USING FUZZY MEASURES OF UNCERTAINTY TO MANAGE COMPLEX SYSTEMS

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Abstract. A complex system usually encompasses agents, as they are known from modern Artificial Intelligence. Within the system dynamics, interactions between agents are affected by perturbations (uncertainty), which often make modeling difficult. This paper introduces a modeling approach based on a function that quantifies uncertainty: the fuzzy measure of ambiguity. A model based on ambiguity minimization has been designed, as succinctly described in this paper. The model might assist the system manager to set an optimal strategy. *Copyright © 2005 IFAC*

Keywords: Multi-Agent Systems, planning, fuzzy uncertainty measures.

1. INTRODUCTION

The concept of *agent* constitutes the foundation of modern Artificial Intelligence (AI) (Russell and Norvig, 1995). Whenever it is a human or a computer routine/program, the *agent* can be understood as an autonomous problem solver that interacts with other agents (usually, heterogeneously distributed in an environment (O'Hare and Jennings, 1996)), in order to progress towards solutions. Capability for self-activation, interaction and evolution are prime features of an agent. A set of interactive agents meant to solve some problem is referred to as *Multi-Agent System* (MAS) (Weiss, 1999). Such a system can be quite complex and appears everywhere human and/or non-human resources are concurrent in finding a solution to a problem (e.g. in a factory or on Internet).

For MAS, stating a problem to solve is equivalent with targeting a global goal. Subsequently, the MAS starts to evolve, according to some strategy (in general adaptive). On the whole, *evolution* means here transition from an initial state through a chain of intermediate states, until the goal is reached in a final state. This motivates the agents to associate or not within working groups referred to as *clusters* or *teams* (Maturana and Norrie, 1996). Thus, a *virtual clustering mechanism* can be observed during the evolution. Clusters are built, modified and/or destroyed any time, describing, in fact, the states parsed by the system. Each possible structure of clusters that covers the agents set will be referred hereafter to as *(clustering) configuration*. Each state of a MAS is described by a configuration covering the agent set. The succession of all states through which MAS transitions until reaching the goal is referred to as *plan* or *(planning) strategy*.

One can hardly predict the MAS dynamics by using simple observations, since various perturbations affect the interactions between agents. Therefore, only an *uncertain* information is available in this aim. When facing the uncertainty in information about MAS dynamics, two sides should be considered: *vagueness* and *ambiguity* (Klir and Folger, 1988).

Vagueness deals with *inconsistent* and *unclear* information. More specifically, a clear distinction between a possible plan reaching the imposed goal and a plan, which, on the contrary, leads the MAS in an opposite direction, is difficult to make. This is the case when, for example, two or more factories are merging and new structures of clusters are constructed. The initial knowledge about the MAS is very parsimonious.

Whenever more information is available, the manager is faced with *ambiguity*. An ambiguous information is quite *consistent*, but the facts are mixed such that all entities seem to be similar. More specifically, the *consistency* means here that several possible plans of MAS can be constructed for a given goal. Nonetheless, it is quite difficult to select one of them, because their descriptions are too general, confuse or non-specific.

Starting from some uncertain information, the problem is to provide models of MAS, useful in selecting the most possible (least ambiguous) plan reaching for a given goal. This problem can be addressed in mathematical terms as follows. One aims to construct a family of possible plans $\mathcal{P} = \{\mathcal{P}_k\}_{k\in\overline{1,K}}$ reaching for a preset goal and a measure of uncertainty U, in order to select the least uncertain (ambiguous) of them:

$$\mathcal{P}_{k_0} = \arg\underset{k\in\overline{1,K}}{\underline{\text{opt}}}\, U(\mathcal{P}_k)\,, \quad \text{where} \quad k_o \in \overline{1,K}\,. \quad (1)$$

The problem of MAS modeling by using vagueness was approached in Ulieru et al. (2000). Within this paper, we are mostly concerned with ambiguity modeling. On the whole, the initial information about the agents of MAS is considered to be sufficiently rich, such that several possible plans can be adopted for a given goal. The solution of problem (1) is founded on Theory of Evidence (Klir and Folger, 1988) and Iterative Deepening Search (IDA*) from AI (Russell and Norvig, 1995). More specifically, *fuzzy measures of uncertainty* (such as *confusion* and *non-specificity*) are employed. The main steps in construction of the required ambiguity measure $U \equiv A$ (from "ambiguity") are succinctly presented hereafter.

2. BUILDING THE FUZZY MODEL

Step 1. *Construct a possibility tree of MAS states*

The possible successions of configurations during the MAS evolution can be represented by means of a tree. Each tree node is assigned to a configuration and comes from a unique parent, corresponding to the previous configuration. A node can have no children (if it is a leaf) or at most $L \geq 1$ children. The root is generated by some initial state. A label can be assigned to the current node, such as: (m,l), where $m \in \overline{0,M}$ is the *depth* and $l \in \overline{1,L_m}$ is the *width*. The maximum depth, $M \geq 1$, is preset and may or may not correspond to a leaf, if plans are very complex. (This limits the tree complexity, which, otherwise, would be unacceptable.) An oriented arc links a parent to a child: $(m-1,l') \triangleright (m,l)$. The arc ending in (m,l) contains two labels: a time cost ($t_{m,l}$ – the time spent by MAS when passing from the previous state to the current one) and a set of N non-temporal costs, ($c_{m,l,n}$ – the cost of actions performed by agent a_n ($n \in \overline{1,N}$) when MAS reaches the current state).

A plan is represented by any path drawn from the root to some leaf. If the leaf does not correspond to the final state, only a *partial plan* is generated. The construction of possibility tree is gradually performed: for each MAS state $S_{m,l}$, one generates all possible subsequent states, starting from the information about clustering capacity of agents. This simple procedure is not based on the prior knowledge of the possible plans. On the contrary, the tree is the basis in construction of possible plans.

MAS are, in general, so complex that it is extremely difficult to figure out in advance what are their final states. For example, in a chess game, there is a very large variety of possible final states, i.e. final positions on the chessboard corresponding to checkmate, stalemate or abandon. It is impossible to consider all the strategies reaching these positions. Therefore, a maximum depth of the possibility tree is imposed, although the goal could not yet be reached. This method can be employed to find the least ambiguous path from the root to a leaf. If the leaf corresponds to a final state of MAS, the procedure stops and the least ambiguous plan was found. If the leaf corresponds to a non-final state of MAS, the method restarts with that leaf as root and so forth, until a final state is reached. Meantime, the MAS can be initiated to evolve according to the partial least ambiguous path found. Thus, the possibility tree changes according to each stage of ambiguity minimization and the initial data should be updated between stages. This approach provides an adaptive mechanism of uncertainty minimization, by accounting that various perturbations may affect not only the data but also the optimum plan, when varying in time.

Step 2. *Construct the family of possible plans*

The collection of all possible plans is $\mathcal{P} = \{\mathcal{P}_k\}_{k\in\overline{1,K}}$. (The number of possible plans, $K \geq 1$, is upper bounded by L^M.) A plan of family $\mathcal{P}$ is expressed as follows: $\mathcal{P}_k = \left\{P_{k,0}\,, P_{k,1}\,, \ldots, P_{k,M_k}\right\}$, $\forall k \in \overline{1,K}$. Here, $P_{k,m}$ is the configuration corresponding to the node $\left(m,l_{k,m}\right)$ of the tree: $P_{k,m} = S_{m,l_{k,m}}$ ($m \in \overline{0,M_k}$), for some $l_{k,m} \in \overline{1,L_m}$ (where $l_{k,0} = 1$). The plan $\mathcal{P}_k$ is thus generated by a path conventionally described as: $(0,1) \triangleright \left(1,l_{k,1}\right) \triangleright \left(2,l_{k,2}\right) \triangleright \cdots \triangleright \left(M_k,l_{k,M_k}\right)$.

Step 3. *Construct possibility distributions for nodes*

Three operations are necessary to complete this step. First, a score is assigned to each agent a_n from the set of N agents $\mathcal{A}_N$, for the current node (m,l):

$$s_{m,l,n} \overset{def}{=} \frac{i_1 N_{n,i_1} + i_2 N_{n,i_2} + \cdots + i_{J_n} N_{n,i_{J_n}}}{i_1 + i_2 + \cdots + i_{J_n}}\,, \quad (2)$$

where $J_n \geq 1$ is the number of clusters including a_n and i_j ($j \in \overline{1,J_n}$) is the number of agents for each cluster. Also, $N_{n,i} \geq 1$ is referred to as *clustering* *number* and represents *the maximum number of clusters with* i *agents that could include the agent* a_n. If $J_n = 1$, the score is simply: $s_{m,l,n} = N_{n,i_1}$. Next, it is suitable to use cost estimators, but if they are unavailable, costs can be set according to MAS specific characteristics. Beside the costs specified in **Step 1**, one defines: $C_{m,l}$ – the non- temporal cost supported by MAS on the path from the initial node to the current one, and $T_{m,l}$ – the time spent by the MAS on that path. Costs can be additive:

$$C_{m,l} = C_{m-1,l_{m-1}} + \sum_{n=1}^{N} c_{m,l,n} \; ; \; T_{m,l} = T_{m-1,l_{m-1}} + t_{m,l} \,. \quad (3)$$

Finally, a possibility distribution $r_{m,l} : \mathcal{A}_N \to [0,1]$ is defined (for $n \in \overline{1,N}$), by normalizing the following map in range [0,1]:

$$\tilde{r}_{m,l}(a_n) \stackrel{def}{=} \frac{\left(s_{m,l,n}\right)^{sign(C_{m-1,l_{m-1}}+c_{m,l,n})}}{\left(T_{m,l}\right)^{sign(C_{m-1,l_{m-1}}+c_{m,l,n})}\left(C_{m-1,l_{m-1}}+c_{m,l,n}\right)} \,. \quad (4)$$

(At least one unit value is necessary, as required by the Possibility Theory (Klir and Folger, 1988).) Definition (4) sets the possibility $r_{m,l}[n]$ of agent a_n to belong to the specific configuration of state $S_{m,l}$. For paid costs ($C_{m-1,l_{m-1}} + c_{m,l,n} > 0$), the possibility varies proportionally to the score ($s_{m,l,n}$) and inverse proportionally to the time cost ($T_{m,l}$), as well as to the non-temporal cost that the MAS would pay due to the agent actions ($C_{m-1,l_{m-1}} + c_{m,l,n}$). For realized gains ($C_{m-1,l_{m-1}} + c_{m,l,n} < 0$), scores and time costs should affect the possibility inversely than for paid costs, but the gains improve it proportionally (negative denominator). One assumes that MAS will reach a state where the clustering capacities of agents and the realized gains are maximum, but the supported costs are minimum. Actually, the possibility values corresponding to paid costs are mapped onto [0,1/2], whereas realized gains lead to values within [1/2,1]. (Different definitions may also be considered.)

Step 4. *Construct consonant bodies of evidence*

It is well known that any possibility distribution can uniquely generate a consonant body of evidence (Klir and Folger, 1988). After decreasingly ordering the distribution $r_{m,l}$ ($r_{m,l}[k_1] = 1 \geq \cdots \geq r_{m,l}[k_N]$), the consonant body of evidence $\left(\Phi_{m,l}, \varphi_{m,l}\right)$ is generated, where the focal elements of $\Phi_{m,l}$ are $F_{m,l,n} = \left\{a_{k_1}, \ldots, a_{k_n}\right\}$, $\forall n \in \overline{1,N}$ and the basic assignment is expressed by:

$$\varphi_{m,l}[n] = r_{m,l}[k_n] - r_{m,l}[k_{n+1}], \; \forall n \in \overline{1,N} \,. \quad (5)$$

For each $n \in \overline{1,N}$, the value $\varphi_{m,l}[n]$ defined in (5) represents the degree of evidence that any agent of $\mathcal{A}_N$ belongs to the focal cluster $F_{m,l,n}$ (with exactly n agents), if MAS would reach the state $S_{m,l}$.

Step 5. *Construct measures of ambiguity.*

The ambiguity can firstly be generated by *confusion*, which focuses on the incapacity to disseminate between characteristics appearing as equally important within the available information. More specifically, the confusion is present especially when the basic assignment takes approximately equal values for a large set of focal elements, because no clear evidence of inclusion in one subset is available. One defines the *ambiguity-confusion measure*, A_{CC}, by composing the following maps:

$$\mathcal{P} \xrightarrow{\mathcal{C}_\psi} \Pi(\mathcal{S}_{M,L}) \xrightarrow{\mathcal{J}} \Pi_{\geq}(\mathcal{S}_{M,L}) \xrightarrow{T} \phi(\mathcal{S}_{M,L}) \xrightarrow{\mathcal{C}_\gamma} \mathbb{R}_+ , \quad (6)$$

where: $\mathcal{S}_{M,L}$ is the collection of all states $S_{m,l}$, $\Pi(\mathcal{S}_{M,L})$ is the set of all possibility distributions, $\Pi_{\geq}(\mathcal{S}_{M,L})$ contains only decreasingly ordered distributions and $\phi(\mathcal{S}_{M,L})$ is the set of all consonant basic assignments. The map $\mathcal{C}_\psi$ assigns a possibility distribution ρ_k to each possible plan $\mathcal{P}_k$, through the confusion measure: $\rho_k(P_{k,m}) = 1 - \alpha_{k,m} / \alpha_{MAX}$, $\forall m \in \overline{1,M_k}$. Here, $\alpha_{k,m}$ are the confusion values of $\left(\Phi_{k,m}, \varphi_{k,m}\right)$ (see (5) and **Step 2**), derived depending on $p_{k,m} = r_{m,l_{k,m}}$:

$$\alpha_{k,m} \stackrel{def}{=} \sum_{n=1}^{N} \left[p_{k,m}[k_{n+1}] - p_{k,m}[k_n]\right] \log_2 \left[1 - p_{k,m}[k_{n+1}]\right] . \quad (7)$$

One can prove that α_{MAX} – the maximum of confusion for consonant bodies of evidence – is smaller than the general upper bound evaluated in Klir and Folger (1988), where no consonance is considered. The larger $\alpha_{k,m}$, the more confuse the configuration $P_{k,m}$ and, thus, the less possible its inclusion into $\mathcal{P}_k$. Each distribution ρ_k is next decreasingly ordered by means of map $\mathcal{J}$, whereas the map T associates ρ_k to its unique (consonant) basic assignment γ_k, like in (5). Finally, a new confusion measure is computed for the whole plan $\mathcal{P}_k$, i.e. $\mathcal{C}_\gamma$, similar to (7). Thus, $\mathcal{C}_\gamma(\gamma_k)$ represents the degree of confusion in information for plan $\mathcal{P}_k$.

Another source of ambiguity is the *non-specificity*, which appears when one member has to be selected from a set and the available information seems to equally point out all its members as candidates. Then, like above, the measure of *ambiguity-non-specificity* A_{UU} is defined by composing the following maps:

$$\mathcal{P} \xrightarrow{\mathcal{U}_p} \Pi(\mathcal{S}_{M,L}) \xrightarrow{\mathcal{J}} \Pi_{\geq}(\mathcal{S}_{M,L}) \xrightarrow{\mathcal{U}_\rho} \mathbb{R}_+ . \quad (8)$$

The map $\mathcal{U}_p$ assigns a possibility distribution ρ_k to $\mathcal{P}_k$, by means of a non-specificity measure. Here, $\alpha_{k,m}$ are the non-specificity values of $\left(\Phi_{k,m}, \varphi_{k,m}\right)$:

$$\alpha_{k,m} \stackrel{def}{=} \sum_{n=1}^{N} \left[p_{k,m}[k_{n+1}] - p_{k,m}[k_n]\right] \log_2 n \,. \quad (9)$$

This time, the maximum is easy to derive: $\alpha_{MAX} = \log_2 N$. The larger $\alpha_{k,m}$, the less specific the configuration $P_{k,m}$ and, thus, the less possible its inclusion into $\mathcal{P}_k$. After ordering decreasingly ρ_k, the non-specificity $\mathcal{U}_\rho$ of a plan $\mathcal{P}_k$ is computed by using an equation similar to (9). Thus, $\mathcal{U}_\rho(\rho_k)$ represents the non-specificity degree of $\mathcal{P}_k$.

Other measures such as the *confusion-non-specificity* ($A_{CU} \equiv \mathcal{U}_\rho \circ \mathcal{I} \circ \mathcal{C}_\psi$) and the *non-specificity-confusion* ($A_{UC} \equiv \mathcal{C}_\gamma \circ T \circ \mathcal{I} \circ \mathcal{U}_p$) can be considered as well. A new couple of measures is obtained by initializing the vagueness minimization procedure (Ulieru et al., 2000) with $\alpha_{k,m}$ given by (7) or (9). These measures are the *confusion-entropy* (V_{CE}) and the *non-specificity-entropy* (V_{UE}).

Step 6. *Optimization.*
By solving problem (1), the 6 measures constructed above (A_{CC}, A_{UU}, A_{CU}, A_{UC}, V_{CE}, V_{UE}) provide 6 least uncertain plans. The minimization concerns the ambiguity (for the first 4 plans) and the vagueness (for the last 2 plans). Since the number of plans, K, is finite, the optimization can be performed by direct comparisons between measure values. But, if K is large, this attempt might be time consuming. An iterative intelligent search for the optimum can be considered as well. Since plans belong to a tree, an adapted version of IDA* (Russell and Norvig, 1995) can be implemented, provided that a good heuristic estimator of ρ_k be defined. One can prove that the AR optimal predictor (Söderström and Stoica, 1989) is an acceptable choice for pure ambiguity measures. But for ambiguity-vagueness measures, this heuristic is not necessarily verifying the basic condition of IDA* (*the cost function, including the heuristic, must be non-decreasing*) and thus the results may be sub-optimal.

Several measures are used because none of them is "perfect". However, from the optimization point of view, some of them can be preferred. In general, it is suitable that an optimization criterion (or a cost function) be smooth (more than one or two times derivable) and to have a unique extreme or, at least, all the extremes with the same value. Ruptures and multiple extremes are unsuitable. If minimization is performed in terms of ρ_k (uniquely assigned to plan $\mathcal{P}_k$), the measures are smooth and exhibit as less minima as possible (all null). If the measures are expressed in terms of $r_{k,m}$, the minimization becomes more complicated for some measures. The ambiguity-non-specificity (A_{UU}) seems to be the best one in this respect. Its minimum is obtained when all $r_{k,m}$ are unitary (*and, thus, the costs among the plan can be minimum*). The next two suitable measures are: confusion-non-specificity (A_{CU}) and the non-specificity-entropy (V_{UE}); they have a unique minimum and, respectively, only null minima on the domain frontier. The other 3 measures are affected by ruptures and/or they have multiple minima (although all null) not only on domain frontier, but also inside the domain.

3. SIMULATION RESULTS

A case study consisting of a MAS with 10 agents has been approached. The agents are: one manager (a_1), two executive directors (a_2 and a_3), three chiefs in charge (a_4, a_5 and a_6) and four resources (a_7, a_8, a_9 and a_{10}), as illustrated in Figure 1. Relationships between agents are described by the symmetric matrix below, referred to as *interactions descriptor*.

$$\mathcal{D} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & -1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & -1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & -1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \quad (10)$$

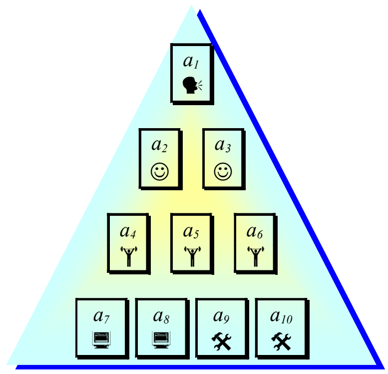

Figure 1. A Multi-Agent System with 10 agents.

The generic matrix element $\mathcal{D}[i, j]$ can be: 1 (when a_i and a_j are *direct agents*, i.e. they interact within the same cluster), –1 (when a_i and a_j are *opposite agents*, i.e. they cannot belong to the same cluster) and 0 (when a_i and a_j are *mediated agents*, i.e. they need a third agent as *mediator*, in order to interact within the same cluster). The interactions descriptor is very useful in evaluation of clustering numbers, through an inference mechanism based on a trivalent logic. The non-temporal costs are considered here as *rates of exchanged messages between agents.* Some messaging weights (evaluated by using $\mathcal{D}$ again) are applied when computing these costs. The time costs are evaluated by adding the duration of transition between a clustering configuration to another (the *reconfiguration duration*) and the *life duration* of each configuration. The possibility tree considered here has $M = 9$ depth levels and $L = 5$ maximum children per node. The maximum number of nodes is 2,441,406 and the maximum number of plans is $K = 1{,}953{,}125$.

The goal of this system is to reach the deepest level with minimum ambiguity, by passing through 9 states, starting from the root. In general, even the user knows very well the MAS, generating the next states starting from a current one is a demanding task. Therefore, beside the states guessed by the user when considering some system characteristics, in this case study, new configurations are produced by using a technique issued from Genetic Algorithms (Mitchell, 1996). Thus, *mutations* and *crossovers* are randomly applied to the matrix describing the current configuration, in order to produce new valid configurations (according to $\mathcal{D}$). From all these configurations, the ones that appear as the most plausible for MAS evolution should be selected. But the user is often still confused and might create the worst situation for optimization algorithm, by selecting the maximum number of children for each current node. One focuses next on this case.

The optimization is performed by an adaptation of IDA*, where the heuristic estimator is an AR predictor of second order (see Söderström and Stoica, 1989), for all 6 measures. Although the pure ambiguity measures match well the predictor, the choice is not very suitable for ambiguity-vagueness measures, because they require to forecast not only possibility distributions of plans but configurations as well. However, the simulations revealed that, for the most part of expanded nodes, these measures are non-decreasing if the AR predictor is used.

The Ambiguity Minimization Procedure has been implemented within MATLAB environment. Before searching for the optimum plans, the procedure is initiated to run with *fast* measures, obtained when dividing the realized ambiguity value by the sum of all $r_{m,l}$ norms associated to the path. This is motivated by 2 main reasons. First, a maximum radius of expansion is produced for the next run (the ambiguity of any optimal plan has to be inferior to the ambiguity of its corresponding *fast plan*). Secondly, since minimizing the ambiguity does not necessarily mean minimizing costs, whereas the norm is proportional with costs, the fast plans reveal a good trade-off ambiguity–costs. The algorithm runs fast with these measures. In average, only one node per level is expanded (see the first row in Table 2).

In Table 1, the ambiguity values corresponding to fast plans and the 6 measures are depicted. The minimum values are underlined, though they are not the best ones. The computational complexity is measured here by using 2 parameters: the number of expanded nodes (the first row in Table 2) and the number of flops (the second row), as accounted by MATLAB environment. The fast plans allow important reductions of computational complexity into the next stage, when the procedure is initiated to run with normal ambiguity measures (without division by possibility distributions norm). The minimum of each column in Table 1 is in fact the maximum radius for expansion within IDA* – the main actor of minimization process. This upper bound involves a mechanism of sub-trees pruning, where all partial paths whose ambiguity values overpass the maximum radius are removed, together with their subsequent tree structures. Also, the maximum radius is upgraded each time a final node is found, provided that the ambiguity of corresponding plan is inferior. The algorithm stops when all nodes with inferior ambiguity that the maximum radius have been expanded. There are 6 optimum plans selected by IDA*. Their ambiguity values are grouped within Table 3. The optimum values lie on main diagonal. The power of IDA* is revealed by the computational complexity of Table 4.

Table 1. Ambiguity values of fast plans.

	A_{CC}	A_{UU}	A_{CU}	A_{UC}	V_{CE}	V_{UE}
$\mathcal{P}^n_{CC}$	$\underline{0.543}$	2.522	$\underline{2.213}$	0.617	67.92	67.96
$\mathcal{P}^n_{UU}$	0.607	$\underline{2.246}$	2.436	0.521	67.9ß	67.98
$\mathcal{P}^n_{CU}$	0.543	2.293	$\underline{2.213}$	0.534	67.90	67.97
$\mathcal{P}^n_{UC}$	0.575	2.248	2.343	$\underline{0.518}$	67.98	67.97
$\mathcal{P}^n_{CE}$	0.580	2.657	2.258	0.626	$\underline{51.82}$	$\underline{51.54}$
$\mathcal{P}^n_{UE}$	0.580	2.657	2.258	0.626	$\underline{51.82}$	$\underline{51.54}$

Table 2. Computational complexity for fast plans.

$\mathcal{P}^n_{CC}$	$\mathcal{P}^n_{UU}$	$\mathcal{P}^n_{CU}$	$\mathcal{P}^n_{UC}$	$\mathcal{P}^n_{CE}$	$\mathcal{P}^n_{UE}$
9	9	9	9	13	13
35,121	34,851	34,851	35,121	94,205	94,703

Table 3. Ambiguity values of optimum plans.

	A_{CC}	A_{UU}	A_{CU}	A_{UC}	V_{CE}	V_{UE}
$\mathcal{P}_{CC}$	$\underline{0.502}$	2.644	2.083	0.635	67.86	67.65
$\mathcal{P}_{UU}$	0.611	$\underline{2.245}$	2.445	0.523	67.97	67.99
$\mathcal{P}_{CU}$	0.515	3.010	$\underline{2.048}$	0.649	67.54	67.70
$\mathcal{P}_{UC}$	0.617	2.274	2.391	$\underline{0.518}$	67.88	67.92
$\mathcal{P}_{CE}$	0.621	2.695	2.375	0.613	$\underline{37.86}$	37.34
$\mathcal{P}_{UE}$	0.621	2.658	2.376	0.600	37.86	$\underline{37.34}$

Table 4. Computational complexity for optimum plans.

	EN	FLOPS	SFW	LFW
$\mathcal{P}_{CC}$	43	159,484	73	173
$\mathcal{P}_{UU}$	276	1,067,195	971	1105
$\mathcal{P}_{CU}$	489	1,888,442	692	1957
$\mathcal{P}_{UC}$	33	124,248	7	133
$\mathcal{P}_{CE}$	89	679,444	158	357
$\mathcal{P}_{UE}$	71	526,891	150	285

Here: **EN** is the number of *expanded nodes*, **SFW** (*short frontier width*) is the maximum width of tree

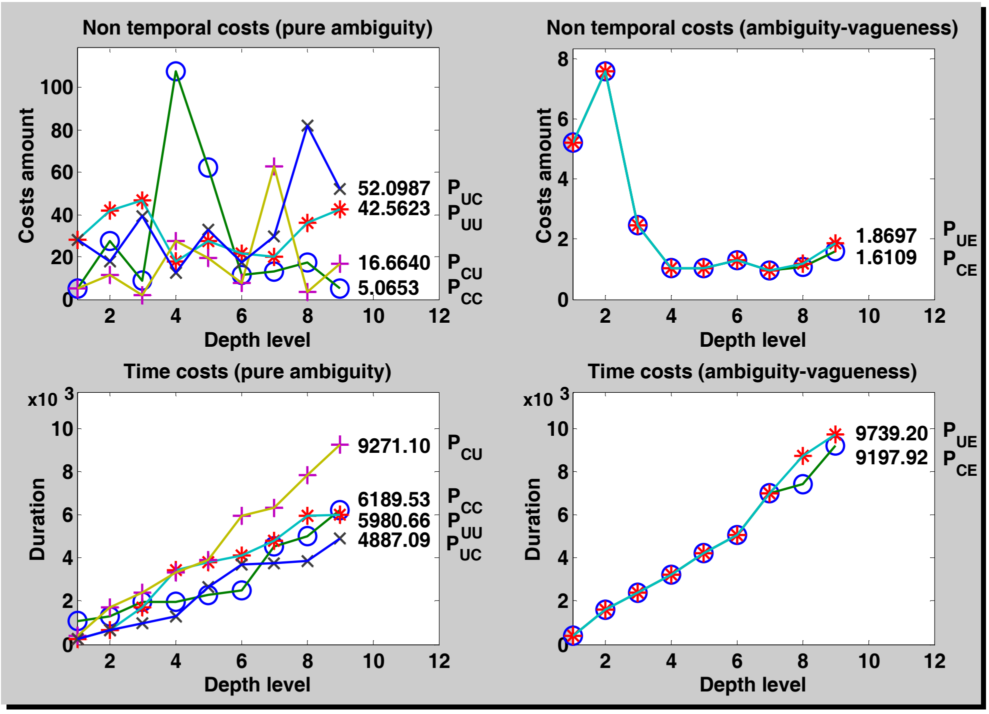

Fig. 2. Evolution of costs among the optimum plans.

dynamic frontier (next nodes for expansion, with evaluated ambiguity) when maximum radius is used and **LFW** (*long frontier width*) is the frontier width without using maximum radius.

Ambiguity-non-specificity (A_{UU}) and confusion-non-specificity (A_{CU}) perform the worst: slow convergence to the optimum (the largest number of flops) and large number of expanded nodes (involving large frontier width). However, the computational complexity is low. Thus, for example, the largest frontier width (1957) takes only 0.4% from the total number of expandable nodes (488,281) or 0.08% from the total number of tree nodes. The non-specificity-confusion (A_{UC}) exhibits the best characteristics. All characteristics are yet balanced by costs diagrams depicted in Figure 2. The variation of non-temporal costs (up) seems to have important oscillations. Actually, the non-temporal costs and the duration to reach a final state are opposite: the MAS preserves small costs with large delays or it will pay more for fastness, as expected. One can note that the ambiguity-non-specificity measure (A_{UU}) realizes a good trade-off between paid costs and duration, being the smoothest one (without costs spikes and relatively constant duration increase). A good result has been obtained by using the ambiguity-confusion measure (A_{CC}) as well (smaller final cost than A_{UU}, but longer final delay), although it provides the largest supported cost, when MAS transitions through the 4-th node. For the other measures, the trade-off is not so suitable. For example, the ambiguity-vagueness measures (V_{CE} and V_{UE}) operate with really low non-temporal costs, but with largest possible delays, whereas the non-specificity-confusion measure (A_{UC}), with minimum delay, led to the largest non-temporal cost with a spike for the 8-th node. Also, note that the ambiguity-vagueness plans are identical. They appeared to be optimum for this application (see Table 1 again), though this property could not be guaranteed by IDA*, as already mentioned.

One can see that every fuzzy measure has strengths and weaknesses. However, the system manager has now the opportunity to select the most suitable managing strategy from a set comprising only 6 plans, depending on the dynamics of costs and delays that can be afforded by the MAS.

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