ZERO OPTIMIZING CONTINUOUS–TIME TRACKING AND DISTURBANCE REJECTING CONTROLLERS

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Abstract: The tuning of PID controllers can essentially be posed as the problem of selecting open-loop zeros such as to obtain a desired system response. In this paper, the general case wherein stable open-loop system zeros can be cancelled is considered, allowing more freedom in placing open-loop zeros, as opposed to just two zeros in the case of a PID controller. Subsequently, optimal open-loop zeros are computed such as to minimize the deviation from a desired reference impulse response, while maintaining the relative degree and the type of the reference system, thus giving the controlled system desired input tracking and disturbance rejection properties. *Copyright ©2005 IFAC*

Keywords: Linear Continuous–Time Systems, Zero Optimizing Controllers, Tracking, Disturbance Rejection

1. INTRODUCTION

Transfer function responses are of considerable interest in the area of control systems and in filter design. Closed–form continuous–time transfer function responses are derived in (Hauksdóttir, 1996) and extended to the case of repeated eigenvalues in (Hauksdóttir and Hjaltadóttir, 2003), (Herjólfsson, 2004), (Ævarsson, 2005) and (Hauksdóttir *et al.*, 2005). Naturally, the closed form lends itself well to analysis and opens up many new interesting applications, e.g., solving for optimal zero locations by minimizing transient responses (Hauksdóttir, 1996); tracking a given reference step response in (Hauksdóttir, 2002, 2004*b*), (Herjólfsson, 2004), (Herjólfsson *et al.*, 2005), (Ævarsson, 2005); and solving the model reduction problem in (Hauksdóttir, 2000), (Ævarsson, 2005), (Ævarsson *et al.*, 2005). The closed–form expressions are further used in the direct computation of coefficients for PID controllers in (Herjólfsson, 2004) and (Herjólfsson and Hauksdóttir, 2003).

Continuous–time transfer function responses are strongly affected, not only by the eigenvalues or poles, but the numerator coefficients, or equivalently, the system's zeros, as well. Controllers that affect open–loop zeros can be designed, one example of such a controller is the well known PID controller. The direct computation of coefficients for PID controllers in (Herjólfsson, 2004) and (Herjólfsson and Hauksdóttir, 2003), essentially involves computation of optimal open-loop zeros tracking a desired reference impulse response. In (Hauksdóttir, 2004*a*), the approach is extended to a more general case wherein stable open-loop zeros are cancelled, allowing more freedom in placing open-loop zeros, as opposed to just two zeros in the case of a PID controller.

In this paper, the work in (Hauksdóttir, 2004*a*) is reformulated and extended to include the case of repeated eigenvalues. The problem is formulated in Section 2, including input tracking and disturbance rejection properties as well as specification

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of the reference system. Linear continuous–time system responses are summarized in Section 3 (Ævarsson, 2005), (Hauksdóttir *et al.*, 2005). The optimal open-loop zeros are computed in Section 4 (Ævarsson, 2005), by minimizing the impulse response deviation between the reference and the actual system, including an example. Conclusions and future work are discussed in Section 5.

2. PROBLEM FORMULATION

Consider the closed-loop control system setup shown in Fig. 1. The plant zeros are given by

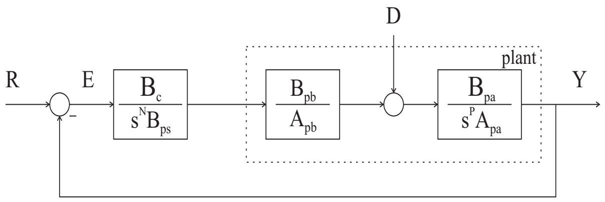

Fig. 1. Closed-loop control system setup.

$$B_p = B_{pb} B_{pa}, \tag{1}$$

where B_p is type zero and the Laplace variable s has been dropped to simplify notation. The term $\frac{1}{s^P}$ belongs to the post-disturbance part of the plant and includes all open-loop pure integrators between the disturbance D and the output Y. All B polynomials are of the generic form

$$B = b_0 s^m + b_1 s^{m-1} + ... + b_m. \tag{2}$$

The plant poles are assumed stable [3] and are given by

$$A_p = A_{pb} A_{pa}. \tag{3}$$

A_p is also type zero, i.e., the term $\frac{1}{s^N}$ grouped by the controller includes all open–loop pure integrators from the input R to the disturbance D. All A polynomials are of the generic form

$$\begin{aligned} A &= s^n + a_1 s^{n-1} + ... + a_n \\ &= (s+\lambda_1)^{d_1}(s+\lambda_2)^{d_2} \cdots (s+\lambda_\nu)^{d_\nu} \end{aligned} \tag{4}$$

The plant is affected by the disturbance input D and its output is Y. The closed-loop control system is driven by the input R and the controller, driven by the error E, is of the form

$$\frac{B_c}{s^N B_{ps}} \tag{5}$$

The controller cancels the stable plant zeros B_{ps} by inverse compensation, the term $\frac{1}{s^N}$ includes all controller and pre-disturbance plant integrators. The controller zeros B_c will be selected such as to optimally track a reference impulse response, maintaining the relative degree and the type of the reference system.

[3] For the case of unstable plant poles, an inner–loop state–feedback type controller can be designed, stabilizing the plant.

2.1 Input Tracking

The input tracking for a setup such as in Fig. 1 is easily obtained in a standard manner. The transfer function from the input R to the error E is given by

$$\begin{aligned} \frac{E}{R} &= \frac{1}{1 + \frac{B_c}{s^N B_{ps}} \frac{B_p}{s^P A_p}} = \frac{1}{1 + \frac{B_c B_{pu}}{s^{N+P} A_p}} \\ &= \frac{s^{N+P} A_p}{s^{N+P} A_p + B_c B_{pu}} \end{aligned} \tag{6}$$

where B_{pu} are the unstable plant zeros. For a unit step input R when $N + P \geq 1$, the steady state error is given by

$$e_{ss} = \lim_{s \to 0} s \frac{s^{N+P} A_p}{s^{N+P} A_p + B_c B_{pu}} \frac{1}{s} = 0. \tag{7}$$

Similarly, the steady state error for a unit ramp input R when $N + P \geq 2$ is given by

$$\begin{aligned} e_{ss} &= \lim_{s \to 0} s \frac{s^{N+P} A_p}{s^{N+P} A_p + B_c B_{pu}} \frac{1}{s^2} \\ &= \lim_{s \to 0} \frac{s^{N+P-1} A_p}{s^{N+P} A_p + B_c B_{pu}} = 0. \end{aligned} \tag{8}$$

2.2 Disturbance Rejection

Likewise, the disturbance rejection for the closed loop is easily obtained in a standard manner. The transfer function from the disturbance input D to the error E is given by

$$\begin{aligned} \frac{E}{D} &= \frac{-\frac{B_{pa}}{s^P A_{pa}}}{1 + \frac{B_c}{s^N B_{ps}} \frac{B_p}{s^P A_p}} = \frac{-\frac{B_{pa}}{s^P A_{pa}}}{1 + \frac{B_c B_{pu}}{s^{N+P} A_p}} \\ &= \frac{-s^N B_{pa} A_{pb}}{s^{N+P} A_p + B_c B_{pu}}. \end{aligned} \tag{9}$$

Then, the steady state error for a unit step disturbance input D when $N \geq 1$ is given by

$$e_{ss} = \lim_{s \to 0} s \frac{-s^N B_{pa} A_{pb}}{s^{N+P} A_p + B_c B_{pu}} \frac{1}{s} = 0. \tag{10}$$

The steady state error for a unit ramp disturbance input D when $N \geq 2$ is given by

$$\begin{aligned} e_{ss} &= \lim_{s \to 0} s \frac{-s^N B_{pa} A_{pb}}{s^{N+P} A_p + B_c B_{pu}} \frac{1}{s^2} \\ &= \lim_{s \to 0} \frac{-s^{N-1} B_{pa} A_{pb}}{s^{N+P} A_p + B_c B_{pu}} = 0. \end{aligned} \tag{11}$$

2.3 Reference System Specification

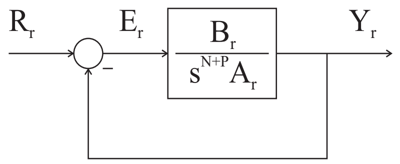

Fig. 2. Closed-loop reference system.

The design criteria is specified as a desired reference closed-loop transfer function and the reference open-loop transfer function is subsequently solved for, see Fig. 2. The transfer function from the input R_r to Y_r is given by

$$\frac{Y_r}{R_r} = \frac{B_r^{cl}}{A_r^{cl}} = \frac{\frac{B_r}{s^{N+P}A_r}}{1+\frac{B_r}{s^{N+P}A_r}} = \frac{B_r}{s^{N+P}A_r + B_r}, \quad (12)$$

where B_r and A_r are type zero. Solving for B_r and A_r gives

$$B_r = B_r^{cl} \quad (13)$$

and

$$A_r = s^{-(N+P)}(A_r^{cl} - B_r^{cl}). \quad (14)$$

In order to ensure that A_r is a regular polynomial in s and that we have an $N+P$ type system with the inner–loop relative degree of α,

$$B_r^{cl} = b_{r0}^{cl}s^{N+P-1} + b_{r1}^{cl}s^{N+P-2} + \cdots + b_{r(N+P-1)}^{cl} \quad (15)$$

and

$$A_r^{cl} = s^{2(N+P)+\alpha-1} + a_{r1}^{cl}s^{2(N+P)+\alpha-2} + \cdots + a_{r(N+P+\alpha-1)}^{cl}s^{N+P} + B_r^{cl}. \quad (16)$$

3. LINEAR CONTINUOUS–TIME SYSTEM RESPONSES

Consider the standard transfer function given by

$$\frac{Y}{U} = \frac{b_0s^m + b_1s^{m-1} + \cdots + b_m}{s^n + a_1s^{n-1} + \cdots + a_n} = \frac{b_0s^m + b_1s^{m-1} + \cdots + b_m}{(s+\lambda_1)^{d_1}(s+\lambda_2)^{d_2}\cdots(s+\lambda_\nu)^{d_\nu}}. \quad (17)$$

It is assumed that the system's eigenvalues $-\lambda_1$, $-\lambda_2$, …, $-\lambda_\nu$ are distinct and repeated d_1, d_2, …, d_ν times, respectively, and furthermore it is assumed that the system is causal, i.e., $m < n$.

The impulse responses are of the generic form (Ævarsson, 2005), (Hauksdóttir *et al.*, 2005)

$$y_I(t) = BH\mathcal{E}(t), \qquad t > 0 \quad (18)$$

where

$$B = \left[\, b_m \ b_{m-1} \ \cdots \ b_0 \,\right] \quad (19)$$

contains the numerator coefficients and

$$H = \begin{bmatrix} h_{01} & h_{02} & \cdots & h_{0\nu} \\ h_{11} & h_{12} & \cdots & h_{1\nu} \\ \vdots & \vdots & & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{m\nu} \end{bmatrix} \quad (20)$$

is an $(m+1)\times n$ matrix. The first line in (20) is given by

$$H_0 = \left[\, h_{01} \ h_{02} \ \cdots \ h_{0\nu} \,\right], \quad (21)$$

where each

$$h_{0i} = \left[\, \kappa_{i1} \ \kappa_{i2} \ \cdots \ \kappa_{id_i} \,\right] \quad (22)$$

contains the partial fraction coefficients of a unity numerator Laplace transform given by

$$Y_b = \frac{1}{s^n + a_1s^{n-1} + \cdots + a_n} = \frac{1}{(s+\lambda_1)^{d_1}(s+\lambda_2)^{d_2}\cdots(s+\lambda_\nu)^{d_\nu}}. \quad (23)$$

The unity numerator partial fraction coefficients are easily computed by

$$\kappa_{id_i} = \prod_{q=1,q\neq i}^{\nu} (-\lambda_i + \lambda_q)^{-d_q} \quad (24)$$

and for $j = 1, 2, 3, \ldots, d_i - 1$

$$\kappa_{ij} = \frac{1}{d_i - j}\sum_{q=1}^{d_i-j} \kappa_{i(j+q)}(-1)^q \sum_{p=1,p\neq i}^{\nu} \frac{d_p}{(-\lambda_i+\lambda_p)^q}, \quad (25)$$

Then,

$$h_{ki} = h_{(k-1)i}W_i, \qquad k = 1, 2, \ldots, m, \quad (26)$$

where

$$W_i = \begin{bmatrix} -\lambda_i & 0 & \cdots & \cdots & 0 \\ 1 & -\lambda_i & \ddots & & \vdots \\ 0 & 1 & -\lambda_i & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 & -\lambda_i \end{bmatrix} \quad (27)$$

is a $d_i \times d_i$ matrix.

Finally, all time dependent factors - effectively linearly independent basis functions, are contained in the $n \times 1$ vector,

$$\mathcal{E}(t) = \begin{bmatrix} \mathcal{E}_1(t) \\ \mathcal{E}_2(t) \\ \vdots \\ \mathcal{E}_\nu(t) \end{bmatrix} \quad (28)$$

where

$$\mathcal{E}_i(t) = \begin{bmatrix} e^{-\lambda_i t} \\ te^{-\lambda_i t} \\ \vdots \\ \frac{t^{(d_i-1)}}{(d_i-1)!}e^{-\lambda_i t} \end{bmatrix}. \quad (29)$$

It should be emphasized that (18) is a general closed–form solution for linear continuous–time system responses corresponding to a general transfer function of the form (17). There are no restrictions, the eigenvalues can be real and/or complex, repeated and/or not and stable and/or unstable.

It should also be noted that

$$\mu = \left[\, \mu_{11} \, \cdots \, \mu_{1d_1} \, \cdots \, \mu_{\nu 1} \, \cdots \, \mu_{\nu d_\nu} \,\right] = BH \quad (30)$$

is a new easily computable recursive form of partial fraction expansion coefficients for the general transfer function of the form (17), given by the well known expression

$$\mu_{ij} = \frac{1}{(d_i-j)!}\frac{d^{d_i-j}}{ds^{d_i-j}}\left[(s+\lambda_i)^{d_i}\frac{Y}{U}\right]\Bigg|_{s=-\lambda_i}. \quad (31)$$

4. OPTIMAL IMPULSE RESPONSE TRACKING

We now wish to match the open-loop impulse responses of the controlled system, see Fig. 3 for a simplified block diagram, and the reference system, see Fig. 2, as closely as possible. Note that we are nonrestrictively assuming that $N+P$ is selected the same in both cases and therefore we consider the open–loop impulse response of the controlled system characterized by the causal transfer function $B_c B_{pu}/A_p$, which has the impulse response

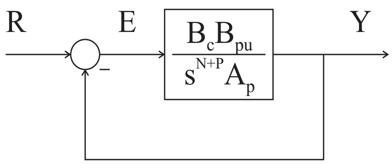

Fig. 3. A simplified block diagram of the controlled system.

$$y_{Icp}(t) = B_c \mathcal{B}_{pu} H_p \mathcal{E}_p(t). \quad (32)$$

Here $\mathcal{B}_{pu}$ is an $(m_c+1) \times (m_c+m_{pu}+1)$ convolution matrix given by (Herjólfsson, 2004), (Herjólfsson and Hauksdóttir, 2003)

$$\mathcal{B}_{pu} = \begin{bmatrix} b^{pu}_{m_{pu}} & \cdots & b^{pu}_0 & 0 & \cdots & 0 \\ 0 & b^{pu}_{m_{pu}} & \cdots & b^{pu}_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & & \ddots & 0 \\ 0 & \cdots & 0 & b^{pu}_{m_{pu}} & \cdots & b^{pu}_0 \end{bmatrix}. \quad (33)$$

Further,

$$H_p = \begin{bmatrix} h_{01} & \cdots & h_{0\nu} \\ h_{11} & \cdots & h_{1\nu} \\ \vdots & & \vdots \\ h_{(m_c+m_{pu})1} & \cdots & h_{(m_c+m_{pu})\nu} \end{bmatrix} \quad (34)$$

is an $(m_c + m_{pu} + 1) \times n$ matrix.

The open–loop impulse response characterized by the causal transfer function B_r/A_r, has the impulse response

$$y_{Ir}(t) = B_r H_r \mathcal{E}_r(t). \quad (35)$$

We then define a cost function measuring the controlled impulse response deviation from the reference impulse response as

$$J = \int_0^\infty (y_{Ir}(t) - y_{Icp}(t))^2 \, dt \quad (36)$$

Differentiating the cost function with respect to B_c and setting the result equal to zero gives

$$\begin{aligned} \frac{\partial J}{\partial B_c} &= \int_0^\infty \frac{\partial}{\partial B_c}(B_r H_r \mathcal{E}_r(t) - B_c \mathcal{B}_{pu} H_p \mathcal{E}_p(t))^2 \, dt \\ &= -2B_r H_r \int_0^\infty \mathcal{E}_r(t)\mathcal{E}_p(t)^T dt \, (\mathcal{B}_{pu} H_p)^T \\ &\quad +2B_c \mathcal{B}_{pu} H_p \int_0^\infty \mathcal{E}_p(t)\mathcal{E}_p(t)^T dt \, (\mathcal{B}_{pu} H_p)^T \\ &= -2\mathcal{D} + 2B_c \mathcal{A} \; = \; 0 \end{aligned} \quad (37)$$

where we have defined

$$\mathcal{D} = B_r H_r \int_0^\infty \mathcal{E}_r(t)\mathcal{E}_p(t)^T dt \, (\mathcal{B}_{pu} H_p)^T \quad (38)$$

and

$$\mathcal{A} = \mathcal{B}_{pu} H_p \int_0^\infty \mathcal{E}_p(t)\mathcal{E}_p(t)^T dt \, (\mathcal{B}_{pu} H_p)^T. \quad (39)$$

The fact that the matrix $\mathcal{A}$ is invertible, is easily seen as the matrix $\mathcal{B}_{pu}$ as in (33) has the same rank as the number of rows in $\mathcal{B}_{pu}$; the matrix H_p as in (34) has the same rank as the number of columns in H_p; and the matrix $\int_0^\infty \mathcal{E}_p(t)\mathcal{E}_p(t)^T dt$ has the same rank as the number of elements in $\mathcal{E}_p(t)$ (see (28) and (29)), since all the element functions of $\mathcal{E}_p(t)$ are linearly independent on $[0, \infty)$.

This gives us the simple closed-form solution

$$B_c = \mathcal{D}\mathcal{A}^{-1}. \tag{40}$$

$\mathcal{E}_p(t)$ and $\mathcal{E}_r(t)$ can be written in a similar manner as given in (28) and (29). Calculating the (ρ,σ)–th element of the (k,j)–th subblock of $\int_0^\infty \mathcal{E}_p(t)\mathcal{E}_p(t)^T dt$, i.e., of the matrix $\int_0^\infty \mathcal{E}_{pk}(t)\mathcal{E}_{pj}(t)^T dt$ is given by

$$\left[\int_0^\infty \mathcal{E}_{pk}(t)\mathcal{E}_{pj}(t)^T dt\right]_{\rho,\sigma} = \frac{\begin{pmatrix}\rho+\sigma-2\\ \rho-1\end{pmatrix}}{(\lambda_{pk}+\lambda_{pj})^{\rho+\sigma-1}}. \tag{41}$$

Similarly, the (ρ,σ)–th element of the (k,j)–th subblock of $\int_0^\infty \mathcal{E}_r(t)\mathcal{E}_p(t)^T dt$, i.e., of the matrix $\int_0^\infty \mathcal{E}_{rk}(t)\mathcal{E}_{pj}(t)^T dt$ is given by

$$\left[\int_0^\infty \mathcal{E}_{rk}(t)\mathcal{E}_{pj}(t)^T dt\right]_{\rho,\sigma} = \frac{\begin{pmatrix}\rho+\sigma-2\\ \rho-1\end{pmatrix}}{(\lambda_{rk}+\lambda_{rj})^{\rho+\sigma-1}}. \tag{42}$$

In general, the relative degree of the inner loop of the controlled system should preferably be selected the same as the relative degree of the inner loop of the reference system, to ease the matching of the two systems.

Example: For demonstration purposes, consider a fictitious twelfth–order type one plant with an input disturbance, where the plant transfer function before the disturbance is given by

$$\frac{B_{pb}}{A_{pb}} = \frac{1}{s^3+11s^2+40s+50} \tag{43}$$

$$= \frac{1}{(s+3+i)(s+3-i)(s+5)}. \tag{44}$$

The plant transfer function after the disturbance has two zeros, an integrator and an eightfold pole in -2, i.e., it is given by

$$\frac{B_{pa}}{sA_{pa}} = \frac{s^2+4s+5}{s(s+2)^8} = \frac{(s+2+i)(s+2-i)}{s(s+2)^8}. \tag{45}$$

It is desired to track a well behaved type-two closed-loop transfer function given by

$$\frac{Y_r}{R_r} = \frac{6s+9}{s^2+6s+9} = 6\frac{s+1.5}{(s+3)^2}, \tag{46}$$

thus having the inner loop

$$\frac{B_r}{s^{(N+P)}A_r} = \frac{6s+9}{s^2}, \tag{47}$$

where $N+P=2$. Then, computing the optimal B_c based on (40) maintaining the same relative degree as the reference system's inner loop, results

b_0	b_2	b_3	b_4	b_5
1.2	33.6	433.9	3370.2	17524.2
b_6	b_7	b_8	b_9	b_{10}
64330.0	170917.1	331536.7	465643.1	462490.1
	b_{11}	b_{12}	b_{13}	
	307972.9	123680.9	22615.1	

Table 1. Optimal controller coefficients, B_c.

in the controller $\frac{B_c}{sB_{ps}}$ where coefficients in the B_c is given in Table 1 and B_{ps} is given by

$$B_{ps} = s^2+4s+5. \tag{48}$$

The zero-pole locations of the open–loop original plant, the compensated inner loop $\frac{B_c}{A_p}$ and the open–loop reference transfer function $\frac{B_r}{A_r}$ are shown in Fig. 4.

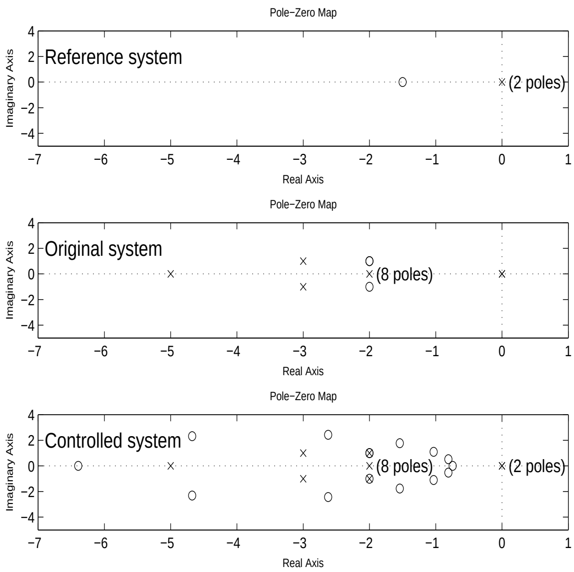

Fig. 4. Open–loop pole/zero locations.

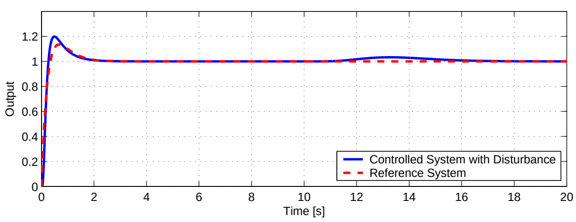

Fig. 5. Closed-loop step responses with the onset of a unit step at $time = 1$ and an onset of a disturbance of 50 at $time = 10$.

Subjecting the closed-loop as in Fig. 1, to a step input at $time = 1$ and to a disturbance of 50 at $time = 10$, results in the response shown in Fig. 5. As may be noted, the controlled system follows the reference system very closely during the step input and the disturbance rejection is excellent.

5. CONCLUSIONS AND FUTURE WORK

The tuning of PID controllers can essentially be posed as the problem of selecting open-loop zeros such as to obtain a desired system response. In this paper, the idealogy behind the PID controller was extended to the general case wherein stable open-loop system zeros can be cancelled, thus allowing more freedom in placing open-loop zeros, as opposed to just two zeros in the case of a PID controller. Subsequently, optimal open-loop zeros were computed such as to minimize the deviation from a desired reference impulse response, while maintaining the relative degree and the type of the reference system, thus giving the controlled system desired input tracking and disturbance rejection properties. Due to the inverse compensation of the plant zeros, the controller is in general causal when the relative degree of the plant and the reference system are similar. In cases when the controller is noncausal, which happens if the plant has a high relative degree and the reference system has a low relative degree, the controller can be realized using poles to limit the high frequency response, as is frequently done in a practical setup of a PID controller.

Excellent results were obtained, wherein a twelfth–order system tracked a well behaved reference system response. The controlled system was shown to have excellent input tracking and disturbance rejection properties.

It is of interest to show that the minimal deviation between the reference and the controlled system does occur when the relative degrees of the two systems are the same. It is further of interest to explore the stability properties of the closed–loop controlled system, in particular to obtain an estimate of the maximum possible deviation between the controlled system and a well-behaved and stable reference closed–loop system, such that stability of the controlled system is guaranteed.

6. ACKNOWLEDGMENTS

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