LOW-COST FLEXIBLE SPEED CONTROL EXPERIMENTS

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Abstract: A set of experiments on speed control realised with $LEGO^{TM}$ elements is presented. Despite their simplicity and low cost, the setups allow to address several interesting control problems. To witness this statement, part of the activity for the course titled 'Engineering and Technology of Control Systems' is described. The characteristics of the proposed experiments make them very suitable both for hands-on and remote use.

Keywords: Education, laboratory, low-cost experiments, motors, speed control.

1. INTRODUCTION

This manuscript describes a set of experiments on speed control, that are realised with extremely low-cost resources, and are used at the Politecnico di Milano in the didactic laboratory for the undergraduate course titled 'Engineering and Technology of Control Systems' (ETCS). In this course, the students are taught the role and theory of operation of the main physical components of control systems, and control structures like disturbance compensation, multivariable decentralised control, split-range, cascade, and decoupling control. This subject is of great interest, as the involved skills are very important for any control engineer (EnTech, 1994; Bialkowski, 2000). Needless to say, experimental activity is very important for ETCS, but since the subject is vast, designing such an activity is not a trivial problem. The ETCS laboratory assignments on control structures are described in (Leva, 2002), while this work focuses on motion control. Some speed control experimental setups are presented, with the following characteristics.

The mechanics is entirely made with $LEGO^{TM}$ elements, and care has been taken to minimise the number of parts used. As a result the setups are very inexpensive, and can be assembled quickly and easily. The responses of the setups are similar to those of industrial speed controls. This is very important, as the various phenomena that appear in the experiments have a relative importance close to that encountered in real applications. The analog electronics for driving the motor and conditioning the speed signal is very simple, so that it can be understood with basic knowledge of electronics. This enhances the students' insight into the experiments, as the overall system's complexity is adequate for their cultural level (Amadi-Echendu and Higham, 1997). Since the experimental setup is *de facto* a toy operating

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at a voltage of 9V, the students can interact freely with all the elements of the mechanical and electronic system, without any safety problem. The software used for the experiments is written in the $LabVIEW^{TM}$ (*http://www.ni.com/labview*) programming environment. This choice makes the analog-to-digital and digital-to-analog interfacing extremely simple, and provides a very user-friendly operator interface, plus offering possibility of enabling the web-access to the experiments almost effortlessly. The control algorithms are completely visible, and can be modified by the students if required.

The presented setups and activity are consistent with the major issues involved in laboratory design for control education (Bristol, 1986; Åström and Lundh, 1992; Kheir *et al.*, 1997). Despite their simplicity, they provide responses that are similar to those encountered when dealing with real drives. Moreover, by modifying some elements, it is easy to obtain responses where the rigid behaviour dominates (as in most industrial drives) or the oscillatory modes due to transmission compliance are prevalent (thus leading to more challenging problems). The setups also allow to treat a variety of problems. It is possible to deal with set point tracking and disturbance and noise rejection, exploiting the two-degrees-of-freedom (2-d.o.f.) structure of industrial regulators—a frequently overlooked issue. Dealing with various problems on the same setup helps breaking the undue connection ‘process control means regulation problems and disturbance rejection, motion control means set point tracking’: the students are led to treat control structures as abstract, conceptual tools, and this is beneficial with respect to the comprehension of the underlying theory. To the authors’ knowledge, the use of LEGO elements for *modulating* control is not common, though - as shown herein - there are very interesting possibilities in that field.

Experiments with the presented setups are short, lasting typically some seconds or tenths of seconds. This allows to experiment with different control synthesis approaches in a reasonably short time (and, for example, (O’Dwyer, 2003) gives an idea of the possibilities with the PID control structure only), and makes the apparatuses very well suited for remote use via the web (though the matter is not treated herein for space reasons).

2. THE LABORATORY SETUP

2.1 The two apparatuses

The first apparatus (figure 1, above) is composed of two LEGO motors, with two pulleys connected by a rubber belt. The first motor (Motor A in figure 1a) is the actuator, while the other (Motor B) serves as tacho generator. A bar lies on the belt and can be lowered manually until it touches two underlying bushes. When the bar lies on the belt the friction is negligible; when the bar is lowered and touches the bushes the friction becomes significant. The role of the bar is illustrated in figure 1 (below).

The second apparatus is shown in figure 2. The actuator (Motor A) and the tacho generator (Motor B) are on the same axle, that drives a second axle through a couple of gears. The second axle has a centred mechanical load, and an eccentric load can be added as shown in figure 2.

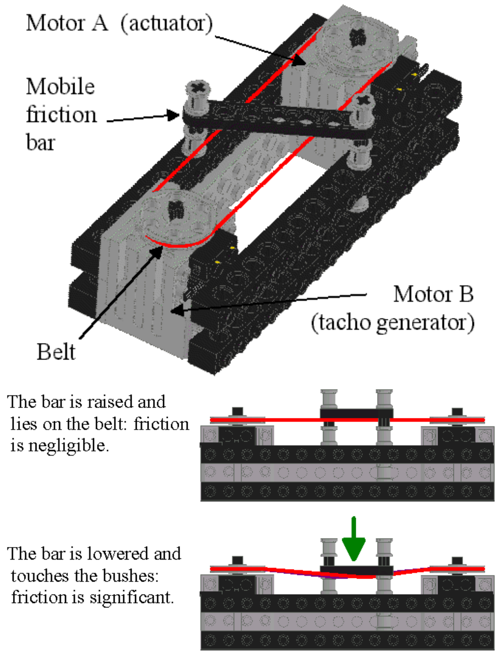

Fig. 1. Assembly of apparatus 1 (above) and role of the friction bar (below).

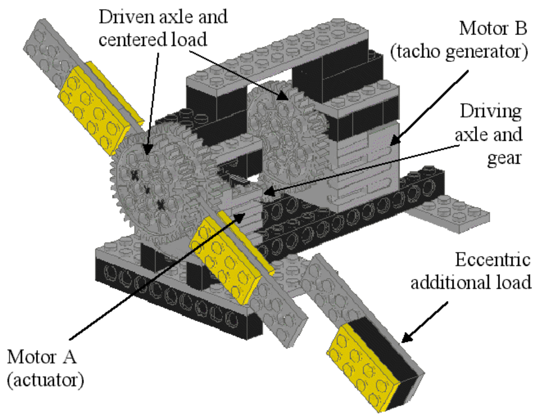

Fig. 2. Assembly of apparatus 2.

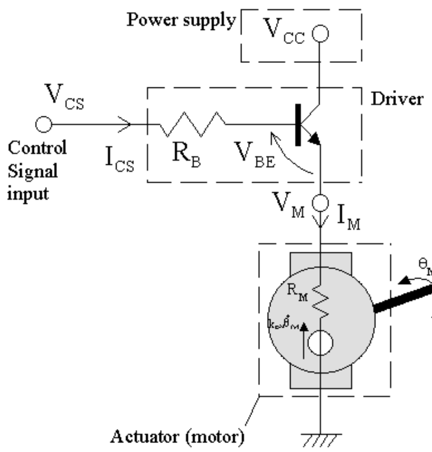

Fig. 3. Electric scheme of the motor and driver.

2.2 Interfacing the apparatuses to the PC

The PC is equipped with an A/D and D/A board, and the signals to and from the apparatuses are analogue voltages in the range 0-10 V. The LEGO motor, when used as a tacho generator, produces a voltage ranging from 0 to 9 V approximately, so there is no need for level adaptation. The interfacing circuitry is the same for both apparatuses. First, the motor driver is considered. Students are reminded that a driver has a 'raw' power input, a control signal input and an output for the actuator, and that the driver's role is to modulate the power released to the actuator according to the control signal.

In the case at hand, this task is accomplished by the simple emitter-follower current amplifier of figure 3, where the power input is the transistor collector, the control input is the base resistor and the actuator output is the transistor emitter. The driver's operation is ruled by the system

$$I_M = \begin{cases} (1+\beta)\dfrac{V_{CS}-V_M-V_{BE}}{R_b} & V_{CS}-V_M \geq V_{BE} \\ 0 & V_{CS}-V_M < V_{BE} \end{cases}$$
$$V_M = R_M I_M + k_{\omega M}\dot{\theta}_M$$
$$\tau_M = k_{\tau M} I_M$$
$$J_M \ddot{\theta}_M = \tau_M - B_M \dot{\theta}_M \tag{1}$$

where β is the transistor current gain, θ_M the motor angular position, J_M the motor inertia, $k_{\omega M}$ the ratio between counterelectromotive force and angular speed, τ_M the motor torque, $k_{\tau M}$ the ratio between torque and motor current, B_M the rotational friction factor, and the other symbols have the meaning shown in figure 3; the motor inductance is neglected for simplicity. It is brought to the students' attention that in (1) the last three equations are electro-mechanical; that is, the behaviour of the system formed by the driver and the motor depends also on its mechanical

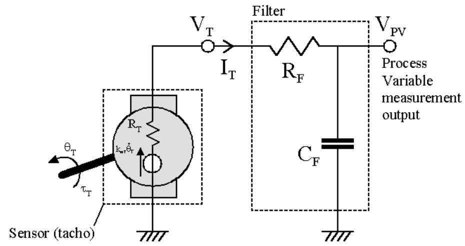

Fig. 4. Electric scheme of the tacho generator and filter.

connection with the remainder of the apparatus. The transistor used is a BC108B, although any low- or mid-power NPN would do the job, and the value of R_b (not critical for the driver's operation) is 10Ω.

Then, the tacho generator is dealt with. To interface it to the PC, the simple passive lowpass filter of figure 4 is used. The system is ruled by

$$\begin{aligned} J_T\ddot{\theta}_T &= \tau_T - B_T\dot{\theta}_T \\ V_T &= R_T I_T + k_{\omega T}\dot{\theta}_T \\ V_{PV} &= V_T - R_F C_F \dot{V}_{PV} \end{aligned} \tag{2}$$

i.e., again, by electromechanical equations. To determine the values of R_F and C_F, several speed responses to a motor command step in the various apparatus configurations are recorded with a digital oscilloscope. Then, the minimum settling time of these responses is taken as a measure of the time scale of the relevant plant dynamics; denoting by t_{set} that sampling time, the time constant $R_F C_F$ of the filter is set to $t_{set}/50$; R_F is chosen so as to be negligible with respect to the input impedance of the A/D card installed in the PC, leading to $R_F = 150\text{k}\Omega$ and $C_F = 100\text{nF}$. The value of the (stabilised) supply voltage Vcc is 9V.

2.3 The control software

The PC application used for the speed control experiments implements a two degree of freedom ISA PID regulator with output derivation (Åström and Hägglund, 1995), i.e., the control law

$$CS(s) = K\left((bSP(s)-PV(s)) + \frac{1}{sT_i}(SP(s)-PV(s)) - \frac{sT_d}{1+sT_d/N}PV(s)\right) \tag{3}$$

where $SP(s)$, $PV(s)$ and $CS(s)$ are, respectively, the Laplace transforms of the set point, the controlled variable and the control signal, K is the PID gain, T_i and T_d are the integral and the derivative time, N is the ratio between T_d and the time constant of a second pole required for the controller properness, and b is the set point weight in the proportional action.

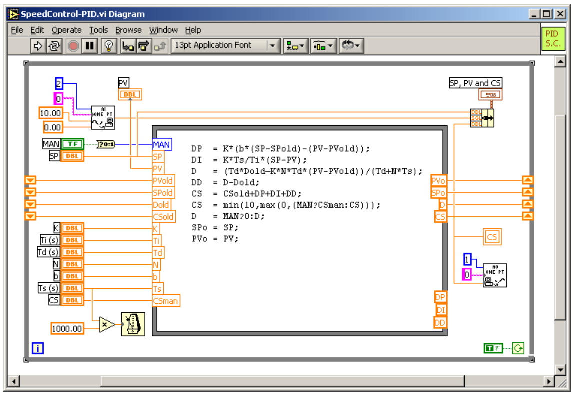

Fig. 5. Block diagram of the LabVIEW PC application for PID speed control.

Discretising (3) in incremental form with the backward difference method, and implementing the antiwindup and the auto/manual switching by a cascaded integrator leads, in the *LabVIEW* 'G' programming language, to the block diagram shown in figure 5.

3. THE LABORATORY ACTIVITY

3.1 Experiences with apparatus 1

Denoting by θ_m and θ_t the motor and tacho angular positions, by J_m and J_t their inertiæ, by B_m and B_t their friction coefficients, by τ_m the motor torque, by V_m the voltage applied to the motor, by T the belt tension, by K_b the belt (linear) elastic coefficient and by r the pulleys' radius, a simple model of apparatus 1 (neglecting the role of the friction bar) can be written as

$$\left\{\begin{array}{rcl} J_m\ddot{\theta}_m &=& \tau_m - Tr - B_m\dot{\theta}_m \\ J_t\ddot{\theta}_t &=& Tr - B_t\dot{\theta}_t \\ T &=& K_b\, r\,(\theta_m - \theta_t) \\ \tau_m &=& K_T I_m \\ I_m &=& \left(V_m - K_\omega\dot{\theta}_m\right)/R_m \end{array}\right. \tag{4}$$

The control variable is the voltage applied on the motor, while the controlled variable is the velocity of the tacho, calculated through its simple algebraic relation with the voltage induced in the tacho itself, i.e.,

$$\dot{\theta}_t = \frac{V_t}{K_\omega} \tag{5}$$

where the tacho voltage V_t is available through the acquisition board). From (4), one can compute the transfer function between the voltage applied to the motor and the tacho velocity, obtaining

$$F(s) = \frac{\mu}{s^3 + \alpha\, s^2 + \beta\, s + \gamma} \tag{6}$$

where

$$\begin{aligned} \alpha &= \frac{B_m}{J_m} + \frac{B_t}{J_t} + \frac{K_t\, K_\omega}{J_m R_m} \\ \beta &= \frac{K_b\, r^2}{J_m} + \frac{K_b\, r^2}{J_t} + \frac{B_m B_t}{J_m\, J_t} + \frac{B_t K_t K_\omega}{J_m\, J_t\, R_m} \\ \gamma &= \left(B_t + B_m + \frac{K_t K_\omega}{R_m}\right)\frac{K_b\, r^2}{J_m J_t} \\ \mu &= \frac{K_b K_t\, r^2}{J_m J_t R_m} \end{aligned} \tag{7}$$

Exploiting the apparatus symmetry, it can be assumed that $J_m = J_t = J$, $B_m = B_t = B$, leading to a transfer function with one real pole and two complex conjugate poles whose damping depends on K_b (i.e., the transmission compliance).

To achieve the desired pedagogical result, the following remarks are made, and the comprehension of these on the part of the students is verified by means of a short discussion in the laboratory. Since the equations of the model are electro-mechanical, there is no way of describing electrical and mechanical phenomena separately, obtaining individually closed and oriented models that are connected together in a block diagram. Model modularity is important, but to achieve it, a non-causal approach is very often required. The model is of the third order, one real pole coming from rigid motion and a couple of complex ones being relative to transmission compliance. It is important to understand the relationships between physical, dimensional data and dynamic behaviour, as such a comprehension is vital for a good control design, but is also precious when designing the physical system to be controlled.

Subsequently, the students perform two series of experiments with the apparatus, the first one with a tight belt (i.e. almost rigid transmission), the second one with a loose belt (i.e. highly flexible transmission), and analyze the results. For space limitations, we only report an experiment of the first type. A simple system model (first order plus delay) is identified by fitting a plant experimental step response to a simulated one. Figure 6 reports an experimental response and a simulated one deriving from the transfer function

$$P(s) = \frac{0.81 e^{-0.03s}}{1 + 0.03s}\,. \tag{8}$$

In all the figures reporting experimental and/or simulated transients, the vertical axis reports the voltage (in V) generated by the tacho.

Students are then asked to tune a PI regulator in the form

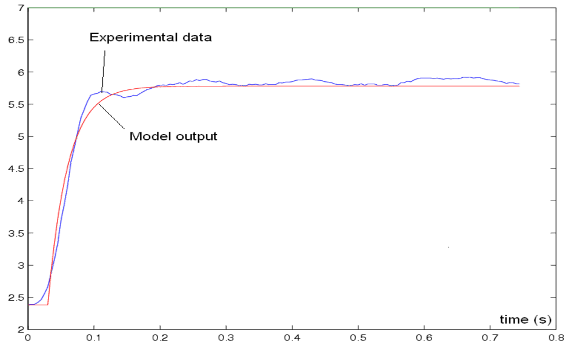

Fig. 6. Experimental and identified open-loop step responses of apparatus 1 with (almost) rigid transmission.

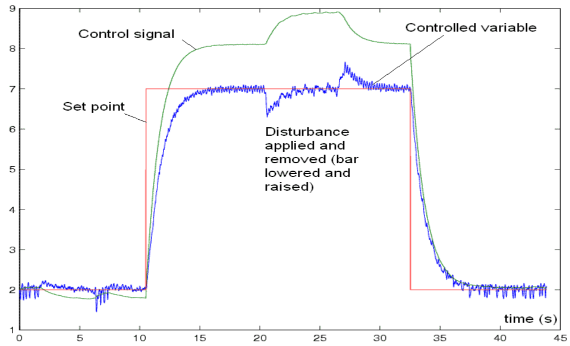

Fig. 7. Experimental and identified closed-loop step responses of apparatus 1 with (almost) rigid transmission.

$$R(s) = K_p \left(1 + \frac{1}{sT_i}\right) \tag{9}$$

on the basis of the identified transfer function, and compare the simulated and physical control system's behaviour. For the considered example, a good choice could be to use a PI regulator. Figure 7 reports such a comparison, where $T_i = 0.03$ and $K = 0.0308$, leading to a phase margin of approximately 40°.

3.2 Experiences with apparatus 2

Denoting by θ_m and θ_l the motor and load angular positions, by J_m the inertia of the motor, the tacho and their connection axis, by J_l the load inertia, by B_m and B_l their friction coefficients, by τ_m the motor torque, by τ_{tm} and τ_{tl} the torque the transmission exert on the motor and on the load, by M the eccentric load mass, by ℓ the distance between the rotation axis and the center of mass of the eccentric load, by n the transmission ratio, a simple (non-linear) model of apparatus 2 can be written as

$$\begin{cases} J_m\ddot{\theta}_m = \tau_m - B_m\dot{\theta}_m - \tau_{tm} \\ J_l\ddot{\theta}_l = -B_l\dot{\theta}_l + \tau_{tl} + Mg\ell\sin(\theta_l) \\ \tau_{tl} = n\tau_{tm} \\ \theta_m = n\theta_l \end{cases} \tag{10}$$

This model can be reformulated as

$$J_{eq}\ddot{\theta}_m = \tau_m - B_{eq}\dot{\theta}_m - \frac{Mg\ell}{n}\sin\left(\frac{\theta_m}{n}\right) \tag{11}$$

where $J_{eq} = J_m + J_l/n^2$,$B_{eq} = B_m + B_l/n^2$. To achieve the pedagogical result, the following remarks are made, and their comprehension is verified. Again, the model is electro-mechanical and there is no separation of its parts into physical blocks. This fact is general. The nonlinearity can be more or less significant depending on the value of n. This is quite an unexpected effect of that design parameter, and helps understanding that a process design choice has often unexpected effects on the complexity of the control problem. The system is nonlinear, and there is no point in linearising it, as there is no equilibrium around which the control must keep the system. The model is very complex unless one assumes that the effect of the nonlinearity can be described with a sinusoidal load disturbance. This is true if the speed is regulated, and therefore can be taken as hypothesis, since the control goal is speed regulation. This interplay of modelling hypotheses and control objectives is quite general, and worth understanding in depth.

Under the hypothesis of regulated speed, the disturbance modelling the nonlinearity is always inside the control band. Therefore, to reject it, a high critical frequency and a large magnitude of the open-loop frequency response below that frequency are required. As a further consequence, it is required to exploit the 2-d.o.f. structure of the regulator, otherwise the tracking properties may be poor in terms of speed or, more frequently, the control upset may be excessive.

Experiences with apparatus 2 are made both with and without the eccentric mechanical load. For space limitations, we only present an experiment of the first type. The FOPDT process model

$$P(s) = \frac{0.92e^{-0.01s}}{1 + 0.055s} \tag{12}$$

is identified with a step experiment (the structure is suitable, as the transmission is almost rigid), and then a PI is tuned to achieve large enough a bandwidth to reject the torque disturbance produced by gravity. Notice that this disturbance *would* be sinusoidal if the rotation speed were regulated perfectly—an interesting example of the interplay between *a priori* hypotheses and control specifications. A typical result is $K = 1.04$, $T_i = 0.055$, leading to a cutoff frequency of 17.4

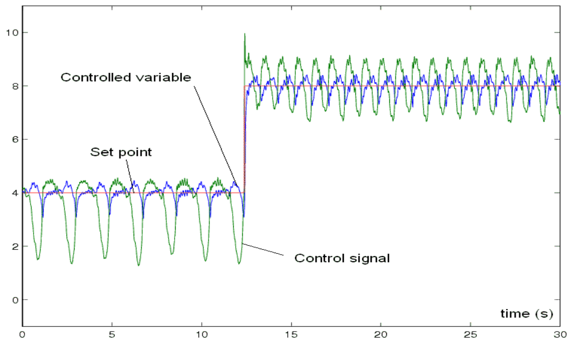

Fig. 8. Experimental and identified closed-loop step responses of apparatus 2 with eccentric load.

r/s. The corresponding set point step response is shown in figure 8: notice the control swin g required for disturbance rejection. The students are instructed to notice how the large bandwidth reflects in a nervous set point response, and to counteract this phenomenon by exploiting the 2-d.o.f. structure of the ISA PID, i.e., by adapting the set point weight b. This experience is very useful, but cannot be presented here for space limitations.

4. CONCLUDING REMARKS

Two experimental setups for speed control, and the corresponding laboratory activity, were presented. The setups are very simple, safe and inexpensive, being composed of LEGO elements and a minimum circuitry (that can be built and fully inspected by the students) for the interfacing with the PC. Various experiments are possible, so as to tackle different problems with sufficient realism. the apparatuses and the laboratory activity are used at the Politecnico di Milano in the undergraduate ETCS course, with satisfactory results in terms of student satisfaction and comprehension.

Plans are underway to make the presented experiments accessible via the web, the relative software has already been developed, and the first experiments are very encouraging: this subject will be treated in future works.

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