MODEL-FOLLOWING ADAPTIVE ROBUST CONTROL FOR A CLASS OF UNCERTAIN SYSTEMS WITH SERIES NONLINEARITIES

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Abstract: The problem of the robust adaptive tracking for a class of uncertain systems with partially known nonlinear uncertainties and series nonlinearities is discussed. The proposed adaptive robust controller guarantees the tracking error of the systems uniformly ultimately bounded, and makes systems with nonlinear inputs the same robust as those with linear input. In contrast to some results in the control literature, the adaptive laws for updating the estimate values of the unknown parameters and the proposed controller are continuous. Moreover, the proposed adaptive control scheme can be easily implemented in practical control due to the continuity of the adaptive laws and the proposed controller. Finally, an illustrative example is given to demonstrate the utilization of the results.

Keywords: Uncertain Systems, Robust Tracking, Adaptive Control, Nonlinear Actuators, Uniformly Ultimately Bounded.

1.INTRODUCTION

In recent years much research effort has been devoted to the problem of robust tracking and output regulation of control systems (Hopp, et al., 1990; Li, et al., 1995; Elmali, et al., 1992; Li, et al., 1997; Hsu, et al., 2001). Hopp and Schmitendort designed a robust asymptotic tracking controller for a class of time–invarying uncertain linear systems (Hopp, T. H. and Schmitendorf, W. E., 1990). Li et al discussed the problem of robust output tracking of a class of single-input/single-output systems with uncertain structures using Lyapunov method (Li, et al., 1995). Elmali and Olgac extended the result to multi-input/ multi-output systems via sliding mode technique (Elmali, H. and Olgac, N., 1992). Li and Krstic studied the optimal design of adaptive tracking controllers for nonlinear systems (Li, Z. and Krstic, M., 1997). Cheng et al proposed a sufficient and generically necessary condition for the solvability of the output regulation problem for affine nonlinear control systems (Hsu Chih-Chin and Fong I-Kong, 2001). More attention has been paid to the research on the problem of robust tracking and output regulation of control systems. (see Haddad, W.M, et al. 1997 and Jong Hyeon Park, et al. 2000.)

These studies concentrate on systems with a “linear” input. They only work under the assumption of linear input, i.e., the system model is indeed linearizable. However, because of the physical limitations in control systems, there are nonlinearities in the control input (for example, saturation function) and these effects cannot be neglected. For example, hydraulic servo and electric servo motors both display such nonlinear characteristics (Truxal 1958). Because they are derived from linear models, the aforementioned studies cannot be applied to the analysis and design on uncertain systems with nonlinear inputs. Therefore, efforts on robust control of uncertain systems with nonlinear inputs have been made, including robust stabilization and robust tracking of linear systems with nonlinear input using variable structure method (Hsu, K. C. 1998a, b, Liu 2001), ultimate boundedness control of linear systems with band-bounded nonlinear actuators (Hsu Chih-Chin and Fong I-Kong, 2001).

So far, most studies on roust control are based on the assumption that the bounds of uncertainties are known and the designed controllers depend on the assumed bounds of uncertainties. However, it is often difficult to estimate the bounds of uncertainties for practical systems. If the actual bounds of the uncertainties exceed the assumed values used in controller design, the stability of the system could not be guaranteed. To ensure the stability, one has to use large bounds of uncertainties in the controller design, which definitely leads to large conservativeness.

In this paper, robust tracking for a class of uncertain systems with nonlinear actuators and un-known bounds of uncertainties is considered. The designed adaptive controller is able to ensure the controlled systems to track the reference model, and makes systems with nonlinear inputs the same robust as those with linear inputs. Finally, an illustrative example is given to demonstrate the utilization of the results.

2.SYSTEM STATEMENT

An uncertain system with nonlinear actuators and unknown bounds of uncertainties can be expressed as

$$\dot{x}(t) = Ax + B(\phi(u) + \xi(x,t)) , \tag{1}$$

where $x \in R^n$ and $u \in R^m$ are the state and the input respectively. And $\xi(x,t)$ is the system uncertainties with certain structure and uncertain parameters. $\boldsymbol{A}$ and $\boldsymbol{B}$ are const matrices with appropriate dimensions. The nonlinear input $\phi(u)$ satisfies $\phi(0)=0$. Assume there is a unique solution to the equation (1) for any initial condition $x(t_0)=x_0$ and input $u(t)$.

Let the reference model take the form of

$$\dot{x}_m = A_m x_m , \tag{2}$$

where $x_m \in R^n$ is the state vector of the reference model.

For the stable tracking of the system (2), the state of the system (2) is assumed to be bounded, i.e. the matrix $\boldsymbol{A_m}$ is stable. Moreover, the following several assumptions must be made.

Assumption 1: The matrix $\boldsymbol{A}$ in the system (1) is Hurwitz.

Assumption 2: For the system (1) and the reference model (2), there exist a matrix $H \in R^{m \times n}$ such that

$$A_m - A = BH , \tag{3}$$

Assumption 3: The nonlinear actuator $\phi(u)$ of the system (1) satisfies

$$u^T \phi(u) \geq h u^T u , \tag{4}$$

where h is the known positive constant.

Assumption 4: The uncertainties $\xi(x,t)$ of the system (1) satisfies

$$\|\xi(x,t)\| \leq \rho^T(x,t)\theta^* , \tag{5}$$

where $\rho^T(\bullet) = (\rho_1(\bullet), \rho_2(\bullet), \cdots, \rho_p(\bullet))^T$, $\theta^* = (\theta_1^*, \theta_2^*, \cdots, \theta_p^*)^T$, where $\rho_i(x,t) > 0, i = 1,2,\cdots,p$ for all x such that $\|x\| > 0$. And the function $\rho_i(\bullet) > 0, \quad i = 1,2,\cdots,p$ is also assumed to be continuous, uniformly bounded with respect to time and locally uniformly bounded with respect to x. $\theta^* \in R^p$ is a bounded constant with unknown bounds.

Remark 1: Equations (3) is a commonly assumption of model-following problem. And Equations (4) is a commonly restriction to discussing the nonlinear actuators. More information of Equations (3), (4) can be found in (Hsu, K. C., 1998a, b).

From Assumption 1, there exists a symmetric positive definite matrix Q such that the equation

$$A^T P + PA = -Q , \tag{6}$$

has positive definite solution P.

For the convenience of discussion, the error vector is defined as $e=x_m-x$. Then, from the equations (1), (2) and Assumption 2, the error equation can be obtained as

$$\dot{e} = Ae + BHx_m - B(\phi(u) + \xi(x,t)) . \tag{7}$$

Moreover, the bounds of the state of the reference model (2) is denoted as

$$\max_t \|Hx_m\| = w^* , \tag{8}$$

It is worth mentioning that w^* is bounded, but may be unknown for the system designer.

Remark 2: From the equation (7), the problem that the system (1) tracks the system (2) is the same as that of uniform ultimate boundedness of the error system (7). Therefore, a controller that makes the error system (7) uniformly ultimately bounded need to be designed Moreover, when $\phi(u)=u$, the model for the system (1) is similar to that one presented in (Hopp, T. H. and Schmitendorf, W. E., 1990). But the uncertainties $\xi(x(t),t)$, which bounds are unknown, are wider than theirs. And the controller presented in this paper will overcome the discontinuity of the ones presented in (Hsu, K. C. 1998a, b; Liu, *et al.*, 2001).

3.THE ROBUST TRACKING CONTROLLER

Under the above assumptions, the robust tracking controller for the error system (7) is presented as

$$u = \varphi(t) B^T Pe\,, \tag{9}$$

where

$$\varphi(t) = \frac{1}{h}\left(\frac{\hat{w}^2(t)}{\|B^T Pe\|\hat{w}(t)+\varepsilon_1} + \frac{(\rho^T(x,t)\hat{\theta}(t))^2}{\|B^T Pe\|\rho^T(x,t)\hat{\theta}(t)+\varepsilon_2}\right) \tag{10}$$

where the scalar ε_1 and ε_2 are positive constants. The scalar h is defined in Assumption 2. The parameters $\hat{w}(t)$ and $\hat{\theta}(t)$ are the estimates of the uncertain items w^* and θ^* satisfying the following adaptive laws

$$\dot{\hat{w}}(t) = -\delta_1\gamma\hat{w}(t) + \gamma\|B^T Pe\|\,, \tag{11}$$

$$\dot{\hat{\theta}}(t) = -\delta_2\Gamma\hat{\theta}(t) + \|B^T Pe\|\Gamma\rho(x,t)\,, \tag{12}$$

where the scalars δ_1, δ_2 and γ are positive constants. Γ is any symmetric positive definite matrix. The parameters $\hat{w}(t_0)=\hat{w}_0$ and $\hat{\theta}(t_0)=\hat{\theta}_0$ are the initial conditions of $\hat{w}(t)$ and $\hat{\theta}(t)$ respectively, which are finite. δ_1, δ_2, γ, Γ, $\hat{w}(t_0)$ and $\hat{\theta}(t_0)$ are design parameters.

Remark 3: In contrast to the controller in (Hopp, T. H. and Schmitendorf, W. E., 1990), one of the outstanding features of the controller defined by the equations (9) and (10) is that there need not know the uncertain bounds of the system (1) for the controller design. And as compared with the controller in (Hsu, K. C. 1998b), the controller in this paper is continuous and parameters-adjustable. Therefore, it can be easily implemented in practical engineering.

On the other hand, letting

$$\tilde{w}(t) = \hat{w}(t) - w^*\,, \tag{13}$$

$$\tilde{\theta}(t) = \hat{\theta}(t) - \theta^*\,. \tag{14}$$

Then the adaptive laws (11) and (12) can be reformed as the following error equations

$$\dot{\tilde{w}}(t) = -\delta_1\gamma\tilde{w}(t) + \gamma\|B^T Pe\| - \delta_1\gamma w^*\,, \tag{15}$$

$$\dot{\tilde{\theta}}(t) = -\delta_2\Gamma\tilde{\theta}(t) + \|B^T Pe\|\Gamma\rho(x,t) - \delta_2\Gamma\theta^*\,. \tag{16}$$

The following theorem can be obtained which shows the uniform ultimate boundedness of the closed-loop system in the equations (7), (9), (15) and (16).

Theorem 1: Consider the error system (7) and the error equations (15) and (16) satisfying Assumptions 1-4. Then the solution $(e,\tilde{w},\tilde{\theta})\,(t;t_0,e(t),\tilde{w}(t),\tilde{\theta}(t))$ to the error system (7) and the error equations (15) and (16) is uniformly ultimately bounded in the presence of the uncertain $\xi(x(t),t)$.

Proof: For the error system (7), the controller (9) and the error equations (15) and (16), a Lyapunov function candidate is defined as

$$V(e,\tilde{w},\tilde{\theta}) = e^T Pe + \gamma^{-1}\tilde{w}^2(t) + \tilde{\theta}^T(t)\Gamma^{-1}\tilde{\theta}(t)\,, \tag{17}$$

where the matrix P is defined by (6). And the parameters γ and Γ are declared in (11) and (12) respectively. Then, taking the derivative of $V(e,\tilde{w},\tilde{\theta})$ along the trajectories of the error system (7), the controller (9) and the error equations (15) and (16) leads to

$$\begin{aligned}\frac{dV(e,\tilde{w},\tilde{\theta})}{dt} &= 2e^T P\dot{e} + 2\gamma^{-1}\tilde{w}(t)\dot{\tilde{w}}(t) + 2\tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}}(t)\\ &= e^T(A^T P + PA)e + 2e^T PBHx_m - 2e^T PB\phi(u)\\ &\quad -2e^T PB\xi(x,t) + 2\gamma^{-1}\tilde{w}(t)\dot{\tilde{w}}(t) + 2\tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}}\,.\end{aligned} \tag{18}$$

It can be got from the equations (4) and (9)

$$u^T\phi(u) = \varphi(t)e^T PB\phi(u) \ge hu^T u = h\varphi^2(t)\|B^T Pe\|^2\,. \tag{19}$$

Furthermore, the inequality (19) can be rewritten as

$$-e^T PB\phi(u) \le -h\varphi(t)\|B^T Pe\|^2\,. \tag{20}$$

Substituting the equations (6) and (10), and the inequality (20) into the equation (18) leads

$$\begin{aligned}\frac{dV(\bullet)}{dt} &\le -e^T Qe + 2e^T PBHx_m - 2h\varphi(t)\|B^T Pe\|^2\\ &\quad -2e^T PB\xi(x,t) + 2\gamma^{-1}\tilde{w}(t)\dot{\tilde{w}}(t) + 2\tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}}(t)\\ &\le -e^T Qe + 2\hat{w}(t)\|B^T Pe\| + 2\|B^T Pe\|\rho^T(x,t)\tilde{\theta}(t)\\ &\quad -\frac{2(\rho^T(x,t)\hat{\theta}(t))^2\|B^T Pe\|^2}{\|B^T Pe\|\rho^T(x,t)\hat{\theta}(t)+\varepsilon_2} - \frac{2\hat{w}^2(t)\|B^T Pe\|^2}{\|B^T Pe\|\hat{w}(t)+\varepsilon_1}\\ &\quad -2\delta_1\tilde{w}^2(t) + 2\delta_1\hat{w}(t)w^* - 2\delta_2\|\tilde{\theta}(t)\|^2\\ &\quad +2\delta_2\|\tilde{\theta}(t)\|\|\theta^*\|\,.\end{aligned} \tag{21}$$

Noting the fact

$$0 \le \frac{ab}{a+b} < b\,, \quad \forall a \ge 0, b > 0\,.$$

Then

$$2\hat{w}(t)\|B^T Pe\| - \frac{2\hat{w}^2(t)\|B^T Pe\|^2}{\|B^T Pe\|\hat{w}(t)+\varepsilon_1} = \frac{2\hat{w}(t)\|B^T Pe\|\varepsilon_1}{\|B^T Pe\|\hat{w}(t)+\varepsilon_1} \le 2\varepsilon_1\,. \tag{22}$$

Similarly

$$2\|B^T Pe\|\rho^T(x,t)\hat{\theta}(t) - \frac{2(\rho^T(x,t)\hat{\theta}(t))^2\|B^T Pe\|^2}{\|B^T Pe\|\rho^T(x,t)\hat{\theta}(t)+\varepsilon_2} \le 2\varepsilon_2\,. \tag{23}$$

On the other hand

$$-2\delta_1\tilde{w}^2(t)+2\delta_1\tilde{w}(t)w^* = -\delta_1\tilde{w}^2(t)-\delta_1(\tilde{w}^2(t)-2\tilde{w}(t)w^*) \le -\delta_1\tilde{w}^2(t)+\delta_1(w^*)^2 . \qquad (24)$$

Similarly

$$-2\delta_2\left\|\tilde{\theta}(t)\right\|^2+2\delta_2\left\|\tilde{\theta}(t)\right\|\left\|\theta^*\right\| \le -\delta_2\left\|\tilde{\theta}(t)\right\|^2+\delta_2\left\|\theta^*\right\|^2 , \quad (25)$$

Inserting the inequalities (22)-(25) into the inequality (21) leads

$$\begin{aligned}\frac{dV(e,\tilde{w},\tilde{\theta}(t))}{dt} &\le -e^T\theta e-\delta_1\tilde{w}^2(t)-\delta_2\left\|\tilde{\theta}(t)\right\|^2 \\ &\quad +2(\varepsilon_1+\varepsilon_2)+\delta_1(w^*)^2+\delta_2\left\|\theta^*\right\|^2 \\ &\le -c\left\|e\right\|^2-\delta_1\tilde{w}^2(t)-\delta_2\left\|\tilde{\theta}(t)\right\|^2 \\ &\quad +2(\varepsilon_1+\varepsilon_2)+\delta_1(w^*)^2+\delta_2\left\|\theta^*\right\|^2 \\ &\le -\tilde{c}\left\|\tilde{e}\right\|^2+\tilde{\varepsilon} , \end{aligned} \qquad (26)$$

where

$$c=\lambda_{\min}(Q) , \qquad (27a)$$

$$\tilde{c}=\min(c,\delta_1,\delta_2) , \qquad (27b)$$

$$\tilde{\varepsilon}=2(\varepsilon_1+\varepsilon_2)+\delta_1(w^*)^2+\delta_2\left\|\theta^*\right\|^2 . \qquad (27c)$$

Thus, from the equality (26), it can be asserted that the solution $(e,\tilde{w},\tilde{\theta})(t;t_0,e(t_0),\tilde{w}(t_0),\tilde{\theta}(t_0))$ to the system (7) and the error equations (15) and (16) is uniformly ultimately bounded in the presence of the uncertain $\xi(x(t),t)$.

Remark 4: It is worth pointing out that the parameters $\tilde{\varepsilon}$, δ_1 and δ_2 can be chosen by the system designer. Therefore, by choosing these parameters correctly, the better tracking performance of the adaptive systems can be guaranteed. In fact, it can be seen from the equation (27) that a smaller ε can be guaranteed by choosing parameters ε_1 , ε_2 , δ_1 and δ_2 which are small enough. However, making δ_1 and δ_2 small will lead to a high adaptive gain, and letting $\varepsilon_1 \to 0$ and $\varepsilon_2 \to 0$, the controller (9) will be reduced to a standard saturation-type controller, resulting in a tradeoffs between the better tracking results and large gains, and the loss of continuity of the controller.

4. SIMULATIONS

Consider an uncertain system with nonlinear actuators described as

$$\dot{x}=\begin{pmatrix}0 & 1\\ -2 & -2\end{pmatrix}x+\begin{pmatrix}0\\1\end{pmatrix}(\phi(u)+\theta(x_1+x_2^2)\sin x_1) , \qquad (28)$$

where

$$\phi(u)=\begin{cases}0.5(e^{|u|}-1)sign(u), & |u|\le 1.5,\\ (0.8|u|+0.54)sign(u)-0.9\sin((5|u|-7.5) \\ \times sign(u))e^{-0.125(|u|-1.5)}, & |u|>1.5.\end{cases}$$

The reference model is

$$\dot{x}_m=\begin{pmatrix}0 & 1\\ -1 & -2\end{pmatrix}x_m . \qquad (29)$$

It can be verified that the system (28) and (29) satisfy Assumption 1-4 (where $H=(1\ \ 0)$). For the equation (6), letting $Q=I$, then

$$P=\begin{pmatrix}0.75 & 0.25\\ 0.25 & 0.125\end{pmatrix}$$

For the equations (9)-(12), setting the uncertain unknown parameter $\theta=-2$, the uncertain known parameter $\rho(x,t)=\left|(x_1+x_2^2)\sin x_1\right|$, the initial conditions $x(0)=(-2,2)^T$ and $x_m(0)=(2,-1)^T$. Fig. 1-2 show the simulation results where the adaptive parameters $\delta_1=0.05$, $\delta_2=0.01$, $\gamma=0.01$ and $\Gamma=0.02$, and the controller parameters $\varepsilon_1=\varepsilon_2=0.01$. Fig. 3-4 show the simulation results where the adaptive parameters $\delta_1=0.2$, $\delta_2=0.1$ and $\gamma=\Gamma=0.1$, and the controller parameters $\varepsilon_1=\varepsilon_2=0.1$ (where the real line represents $e_1=x_{m1}-x_1$, the broken line represents $e_2=x_{m2}-x_2$).

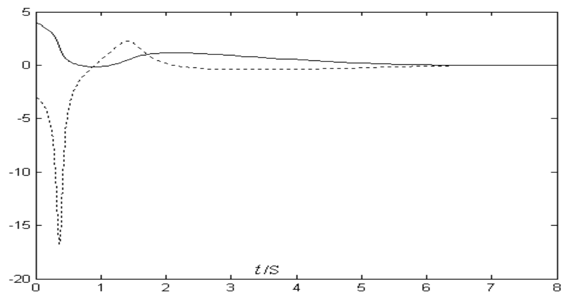

Fig. 1: The error trajectory ($\delta_1=0.05$, $\delta_2=0.01$, $\gamma=0.01$, $\Gamma=0.02$, $\varepsilon_1=\varepsilon_2=0.01$)

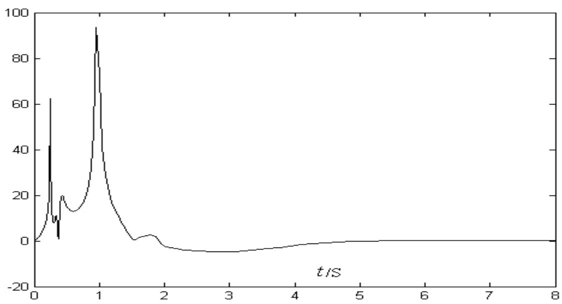

Fig. 2: The control trajectory ($\delta_1=0.05$, $\delta_2=0.01$, $\gamma=0.01$, $\Gamma=0.02$, $\varepsilon_1=\varepsilon_2=0.01$)

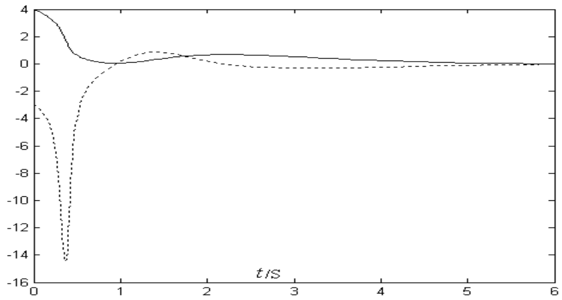

Fig. 3: The error trajectory ($\delta_1 = 0.2$, $\delta_2 = 0.1$, $\gamma = \Gamma = 0.1$, $\varepsilon_1 = \varepsilon_2 = 0.1$)

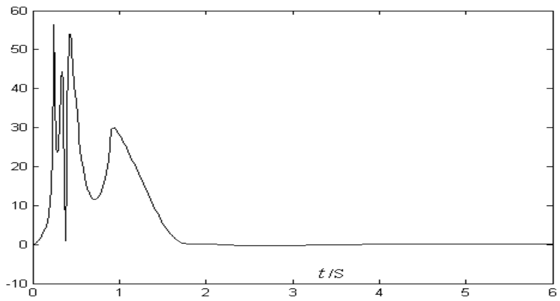

Fig. 4: The Control trajectory ($\delta_1 = 0.2$, $\delta_2 = 0.1$, $\gamma = \Gamma = 0.1$, $\varepsilon_1 = \varepsilon_2 = 0.1$)

5. CONCLUSION

The problem of the robust adaptive tracking for a class of uncertain systems with partially known nonlinear uncertainties and series nonlinearities is discussed. The proposed adaptive robust controller guarantees the tracking error of the systems uniformly ultimately bounded, and makes systems with nonlinear inputs the same robust as those with linear input. In contrast to some results in the control literature, the adaptive laws for updating the estimate values of the unknown parameters and the proposed controller are continuous. Moreover, the proposed adaptive control scheme can be easily implemented in practical control due to the continuity of the adaptive laws and the proposed controller.

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REFERENCES

Elmali, H. and Olgac, N., (1992). Robust output tracking control of nonlinear MIMO systems via sliding mode technique, Automatica, 1992, 28, 145-151.

Haddad, W.M., Chellaboina, V.S. and Fausz, J.L.; Leonessa, A., (1997). Optimal nonlinear robust control for nonlinear uncertain cascade systems., Proc. of ACC, 1997, Volume: 1, 4-6, June, 1997, 403 – 407.

Hopp, T. H. and Schmitendorf, W. E., (1990). Design of a linear controller for robust tracking and model following, ASME J. on Dynamic Systm. Meas. and Control, 1990, 112, 552-558.

Hsu Chih-Chin and Fong I-Kong, (2001). Ultimate boundedness control of linear systems with bandbounded nonlinear actuators and additive measurement noise, Systems & Control Letters, 2001, 43, 329-326.

Hsu, K. C., (1998a). Variable structure control design for uncertain dynamic systems with sector nonlinearities, Automatica, 1998, 34(4): 505-508.

Hsu, K. C., (1998b). Decentralized variable-structure model-following control design for interconnected systems with series nonlinearities, International Journal of Systems Science, 1998, 29(4): 365-372.

Jong Hyeon Park and Jongguk Yim. (2000). Robust control of cascaded nonlinear systems. Intelligent Robots and Systems, 2000. (IROS 2000). Proc. of IEEE/RSJ International Conference on, Volume: 1, 2000:33 – 38.

Li, Z., Chai, T. Y., Wen, C. and Soh, C. B., (1995). Robust output tracking control for nonlinear uncertain systems, System Control Letters, 1995, 25, 53-61.

Li, Z. and Krstic, M., (1997). Optimal design of adaptive tracking controllers for nonlinear systems, Automatica, 1997, 33, 1459-1473.

Liu F. L., Wu H., Ma C. G., Li J. B. and Zhu J., (2001). Decentralized dynamic output feedback control for a class of similar interconnected large-scale systems with nonlinear input, Proc. of World Multiconference on systemics, Cyberne-tics and Informatics, Orlando, USA, 2001, Vol., IX, Part 1, 142-147.

Truxal, J. G., (1958). Control Engineer's Handbook, New York: McGraw-Hill.