_i(z_i)$, we choose

$$\begin{aligned}\alpha_1 &= -(c_1 + \frac{1}{4})(|z_1| - \delta_1)^n sg_1(z_1) - \hat{\theta}^T \phi_1 \\ &\quad -(\delta_2 + 1) sg_1(z_1)\end{aligned} \tag{21}$$

$$\begin{aligned}\alpha_i &= -(c_i + \frac{5}{4})(|z_i| - \delta_i)^{n-i+1} sg_i(z_i) \\ &+ \sum_{j=1}^{i-1} [\frac{\partial \alpha_{i-1}}{\partial x_j} x_{j+1} + \frac{\partial \alpha_{i-1}}{\partial y_r^{(j-1)}} y_r^{(j)}] + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i \\ &- \big(\hat{\theta}^T - \sum_{k=2}^{i-1} (|z_k| - \delta_k)^{n-i+1} \eta_k sg_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}}\big) \cdot \\ &\big(\ \phi_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \phi_k\big) - (\delta_{i+1} + 1) sg_i(z_i)\end{aligned} \tag{22}$$

$$\tau_1 = \phi_1 (|z_1| - \delta_1)^n \eta_1 sg_1(z_1) \tag{23}$$

$$\begin{aligned}\tau_i &= (\phi_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \phi_k)(|z_i| - \delta_i)^{n+1-i} \eta_i sg_i(z_i) \\ &\quad + \tau_{i-1}\end{aligned} \tag{24}$$

where c_i is a positive design parameter. The derivative of V_i is given by

$$\begin{aligned}\dot{V}_i &\leq -\sum_{i=1}^{i} c_i (|z_i| - \delta_i)^{2(n-i+1)} \eta_i + \tilde{\theta}^T(\tau_i - \Gamma^{-1}\dot{\hat{\theta}}) \\ &\quad + (\sum_{k=2}^{i-1} (|z_k| - \delta_k)^{n-k+1} sg_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}})(\Gamma \tau_n - \dot{\hat{\theta}}) \\ &\quad + (|z_i| - \delta_i)^{n-i+1}(|z_{i+1}| - \delta_{i+1} - 1)\eta_i\end{aligned} \tag{25}$$

• *Step n*:
The derivative of z_n can be written as

$$\dot{z}_n = \phi_0(x(t)) + \theta^T \phi_n(x(t)) + N(u) - y_r^{(n)} - \dot{\alpha}_{n-1} \tag{26}$$

Using (13), we get

$$\begin{aligned}\dot{z}_n &= v - \hat{W}^T \sigma(M^T u + V) M^T \tilde{W}_0^T \sigma(M_0, v, N_0) \\ &\quad + \tilde{W}^T \sigma(M, u, V) M^T \hat{\omega}_{NN} + \phi_0(x(t)) \\ &\quad + \theta^T \phi_n(x(t)) - y_r^{(n)} - \dot{\alpha}_{n-1} + d(t)\end{aligned} \tag{27}$$

The control law is designed as follows:

$$\begin{aligned}v &= -(c_n + 1)(|z_n| - \delta_n) sg_n(z_n) - \hat{\theta}^T \phi_n - \phi_0 \\ &- sg_n(z_n)\hat{\beta} Y + \sum_{j=1}^{n-1} \frac{\partial \alpha_{i-1}}{\partial x_j} x_{j+1} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \\ &+ \sum_{j=1}^{n-1} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j-1)}} y_r^{(j)} - \big(\phi_n - \sum_{k=1}^{n-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \phi_k\big) \\ &\big(\ \hat{\theta}^T - \sum_{k=2}^{n-1} (|z_k| - \delta_k)^{n-k+1} sg_k \eta_k \cdot \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}}\big)\end{aligned} \tag{28}$$

and

$$u(t) = v + \hat{\omega}_{NN}(v) \tag{29}$$
$$\hat{\omega}_{NN}(v) = \hat{W}_0^T \sigma(M_0, v, N_0) \tag{30}$$

The parameter update laws are designed as follows

$$\dot{\hat{\theta}} = \Gamma \tau_n \quad (31)$$

$$\dot{\hat{\beta}} = \Gamma_1(|z_n| - \delta_n)\eta_n Y \quad (32)$$

$$\dot{\hat{W}} = \Gamma_2(|z_n| - \delta_n)\eta_n sg_n \sigma(M, u, N)M^T \hat{\omega}_{NN} \quad (33)$$

$$\dot{\hat{W}}_0 = -\Gamma_3(|z_n| - \delta_n)\eta_n sg_n \hat{W}^T \sigma(M, u, N) \cdot M^T \sigma(M_0, v, N_0) \quad (34)$$

where c_n is a positive parameter, and $\Gamma, \Gamma_1, \Gamma_2, \Gamma_3$ are positive definite matrices. We choose the Lyapunov function as follows:

$$V_n = \sum_{i=1}^{n} \frac{1}{n-i+2}(|z_i| - \delta_i)^{n-i+2}\eta_i + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1}\tilde{\theta} + \frac{1}{2}\tilde{W}^T \Gamma_2^{-1}\tilde{W} + \frac{1}{2}\tilde{W}_0^T \Gamma_3^{-1}\tilde{W}_0 + \frac{1}{2}\tilde{\beta}^T \Gamma_1^{-1}\tilde{\beta} \quad (35)$$

Since $|d(t)| \leq \beta^T Y$, by using adaptive laws (31-32), the derivative of V_n along (35) is given by

$$\begin{aligned}
\dot{V}_n \leq & -\sum_{i=1}^{n} c_i(|z_i| - \delta_i)^{2(n-i+1)}\eta_i + \tilde{\theta}^T(\tau_n - \Gamma^{-1}\dot{\hat{\theta}}) \\
& +\Big(\sum_{k=2}^{n-1}(|z_k| - \delta_k)^{n-k+1} sg_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}}\Big)(\Gamma\tau_n - \dot{\hat{\theta}}) \\
& +\tilde{W}^T\big[(|z_n| - \delta_n)\eta_n sg_n \sigma(M, u, N)M^T \hat{\omega}_{NN} \\
& -\Gamma_2^{-1}\dot{\hat{W}}\big] - \tilde{W}_0^T\big[\Gamma_3^{-1}\dot{\hat{W}}_0 + (|z_n| - \delta_n)\eta_n sg_n \\
& \cdot\hat{W}^T\sigma(M, u, N)M^T\sigma(M_0, v, N_0)\big] \\
& +\tilde{\beta}^T\big[(|z_n| - \delta_n)\eta_n Y - \Gamma_1^{-1}\dot{\hat{\beta}}\big] \\
= & -\sum_{i=1}^{n} c_i(|z_i| - \delta_i)^{2(n-i+1)}\eta_i \quad (36)
\end{aligned}$$

The boundedness of V_n can now be established. Integrating both sides of (36) gives

$$V_n(t) + \int_0^t \sum_{i=1}^{n} c_i(|z_i| - \delta_i)^{2(n-i+1)}\eta_i d\tau \leq V(0) \quad (37)$$

It implies $V_n(t)$ is bounded. The boundedness of V_n further implies that $z_i, i = 1, \ldots, n$, $\tilde{\theta}$, $\tilde{W}, \tilde{W}_0, \tilde{\beta}$ are bounded. From the Barbalat's lemma, we can conclude that for $1 \leq i \leq n, lim_{t\to\infty}(|z_i| - \delta_i)^{n-i+1}f_i = 0$, which implies that $lim_{t\to\infty}\dot{\hat{\theta}} = 0, lim_{t\to\infty}\dot{\hat{W}} = 0, lim_{t\to\infty}\dot{\hat{W}}_0 = 0$, and $lim_{t\to\infty}\dot{\hat{\beta}} = 0$, and in particular, $y - y_r$ converges to $[-\delta_1, \delta_1]$.
These results obtained for the above analysis are now summarized in the following theorem.

Theorem 2. Consider the uncertain nonlinear system (1) satisfying Assumptions 1-2. With the application of controller (29) and the parameter update laws (31) to (34), the following statements hold:
• The resulting closed loop system is globally stable.
• The tracking error approaches δ_1 asymptotically, i.e.,

$$\lim_{t\to\infty} |y(t) - y_r(t)| = \delta_1 \quad (38)$$

• The transient tracking error performance is given by

$$\| y(t) - y_r(t) \|_2 \leq \delta_1 + c_1^{-1/2n}\big(\| \theta(0) \|^2_{\Gamma^{-1}} + \| \beta(0) \|^2_{\Gamma_1^{-1}} + \| W(0) \|^2_{\Gamma_2^{-1}} + \| W_0(0) \|^2_{\Gamma_3^{-1}} \big)^{1/2n}$$

with $z_i(0) = \delta_i, i = 1, \ldots, n,$

Proof: From (37), we have

$$\| |z_1| - \delta_1 \|_2^{2n} = \int_0^\infty |(|z_1| - \delta_1)|^{2n} d\tau \leq \frac{1}{c_1}V(0)$$

Be setting $z_i(0) = \delta_i, i = 1, \ldots, n,$, the bound is given by

$$\| z_1 \|_2 \leq c_1^{-1/2n}\big(\| \theta(0) \|^2_{\Gamma^{-1}} + \| W(0) \|^2_{\Gamma_2^{-1}} + \| W_0(0) \|^2_{\Gamma_3^{-1}} + \| \beta(0) \|^2_{\Gamma_1^{-1}} \big)^{1/2n} + \delta_1$$

Remark 3. From Theorem 2, the following conclusions can be obtained:
• The transient performance depends on the initial estimate errors $\tilde{\theta}(0)$, $\tilde{W}(0)$, $\tilde{W}_0(0)$, $\tilde{\beta}(0)$ and explicit design parameters. The closer the initial estimates $\hat{\theta}(0)$, $\hat{W}(0)$, $\hat{W}_0(0)$, $\hat{\beta}(0)$ to the true values θ, W, W_0 and β, the better the transient performance.
• The bound for $\| y(t) - y_r(t) \|_2$ is an explicit function of design parameters and thus is computable. We can decrease the effects of the initial error estimates on the transient performance by increasing the adaptation gains $\Gamma, \Gamma_1, \Gamma_2, \Gamma_3$.
• The reduction of the error is at the expense of increasing the control signal.

5. SIMULATION STUDIES

In this section, we illustrate the above methodology on a system which is described by

$$\dot{x} = a\frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} + w \quad (39)$$

$$w = N(u) = \begin{cases} m(u(t) - b_r) & u(t) \geq b_r \\ 0 & b_l < v(t) < b_r \\ m(u(t) - b_l) & u(t) \leq b_l \end{cases} \quad (40)$$

where w represents the output of the dead-zone nonlinearity. The actual parameter value is $a = 1$. The parameters of the dead-zone are $b_r = 0.5, b_l = -0.6$ and $m = 1$. The objective is to control the system state x to follow a desired trajectory $y_r(t) = 2.5sin(t)$.
In the simulation study, the robust adaptive control law (29) and the parameter update laws (31) to (34) were used. NN I and NN II have $L = 10$ layer nodes. The parameters are chosen as $c_1 = 0.8, \Gamma = 0.1, \Gamma_1 = 0.2I_3, \Gamma_2 = 2I_{11}, \Gamma_3 = I_{11}$. Simulation results presented in Fig. 2 and Fig. 3 are the system tracking error and input. Clearly, all the results verify our theoretical findings and demonstrate the effectiveness of the proposed control scheme.

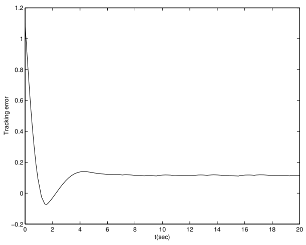

Fig. 2. Tracking error

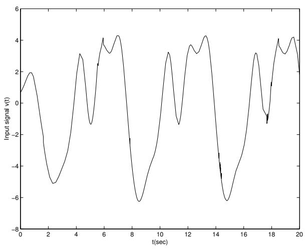

Fig. 3. Control signal $u(t)$

6. CONCLUSION

In this paper, a new adaptive control architecture is proposed for a class of nonlinear uncertain systems containing nonsmooth nonlinearity in the actuator device. Using backstepping technique, the control is designed by introducing certain well defined sign functions and NN approximations. The proposed adaptive control law not only can guarantee global stability, but also transient performance. For the design and implementation of the controller, no restricted assumptions are assumed on the unknown system parameters and nonlinearity.

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