

## DESIGN OF AUGMENTED FAULT DETECTION FILTER FOR FAULT TOLERANT CONTROL

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Abstract : In this paper, a new Fault-Tolerant Control System strategy is presented for linear dynamic systems. The detection and isolation of each jump is achieved by the Kalman filter designed under a dead beat constraint allowing the maximum adaptability of the fault magnitude estimation in the case of abrupt change. By using the state estimation corrected by the estimation of jumps we will describe a integrated Fault Tolerant Control via a particular LQG approach. *Copyright © 2005 IFAC*

Keywords : Detection filter, fault accommodation, disturbance rejection, winding machine.

### 1. INTRODUCTION

Due to an increasing complexity of modern engineering systems, as well as the need for reliability, safety and efficient operation of industrial systems, a great deal of attention has been focused on the subject of the fault tolerant control (FTC). The aim of Fault Tolerant Control is to maintain current performances closed to a desired one and to preserve the stability conditions in presence of component and/or instrument faults. In addition, reduced performances could be accepted as a trade-off.

A natural way to cope with the FTC problem is to modify the controller parameters according to an on-line identification of the system parameters when a fault occurs. However, due to difficulties inherent to the on-line multivariable identification in closed-loop systems, such as the lack of excitation signals or the presence of noise, we propose an alternative solution based on the computation of a new control law to be added to the nominal one. But since this new control

law is not the same for both cases, an FDI module is necessary to isolate the faulty element accurately.

The existing methods of reconfigurable controller design can be classified as linear quadratic regulator (D.P. Looze, 1985), eigenstructure assignment (J. Jiang, 1994), multiple model (P.S. Maybeck, 1991), adaptive control (M. Bodson, 1997) (Wu et al, 2000), pseudoinverse (A.K. Caglayan, 1988). The reconfiguration function may be active ((Jiang, 1998), (Noura, 2000)) or passive (Zhao, 1998). A classical way to achieve fault tolerant control relies on supervised control where an FDI unit provides information about the location and time occurrence of any fault. Faults are compensated via an appropriate control law triggered according to diagnosis of the system. Nevertheless, it is to be noticed that only few methods have been applied to real plants (Ballé et al. 1998), (Noura et al.2000).

Fault diagnosis implies to design residuals that are close to zero in fault-free situations and clearly

deviate from zero in the presence of faults. Residual must possess the ability to discriminate between all possible modes of faults, which explains the use of enhancing fault isolability. The fault detection filter which is proposed in this paper is a special full-order state observer which generates output residuals having directional properties in response to each fault. First developed by Beard (1971), the fault detection filter (FDF) has been revisited by Massoumnia (1986) from the geometric state-space control theory and by White and Speyer (1987) in the context of eigenstructure assignment. To apply the fault detection filter in stochastic systems, a new interpretation of the fault detection filter have been suggested by Park and Rizzoni (1994a) before its optimization in (1994b), however, the treatment of multiple faults, convergence and stability conditions of the filter was not studied. Sauter and Hamelin have studied the fault detection filter in frequency domain (1999). Further improvements were suggested by Liu and Si (1997) and Keller and Sauter (2000). Recently, Chen and Speyer (2002) have proposed a new robust multiple fault detection filter which is derived by solving an optimization problem in the context where we can not achieve a perfect decoupling. The fault isolation filter (FIF) presented here is very similar to the predictor structure of the standard Kalman filter allowing the establishment of its convergence and stability conditions.

The paper is organized as follows. Section 2 presents the augmented fault isolation filter designed under dead beat constraints. Section 3, propose the design of control law with a maximum adaptivity. The last section, illustrates the application of the fault tolerant r to the winding system.

This work is related to IFATIS project.

## 2. AUGMENTED FAULT DETECTION FILTER

Consider the following discrete time linear system

$$x_{k+1} = Ax_k + Bu_k + Fv_k + w_k \quad (1.a)$$

$$y_k = Cx_k + v_k \quad (1.b)$$

where  $x_k \in \mathfrak{R}^n$  is the state vector,  $y_k \in \mathfrak{R}^m$  is the output vector,  $u_k \in \mathfrak{R}^p$  is the input vector.  $F = [f_1 \dots f_i \dots f_q]$  is fault distribution matrix,  $v_k \in \mathfrak{R}^q$  is the bias fault. The state and measurement noises  $w_k$  and  $v_k$  are zero mean uncorrelated random sequences with

$$E \left( \begin{bmatrix} w_k \\ v_k \end{bmatrix} \begin{bmatrix} w_j^T & v_j^T \end{bmatrix} \right) = \begin{bmatrix} W & 0 \\ 0 & I \end{bmatrix} \delta_{kj} \quad \text{where } W \geq 0 \quad \text{and} \\ V > 0.$$

The initial state  $x_0$ , uncorrelated with  $w_k$  and  $v_k$ , is a gaussian random variable with  $E\{x_0\} = \bar{x}_0$  and  $E\{(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T\} = \bar{P}_0$ .

We suppose the bias model has the following form

$$v_{k+1} = v_k + \Delta v \delta_{k,r} \quad (2)$$

where  $r$  is the unknown occurrence time of the impulsive change,  $\Delta v$  the impulsive jump magnitude and  $\delta_{k,r}$  the Kronecker operator.

the term isolation. Generation of residuals having directional properties in response to a particular faults is an attractive way for under (1) and (2) the system  $h_q$  can be rewritten as follow

$$\begin{bmatrix} \bar{x}_{k+1} \\ v_{k+1} \end{bmatrix} = \begin{bmatrix} A & F \\ 0 & I \end{bmatrix} \begin{bmatrix} x_k \\ v_k \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} 0_{n,q} \\ I \end{bmatrix} \Delta v \delta_{k,r} + \begin{bmatrix} I \\ 0 \end{bmatrix} w_k \quad (3.a)$$

$$y_k = [C \quad 0] \begin{bmatrix} x_k \\ v_k \end{bmatrix} + v_k \quad (3.b)$$

where  $\Delta v$  can be considered as an impulsive unknown input, with

$$\bar{A} = \begin{bmatrix} A & F \\ 0 & I \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \bar{C} = [C \quad 0], \quad \bar{F} = \begin{bmatrix} 0 \\ I \end{bmatrix}, \\ \bar{I} = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

Consider the following observer

$$\hat{X}_{k+1} = \bar{A}\hat{X}_k + \bar{B}u_k + K(y_k - \bar{C}\hat{X}_k) \quad (4.a)$$

$$\Delta \hat{v} = L(y_k - \bar{C}\hat{X}_k) \quad (4.b)$$

where  $\hat{X}_k$  is the state of the filter,  $\Delta \hat{v}$  the output of the filter and where  $L \in \mathfrak{R}^{q,m}$  and  $K \in \mathfrak{R}^{n+q,m}$  are unknown matrices that we will design in order to solve the fault detection and isolation problem.

From (3) and (4), the estimation error  $\Xi_k = X_k - \hat{X}_k$  and the output of the filter  $\Delta \hat{v}$  propagate as

$$\Xi_{k+1} = (\bar{A} - K\bar{C})\Xi_k + \bar{F}\Delta v_k + \bar{I}w_k - K v_k \quad (5.a)$$

$$\Delta \hat{v} = L(\bar{C}\Xi_k + v_k). \quad (5.b)$$

Let us define the detectability indexes first introduced by Liu and Si (1997) and later used by Keller (1999). The linear time invariant system (3) has the following set of fault detectability indexes  $\rho = \{\rho_1 \dots \rho_i \dots \rho_q\}$

$$\text{where } \rho_i = \min \{v : \bar{C} \bar{A}^{v-1} \bar{f}_i \neq 0, v = 1, 2, \dots\}.$$

Define the fault detectability matrix  $\bar{\Psi} = \bar{C}\bar{D}$

$$\text{with } \bar{D} = \begin{bmatrix} \bar{A}^{\rho_1-1} \bar{f}_1 & \dots & \bar{A}^{\rho_i-1} \bar{f}_i & \dots & \bar{A}^{\rho_q-1} \bar{f}_q \end{bmatrix} \quad (6)$$

Under the following assumptions

$$\text{rank}(\bar{\Psi}) = q \quad (7)$$

the goal of this paper is to compute  $K$  and  $L$  so that

$$W(z) = L\bar{C}(zI - (\bar{A} - K\bar{C}))^{-1}\bar{F} = \begin{pmatrix} z^{-\rho_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & z^{-\rho_q} \end{pmatrix} \quad (8.a)$$

where (8.a) ensures the diagonal structure of transfer from faults to residuals allowing the multiple faults isolation.

After having parameterized (8.a), the remaining design of freedom will be used to minimize the trace of the fault estimation error covariance matrix  $P_k$  given by

$$P_k^{\Delta v} = E \left( (\Xi_k^{\Delta v} - E(\Xi_k^{\Delta v})) (\Xi_k^{\Delta v} - E(\Xi_k^{\Delta v}))^T \right) \quad (8.b)$$

with  $e_k^{\Delta v} = \Delta v - \Delta \hat{v}$  where

$$E(\Delta \hat{v}) = \begin{bmatrix} \Delta v_{k-p_l}^l & \dots & \Delta v_{k-p_l}^i & \dots & \Delta v_{k-p_q}^q \end{bmatrix}^T \quad \text{under}$$

(8.a).

**Theorem 2.1: Parameterization of the FIF.**

Under (7), the solutions of (8.a) can be parameterized as

$$K = \omega \Pi + \bar{K}_k \Sigma \quad (9.a)$$

$$L = \Pi + \bar{L}_k \Sigma \quad (9.b)$$

with  $\Sigma = \beta(I - \bar{\Psi}\Pi)$ ,  $\Pi = \bar{\Psi}^+$  and  $\omega = \bar{A}\bar{D}$  and  $\bar{\Psi} = \bar{C}\bar{D}$  where  $\beta$  is an arbitrary matrix chosen so that  $\text{rank}(\Sigma) = m - q$  and  $\bar{K}_k \in \mathfrak{R}^{n+q, m-q}$ ,  $\bar{L}_k \in \mathfrak{R}^{n, m-q}$  are the time-varying free parameters.

**Proof:** We have

$$\begin{aligned} W(z) &= L\bar{C}(zI - (\bar{A} - K\bar{C}))^{-1}\bar{F} \\ &\stackrel{\Delta v \rightarrow \Delta \hat{v}}{=} \sum_{k \geq 0} L\bar{C}(\bar{A} - K\bar{C})^k \bar{F} z^{-k-1} \\ &= \sum_{k \geq 0} z^{-k-1} [\dots \mid L\bar{C}(\bar{A} - K\bar{C})^k \bar{f}_i \mid \dots] \end{aligned} \quad (10)$$

where

$$\begin{aligned} &\sum_{k \geq 0} z^{-k-1} L\bar{C}(\bar{A} - K\bar{C})^k \bar{f}_i = z^{-1} L\bar{C}\bar{f}_i + \\ &+ z^{-2} L\bar{C}(\bar{A} - K\bar{C})\bar{f}_i + z^{-3} L\bar{C}(\bar{A} - K\bar{C})^2 \bar{f}_i + \dots \end{aligned} \quad (11)$$

Substituting (11) in (10), we obtain

$$\begin{aligned} W(z) &= \\ &\stackrel{\Delta v \rightarrow \Delta \hat{v}}{=} \left[ \dots \mid L\bar{C}\bar{A}^{p_l-1} \bar{f}_i z^{-p_l} + \sum_{k \geq 0} L\bar{C}(\bar{A} - K\bar{C})^{k+1} \bar{A}^{p_l-1} \bar{f}_i z^{-k-l-p_l} \mid \dots \right] \end{aligned} \quad (12)$$

So, if the observer's gain  $K$  satisfies the algebraic

$$\text{constraint } (\bar{A} - K\bar{C}) \begin{bmatrix} \dots \mid \bar{A}^{p_l-1} \bar{f}_i \mid \dots \end{bmatrix} = 0 \quad (13)$$

then (12) gives

$$\begin{aligned} W(z) &= \left[ \dots \mid L\bar{C}\bar{A}^{p_l-1} \bar{f}_i z^{-p_l} \mid \dots \right] \\ &\stackrel{\Delta v \rightarrow \Delta \hat{v}}{=} \\ &= L\bar{\Psi} \begin{pmatrix} z^{-p_l} & 0 & \dots & \dots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & z^{-p_l} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & \dots & 0 & z^{-p_q} \end{pmatrix} \end{aligned} \quad (14)$$

and (8.a) is satisfied under

$$L\bar{\Psi} = I \quad (15)$$

Under the condition (7), the solutions of the eigenstructure assignment (13) and algebraic constraint (15) can be parameterized as

$$K = \omega \Pi + \bar{K}_k \Sigma \quad (16)$$

$$L = \Pi + \bar{L}_k \Sigma \quad (17)$$

closing the proof.

We are going to compute  $\bar{K}_k$  and  $\bar{L}_k$  so that the trace of the fault estimation error covariance matrix  $P_k^{\Delta v}$  given by (8.b) is minimized.

**Theorem 2.2: The Robust Fault Isolation Filter (RFIF)**

Under the stability and convergence conditions given by

$$\text{rank} \begin{bmatrix} zI - \bar{A} & \bar{D} \\ \bar{C} & 0 \end{bmatrix} = n + q, \forall z \in C, |z| \geq 1. \quad (18.a)$$

$$\text{rank}[-e^{jw}I + \bar{A} \quad \bar{D} \quad W^{1/2}] = n, \forall w \in [0, 2\pi]. \quad (18.b)$$

The RFIF is described by

$$\hat{X}_{k+1} = \bar{A}\hat{X}_k + \bar{B}u_k + (\omega \Pi + \bar{K}_k \Sigma)(y_k - C\hat{X}_k) \quad (18.c)$$

$$\begin{aligned} \bar{P}_{k+1} &= (\bar{A} - (\omega \Pi + \bar{K}_k \Sigma)\bar{C})\bar{P}_k (\bar{A} - (\omega \Pi + \bar{K}_k \Sigma)\bar{C})^T \\ &+ W + (\omega \Pi + \bar{K}_k \Sigma)(\omega \Pi + \bar{K}_k \Sigma)^T \end{aligned} \quad (18.d)$$

$$\Delta \hat{v}_k = (\Pi + \bar{L}_k \Sigma)(y_k - C\hat{X}_k) \quad (18.e)$$

$$P_k^{\Delta v} = (\Pi + \bar{L}_k \Sigma)H_k (\Pi + \bar{L}_k \Sigma)^T \quad (18.f)$$

with

$$\bar{K}_k = \bar{A}_b \bar{P}_k \bar{C}_b^T (C_b \bar{P}_k C_b^T + V_b)^{-1} \quad (18.g)$$

$$\bar{L}_k = -\Pi H_k \Sigma^T (\Sigma H_k \Sigma^T)^{-1} \quad (18.h)$$

$$H_k = \bar{C} \bar{P}_k \bar{C}^T + I \quad (18.i)$$

where

$$\bar{A}_b = \bar{A} - \omega \Pi \bar{C}, \quad \bar{C}_b = \Sigma \bar{C} \quad \text{and} \quad V_b = \Sigma \Sigma^T.$$

**Proof.** From equations (5), we can write  $\mathcal{E}_k$  as

$\mathcal{E}_k = \bar{\mathcal{E}}_k + \tilde{\mathcal{E}}_k$  where  $\bar{\mathcal{E}}_k$  is the estimation error without fault and  $\tilde{\mathcal{E}}_k$  the estimation error taking into account the presence of faults. We have

$$\begin{aligned} \bar{\mathcal{E}}_{k+1} + \tilde{\mathcal{E}}_{k+1} &= (\bar{A} - K_k \bar{C})(\bar{\mathcal{E}}_k + \tilde{\mathcal{E}}_k) + \bar{F}\Delta v_k + \\ &+ \bar{F}w_k - K_k v_k \end{aligned} \quad (19)$$

or

$$\bar{\mathcal{E}}_{k+1} = (\bar{A} - K_k \bar{C})\bar{\mathcal{E}}_k + \bar{F}w_k - K_k v_k \quad (20.a)$$

$$\tilde{\mathcal{E}}_{k+1} = (\bar{A} - K_k \bar{C})\tilde{\mathcal{E}}_k + \bar{F}\Delta v_k \quad \text{with} \quad \tilde{\mathcal{E}}_0 = 0. \quad (20.b)$$

From theorem 2.1, the faults estimation can be given by

$$\begin{aligned} \Delta \hat{v}_k &= L\bar{C}\bar{\mathcal{E}}_k \\ &= L(\bar{C}\bar{\mathcal{E}}_k + v_k) + \left[ \Delta v_{k-p_l}^l \quad \dots \quad \Delta v_{k-p_l}^i \quad \dots \quad \Delta v_{k-p_q}^q \right]^T \end{aligned} \quad (21)$$

So,  $E(\Delta \hat{v}_k) = \left[ \Delta v_{k-p_l}^l \quad \dots \quad \Delta v_{k-p_l}^i \quad \dots \quad \Delta v_{k-p_q}^q \right]^T$  and

let  $e_k^{\Delta v} = \Delta v_k - E(\Delta v_k)$  the fault estimation errors which propagates as

$$\bar{\mathcal{E}}_{k+1} = (\bar{A} - (\omega \Pi + \bar{K}_k \Sigma)\bar{C})\bar{\mathcal{E}}_k + \bar{F}w_k - (\omega \Pi + \bar{K}_k \Sigma)v_k \quad (22.a)$$

$$\mathcal{E}_k^{\Delta v} = (\Pi + \bar{L}_k \Sigma)(\bar{C}\bar{\mathcal{E}}_k + v_k). \quad (22.b)$$

The estimation errors covariance matrices  $\bar{P}_k = E\{\bar{\mathcal{E}}_k \bar{\mathcal{E}}_k^T\}$  and  $P_k^{\Delta v} = E\{e_k^{\Delta v} e_k^{\Delta v T}\}$  satisfy

$$\begin{aligned} \bar{P}_{k+1} &= (\bar{A} - (\omega \Pi + \bar{K}_k \Sigma)\bar{C})\bar{P}_k (\bar{A} - (\omega \Pi + \bar{K}_k \Sigma)\bar{C})^T \\ &+ W + (\omega \Pi + \bar{K}_k \Sigma)(\omega \Pi + \bar{K}_k \Sigma)^T \end{aligned} \quad (23.a)$$

$$P_k^{\Delta v} = (\Pi + \bar{L}_k \Sigma)(\bar{C}\bar{P}_k \bar{C}^T + I)(\Pi + \bar{L}_k \Sigma)^T \quad (23.b)$$

$$H_k = \bar{C} \bar{P}_k \bar{C}^T + I. \quad (23.c)$$

The traces of  $\bar{P}_{k+1}$  and  $P_k^{\Delta v}$  are minimized with respect to  $\bar{K}_k$  and  $\bar{L}_k$  if and only if

$$-(\bar{A} - (\omega\Pi + \bar{K}_k \Sigma) \bar{C}) \bar{P}_k \bar{C}^T \Sigma^T + (\omega\Pi + \bar{K}_k \Sigma) \Sigma^T = 0 \quad (24.a)$$

$$(\Pi + \bar{L}_k \Sigma) H_k \Sigma^T = 0 \quad (24.b)$$

The solution of (24) gives

$$\bar{K}_k = (\bar{A}_b \bar{P}_k \bar{C}_b^T - \omega \Pi H_k) \Sigma^T (\Sigma H_k \Sigma^T)^{-1} \quad (25.a)$$

$$\bar{L}_k = -\Pi H_k \Sigma^T (\Sigma H_k \Sigma^T)^{-1} \quad (25.b)$$

Since  $\Pi \Sigma^T = 0$ , (25.a) can be rewritten  $\bar{K}_k = \bar{A}_b \bar{P}_k \bar{C}_b^T (\bar{C}_b \bar{P}_k \bar{C}_b^T + V_b)^{-1}$  where  $\bar{A}_b = \bar{A} - \omega \Pi \bar{C}$ ,  $\bar{C}_b = \Sigma \bar{C}$  and  $V_b = \Sigma \Sigma^T$ . So, the stability and convergence conditions of the augmented FDF are deduced from the results given in (Keller, 1999) then in (Jamouli et al.2003).

### 3. FAULT TOLERANT CONTROL WITH MAXIMUM ADAPTIVITY

The general concept of this approach is illustrated by Fig.1 The FDI module consists of residual generation and residual evaluation. Second stage is performance evaluation and the third stage is represented by the reconfiguration mechanism. Fault detection and isolation must be achieved as soon as possible to avoid huge losses in system performance or even catastrophic consequences. Once the FDI module indicates which sensor or actuator is faulty, the fault magnitude is estimated and a new control law is added to the nominal one to thwart the fault effect on the system.

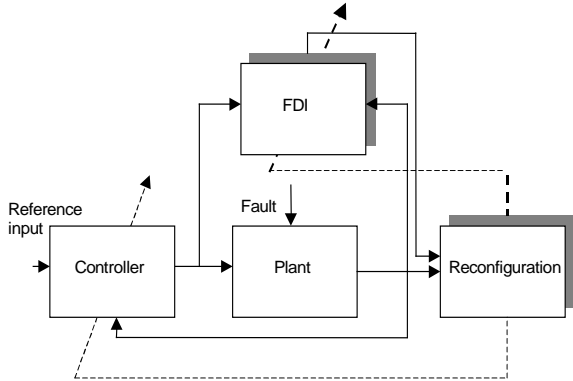


Fig. 1: Architecture of the fault tolerant controller

We recall that the filter designed in the last paragraph have a dead beat structure. When an impulsive jump fault occurs  $d_k = \Delta v_k \delta_{k,r}$  at time  $r$ , the estimation error of the filter affected by  $\Delta v_k$  decreased to zero in a minimal time. This property will be used for the design of the control law with a maximum adaptivity to abrupt changes  $d_k$ .

We propose to design a control law of the form

$$u_k = -L X_k = -\begin{bmatrix} L^x & L^y \end{bmatrix} \begin{bmatrix} x_k \\ v_k \end{bmatrix} \quad (26)$$

or by the separation principle to compute a control law  $u_k = -L X_k$  for the following system

$$X_{k+1} = \bar{A} X_k + \bar{B} u_k \quad (27.a)$$

$$\bar{y}_k = \bar{C} X_k. \quad (27.b)$$

Consider  $u_k = u_k^n - G v_k$  where  $u_k^n = -\bar{L} \bar{x}_k$  is a nominal control law designed on the jump-free system.

$$\bar{x}_{k+1} = \bar{A} \bar{x}_k + B u_k \quad (28.a)$$

$$y_k = C \bar{x}_k. \quad (28.b)$$

From the state transformation

$$\begin{bmatrix} \bar{x}_k \\ v_k \end{bmatrix} = \begin{bmatrix} I & T \\ 0 & I \end{bmatrix} \begin{bmatrix} x_k \\ v_k \end{bmatrix} \quad (29)$$

$$\text{and the law control } u_k = u_k^n - G v_k. \quad (30)$$

The noise-free system (27) controlled with (30) can be equivalently rewritten

$$\begin{bmatrix} \bar{x}_{k+1} \\ v_{k+1} \end{bmatrix} = \begin{bmatrix} A & (I-A)T+F \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{x}_k \\ v_k \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} (u_k^n - G v_k) \quad (31.a)$$

$$y_k = \begin{bmatrix} C & -CT \end{bmatrix} \begin{bmatrix} \bar{x}_k \\ v_k \end{bmatrix}. \quad (31.b)$$

The influence of jumps  $v_k$  on  $y_k$  is asymptotically rejected if and only if  $T$  and  $G$  satisfy the following algebraic equations

$$(A-I)T + BG = F \quad (32.a)$$

$$CT = 0 \quad (32.b)$$

rewritten in the matricial form

$$\begin{bmatrix} T^T & G^T \end{bmatrix} \begin{bmatrix} (A-I)^T & C^T \\ B^T & 0 \end{bmatrix} = \begin{bmatrix} F^T & 0 \end{bmatrix}. \quad (33)$$

Under the existence condition of a solution of (33) given by

$$\text{rank} \begin{bmatrix} A-I & B & F \\ C & 0 & 0 \end{bmatrix} = \text{rank} \begin{bmatrix} I-A & B \\ C & 0 \end{bmatrix} \quad (34)$$

the solution of (33) gives

$$G = \begin{bmatrix} C(I-A)^{-1} B \end{bmatrix}^+ C(I-A)^{-1} F \quad (35)$$

where  $T = (I-A)^{-1}(BG-F)$ . Substituting (35) in (31) gives

$$\bar{x}_{k+1} = (A - B\bar{L}) \bar{x}_k \quad (36.a)$$

$$y_k = C \bar{x}_k \quad (36.b)$$

where  $A - B\bar{L}$  is stable. The control law

$$u_k = \begin{bmatrix} \bar{L} & G \end{bmatrix} \begin{bmatrix} \bar{x}_k \\ v_k \end{bmatrix} \quad (37)$$

is a stabilisante control law which reject the jumps uncontrollable modes  $v_k$ . We have

$$u_k = -\begin{bmatrix} \bar{L} & G \end{bmatrix} \begin{bmatrix} I & T \\ 0 & I \end{bmatrix} \begin{bmatrix} I & -T \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{x}_k \\ v_k \end{bmatrix} \quad (38)$$

$$= -\begin{bmatrix} \bar{L} & \bar{L}T + G \end{bmatrix} \begin{bmatrix} x_k \\ v_k \end{bmatrix} \quad (39)$$

by the separation principle,

$$u_k = -\begin{bmatrix} L^x & L^y \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{v}_k \end{bmatrix} \quad \text{with} \quad L^x = \bar{L} \quad \text{and}$$

$$L^y = \bar{L}T + G$$

is a stabilisate control law which rejects in a minimum time the effect of impulsionnel signal  $d_k = \Delta v_k \delta_{k,r}$ . It is obvious that performances and robustness of our control law depend on nominal control law  $u_k^n = -\bar{L} \bar{x}_k$ .

### 4. APPLICATION

The Fault Detection and Isolation filter proposed here has been applied to a winding machine (fig. 2) representing a subsystem of many industrial systems as sheet and film processes, steel industries, and so on. The system is composed of three reels driven by DC motors ( $M_1$ ,  $M_2$ , and  $M_3$ ), gears reduction

coupled with the reels, and a plastic strip. Motor  $M_1$  corresponds to the unwinding reel,  $M_3$  is the rewinding reel, and  $M_2$  is the traction reel. The angular velocity of motor  $M_2$  ( $\Omega_2$ ) and the strip tensions between the reels ( $T_1$ ,  $T_3$ ) are measured using a tachometer and tension-meters, respectively. Each motor is driven by a local controller. Torque control is achieved for motors  $M_1$  and  $M_3$ , while speed control is realised for motor  $M_2$ .

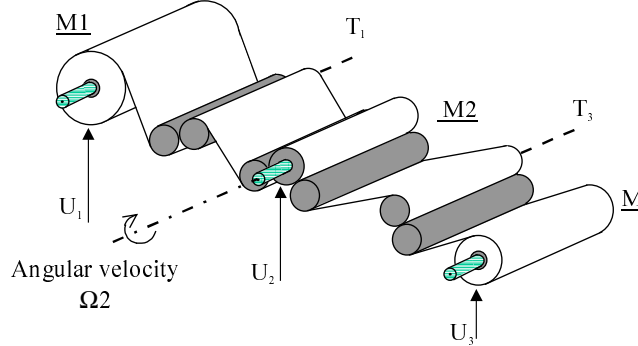


Fig. 2. The winding machine

The control inputs of the three motors are  $U_1$ ,  $U_2$ , and  $U_3$ .  $U_1$  and  $U_3$  correspond to the current set points  $I_1$  and  $I_3$  of the local controller.  $U_2$  is the input voltage of motor  $M_2$ . In winding processes, the main goal usually consists of controlling tensions  $T_1$  and  $T_3$  and the linear velocity of the strip. Here the linear velocity is not available for measurement, but since the traction reel radius is constant, the linear velocity can be controlled by the angular velocity  $\Omega_2$ .

With the sampling interval is  $T_s = 0.1$  s., the linearized model of the winding machine around the operating point  $(u_0, y_0)$  is given by the following discrete state-space representation:

$$u_0 = [-0.15 \ 0.6 \ 0.15]^T \quad y_0 = [0.6 \ 0.55 \ 0.4]^T$$

$$x = \begin{bmatrix} T_1 \\ \Omega_2 \\ T_3 \end{bmatrix}, u = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}, A = \begin{bmatrix} 0.4126 & 0 & -0.0196 \\ 0.0333 & 0.5207 & -0.0413 \\ -0.0101 & 0 & 0.2571 \end{bmatrix},$$

$$B = \begin{bmatrix} -1.7734 & 0.0696 & 0.0734 \\ 0.0928 & 0.4658 & 0.1051 \\ -0.0424 & -0.093 & 2.0752 \end{bmatrix}$$

$C$  is the identity matrix  $I_3$ . The system described by these matrices is completely observable and controllable.

A nominal control law is first set up according to the LQI technique such that the following cost function is minimised:

$$J = \frac{1}{2} \sum_{k=0}^N (\tilde{x}_k^T Q \tilde{x}_k + u_k^T R u_k).$$

The weighting matrices  $Q$  and  $R$  are respectively nonnegative symmetric and positive definite symmetric,  $Q = 0.05I_3$  and  $R = 0.1I_3$ .

In the first scenario the effectiveness of the second actuator  $M_2$  acting on the strip velocity is reduced by 5% and appears at instant 30s. According to the actuator fault description given earlier, this fault corresponds to a coefficient  $\alpha = +0.95$  and

appears abruptly on the system. In the second scenario, the same kind of fault, with a reduction of control effectiveness of 20% is applied to the actuator  $M_1$  acting on the strip tension at instant 82s during a ramp change of the strip tension. The system outputs are displayed on Fig. 3

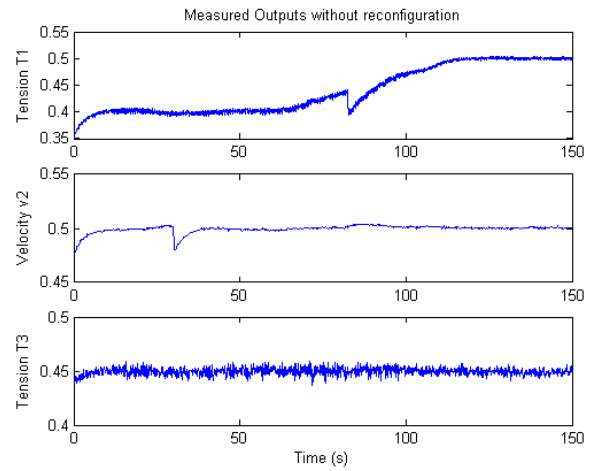


Fig3. System outputs with loss of control effectiveness applied respectively at  $t=30$ s on second actuator (motor  $M_2$ ) and at  $t=82$ s on first actuator (motor  $M_1$ )

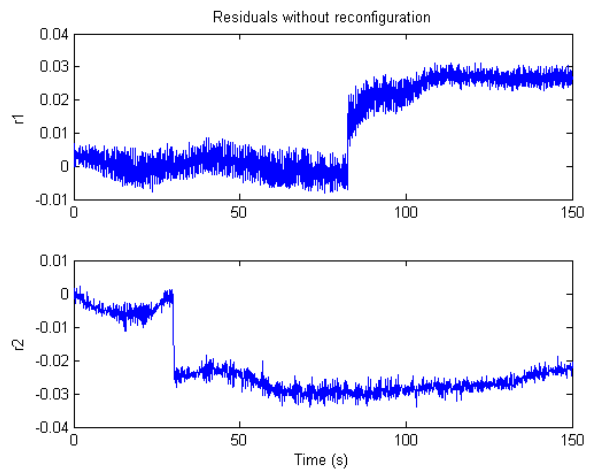


Fig4. Reponse of the isolation filter; two residuals are generated

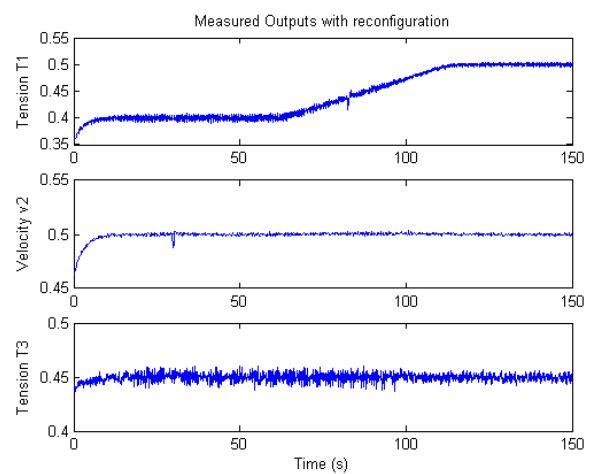


Fig5. System outputs with control reconfiguration

Fig 5 clearly demonstrate the FTC method's ability to compensate for such actuator faults. Indeed, since an actuator fault acts on the system as a perturbation, and due to the presence of the integral error in the

controller, the system outputs again reach their nominal values even without fault compensation. It shows that, without FTC, the strip tension, which is the output more affected by the fault, reaches its corresponding reference input about 7s after the fault occurrence, whereas it takes only 1s using the FTC method.

#### 4. CONCLUSION

In this paper, we have shown that the FDF can be integrated in harmony with a control law reconfiguration mechanism to maintain the steady state performances of unknown disturbance rejection in the event of failures of abrupt changes. The results show that once the fault appears, it's easy to reduce its effects on the system in minimum time. We can show that this approach tends towards to LTR control law. However, the limits of this method are reached when there is complete loss of an actuator; in this case, only a hardware redundancy is effective and could ensure performance reliability.

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