DESIGN OF LONGITUDINAL CONTROL SYSTEM FOR A NONLINEAR F-16 FIGHTER USING MSS METHOD

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Abstract: In automatic flight control system or autopilots, multiple specifications criteria are needed to be satisfied concurrently, such as good holding (small static altitude holding error), fast response, smooth transition (less oscillation, overshoot). So how to design the MSS (Multiple Simultaneous Specification) controller effectively and practically is a very significant and challenging job. Liu proposed a MSS controller design method (Liu and Mills, 2000). In this paper, we further apply the method together with the fine-tuning technique to the 6DoF nonlinear F-16 fighter longitudinal control channel. Simulation results show its applicability to nonlinear flight control system.

Keywords: MSS control, flight control, longitudinal control channel, pitch attitude control, speed control.

1. INTRODUCTION

Aircraft manufacturers have reached a high level of expertise and experience in flight control. The current design and analysis techniques applied in industry enable flight control engineers to address virtually any realistic design challenge. However, the design and implementation of flight control laws is still a very complex task and the many design problems that have to be considered make it a costly and lengthy process. For example, in automatic flight control system or autopilots, multiple specifications criteria are needed to be satisfied concurrently, such as good holding (small static altitude holding error), fast response, smooth transition (less oscillation, overshoot). So how to design the MSS (Multiple Simultaneous Specification) controller effectively and practically is a very significant and challenging job. Liu proposed a MSS controller design approach for the above problem (Liu and Mills, 2000). In this paper, we apply the method further to a more practical environment-6DoF nonlinear F-16 fighter longitudinal control channel.

In the longitudinal control channel of the F-16 fighter, the pitch control loop and speed control loop are considered for the flight control integration (Etkin, B.,1982). The pitch attitude control channel is the basic longitudinal autopilot channel; it controls the pitch angle by applying appropriate deflections of the elevator if the actual pitch angle differs from the desired reference value. The speed control channel is also an autopilot channel; it maintains a constant speed or Mach number through coordinated control of throttle and elevator. For the longitudinal control loop, we need to design proper controller to satisfy multiple objectives. Here we apply the MSS controller design method together with the fine-tuning technique to obtain the final controller for the 6DoF nonlinear F-16 fighter.

The rest of the paper is organized as follows. In Section 2 we review the necessary theoretical background of the MSS controller design method. And in Section 3, we give the 6DoF F-16 fighter nonlinear model and the linear model at the trimmed operation point. Section 4 gives the design implementation of the individual control channel and the integrated control of the longitudinal channels of the F-16 linear model, and the simulation results of linear model and non-linear model. The conclusions and on-going/future research are given in Section 5.

2. MSS CONTROLLER DESIGN METHOD

A general framework for control system includes the plant represented by a transfer matrix P, an exogenous input w and actuator input u, a controller represented by a transfer matrix K, and a regulated output z and sensor output y, as shown in figure 1 (Boyd and Barratt, 1991).

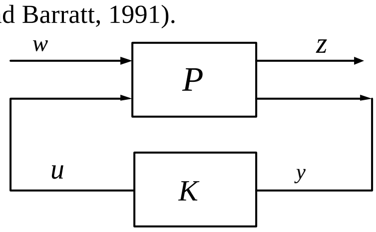


Fig.1. Control system frame work

We partition the plant transfer matrix P as

$$P = \begin{bmatrix} P_{zw} & P_{zu} \\ P_{yw} & P_{yu} \end{bmatrix} \quad (1)$$

Hence

$$\begin{aligned} z &= P_{zw}w + P_{zu}u \\ y &= P_{yw}w + P_{yu}u \end{aligned} \quad (2)$$

where P_{ij} is the transfer matrix from j to i, $i = z, y$; $j = u, w$. Now suppose the controller is operating, so that we have

$$u = Ky \quad (3)$$

We can solve for z in term of w to get

$$z = (P_{zw} + P_{zu}K(I - P_{yu}K)^{-1}P_{yw})w \quad (4)$$

that is, the closed-loop transfer matrix H can be represented as

$$H = P_{zw} + P_{zu}K(I - P_{yu}K)^{-1}P_{yw} \quad (5)$$

Many control design specifications are convex functions with respect to the closed-loop transfer matrices H (Boyd and Barratt, 1991), that is, all performance specifications can be considered simultaneously as functions in terms of H, which are evaluated under every different controller K. If there are n convex specifications required to be satisfied simultaneously, denoted as

$$\begin{aligned} f_1(H) &\le a_1 \\ f_2(H) &\le a_2 \\ &\cdots \\ f_n(H) &\le a_n \end{aligned} \quad (6)$$

where a_i ($i = 1,2,...,n$) denote the expected specification value, then a MSS control problem can be formalized as: Design a controller K such that all the specifications hold simultaneously. We call such a controller a satisfactory controller. Liu proposed the convex combination method (Liu, 2001) to obtain the MSS controller.

3. NONLINEAR F-16 MODEL AND LINEARIZATION

Now we apply the proposed MSS controller design method to a 6DoF F-16 fighter. In the following, we give the non-linear ordinary differential equations (ODE) describing the motion of a F-16 fighter (Luat, T. Nguyen, *et al*., 1979):

$$\begin{aligned}
\dot{u} &= rv - qw - g\sin q + \frac{\bar{q}s}{m}C_{X,t} + \frac{T}{m} \\
\dot{v} &= pw - ru + g\cos q\sin j + \frac{\bar{q}s}{m}C_{Y,t} \\
\dot{w} &= qu - pv + g\cos q\cos j + \frac{\bar{q}s}{m}C_{Z,t} \\
\dot{p} &= \frac{I_Y - I_Z}{I_X}qr + \frac{I_{XZ}}{I_X}(\dot{r} + pq) + \frac{\bar{q}sb}{I_X}C_{l,t} \\
\dot{q} &= \frac{I_Z - I_X}{I_Y}pr + \frac{I_{XZ}}{I_Y}(r^2 - p^2) + \frac{\bar{q}s\bar{c}}{I_Y}C_{m,t} - H_e r \\
\dot{r} &= \frac{I_X - I_Y}{I_Z}pq + \frac{I_{XZ}}{I_Z}(\dot{p} - qr) + \frac{\bar{q}sb}{I_Z}C_{n,t} - H_e q
\end{aligned} \quad (7)$$

where u, v, w and p, q, r are the body-axes components of linear velocities and rotational velocities, respectively; yaw angle y, pitch angle q, and roll angle j, that is, the Euler angles denote the attitudes of the aircraft with respect to the Earth; g is acceleration due to gravity, m is airplane mass; $\bar{q}$ is the free-stream dynamic pressure; s denotes wing area, b is wing span, $\bar{c}$ is wing mean aerodynamic chord. T is the engine thrust, He is the engine angular momentum; $I_X, I_Y, I_Z, I_{XY}, I_{XZ}, I_{YZ}$ are inertia tensor; the coefficient $C_{X,t}, C_{Y,t}, C_{Z,t}, C_{l,t}, C_{m,t}, C_{n,t}$, are the total aerodynamic coefficient, which were derived from low-speed static and dynamic wind-tunnel tests conducted with subscale models of the F-16 in wind-tunnel facilities at the NASA Ames and Langley Research Centres fighter (Luat, T. Nguyen, *et al*., 1979). The motion equations and the below kinematic equations together make up the 12 independent ODEs, which is the F-16 nonlinear ODE model.

$$
\begin{aligned}
\dot{y} &= \frac{q\sin j + r\cos j}{\cos q} \\
\dot{q} &= q\cos j - r\sin j \\
j &= p + \dot{y}\sin q \\
\dot{x}_e &= \{u\cos q + (v\sin j + w\cos j)\}\cos y - \\
&\quad (v\cos j - w\sin j)\sin y \\
\dot{y}_e &= \{u\cos q + (v\sin j + w\cos j)\}\sin y - \\
&\quad (v\cos j - w\sin j)\cos y \\
\dot{z}_e &= -u\sin q + (v\sin j + w\cos j)\cos q
\end{aligned} \tag{8}
$$

where the Euler angles y , q , and j denote the attitudes of the aircraft with respect to the earth; x_e, y_e, z_e denote the position of the aircraft with respect to the earth-fixed reference frame.

In order to design the MSS controller for the non-linear F-16 fighter, first, we need to obtain the linearized model, and synthesize the linear controller based on linear system theory. Then, with the non-linear fighter model, we apply and fine-tune the linear controller to derive the proper controller. In this paper, we apply the MSS controller design method to design the initial controller for the individual loop and then integrate the MSS controllers using the open-loop combination method (Liu, 2001). According to the simulation results of the nonlinear model, the trial-and-error method is used to add the derivative control term to the initial controller, and the proper controller for the nonlinear system is finally obtained. At the beginning, we need to find the steady-state flight conditions which can be used as 'operating points' for the linearization, and as initial conditions for simulations. In this paper, we consider the steady wing-level flight of F-16 fighter at an altitude of 5000m and Mach number is 0.6. Then we compute the steady-state flight conditions and linearize the ODE nonlinear model by small-perturbation methods.

The linearized F-16 system described by the state-space matrices A, B, C, D , can be denoted by the standard Matlab LTI system for study convenience. We consider two inputs two outputs subsystem, whose inputs are "deflection of elevator d_e ", "deflection of engine thrust d_t ", the outputs are "pitch attitude q ", "airspeed u along x-axes" and the state vector x is $[u, v, w, p, q, r, y, q, j, x_e, y_e, z_e]$ .Because the matrix A is a 12×12 matrix, the denominator of the transfer functions is of order 12. It is true that under the conditions of small perturbations from steady-state, wings-level, non-sideslipping flight, the rigid-aircraft equations of motion could be split into two uncoupled sets. These are the longitudinal equations that involve u, w, q, q and the lateral-directional equations that involve v, j, p, r . It is possible to extract simplified sub-matrix A_{Lo} from A by specifying a vector with the element number of the required state variables. Similarly, the sub-matrices B_{Lo}, C_{Lo}, D_{Lo} can be obtained from B, C, D , respectively. Then the derived simplified longitudinal system has the following expression:

$$
\begin{aligned}
\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{q} \end{bmatrix} &= A_{Lo} \begin{bmatrix} u \\ w \\ q \\ q \end{bmatrix} + B_{Lo} \begin{bmatrix} d_t \\ d_e \end{bmatrix} \\
\begin{bmatrix} u \\ q \end{bmatrix} &= C_{Lo} \begin{bmatrix} u \\ w \\ q \\ q \end{bmatrix} + D_{Lo} \begin{bmatrix} d_t \\ d_e \end{bmatrix}
\end{aligned} \tag{9}
$$

where

$$
A_{Lo} = \begin{bmatrix} -0.0083 & 0.0463 & -9.3614 & -9.7888 \\ -0.0609 & -0.8214 & 180.9018 & -0.4887 \\ 0.0023 & -0.0466 & -1.6248 & 0 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix}
$$

$$
B_{Lo} = \begin{bmatrix} 1.0748 & 0.0399 \\ 0 & -0.3589 \\ 0 & -0.2012 \\ 0 & 0 \end{bmatrix}; \quad C_{Lo} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
D_{Lo} = 0
$$

The transfer functions of the F-16 longitudinal control system are then derived at last.

$$
\begin{aligned}
G_{ud_t} &= \frac{1.0748\,(s - 0.002331)\,(s^2 + 2.448s + 9.777)}{(s^2 + 0.01017s + 0.00474)(s^2 + 2.444s + 9.787)} \\
G_{qd_t} &= \frac{0.0025024\,(s + 2.042)}{(s^2 + 0.01017s + 0.00474)(s^2 + 2.444s + 9.787)} \\
G_{ud_e} &= \frac{0.03988\,(s + 48.22)\,(s^2 + 1.044s + 0.758)}{(s^2 + 0.01017s + 0.00474)\,(s^2 + 2.444s + 9.787)} \\
G_{qd_e} &= \frac{-0.2012\,(s + 0.7349)\,(s + 0.0112)}{(s^2 + 0.01017s + 0.00474)\,(s^2 + 2.444s + 9.787)}
\end{aligned}
$$

4. INTEGRATED PITCH/SPEED AUTOPILOT DESIGN AND SIMULATION

Now we consider the integrated longitudinal control system of aircraft (Liu, 2002), shown in figure 2.

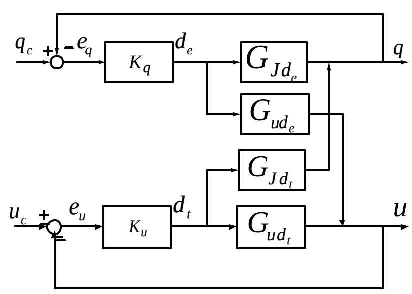


Fig.2. Integrated longitudinal control system

Assume that the overall multiple performance requirements are: the pitch attitude and speed control both have good design criteria in term of tracking (small steady state error and fast setting time) and safety (acceptable overshoot). The cross effect is represented by the simulation stop time value under the cross step command.

$$
\begin{aligned}
f_1(H) &= f^u_{overshoot} = (\max |u(t)|_{t\ge 0, u_c=1(t)} - 1) && \le a_1;\\
f_2(H) &= f^u_{settle} = |u(T)-1| && \le a_2;\\
f_3(H) &= f^q_{overshoot} = (\max |q(t)|_{t\ge 0, q_c=1(t)} - 1) && \le a_3;\\
f_4(H) &= f^q_{settle} = |q(T)-1| && \le a_4;\\
f_5(H) &= f^u_{cross} = |u(T)|_{q_c=1(t)} && \le a_5;\\
f_6(H) &= f^q_{cross} = |q(T)|_{u_c=1(t)} && \le a_6
\end{aligned}
\tag{10}
$$

where T is the simulation end time. The desired specification values of the F-16 fighter longitudinal control system are defined by:
$a_1 = 0.03$; $a_2 = 0.005$; $a_3 = 0.3$; $a_4 = 0.02$; $a_5 = 0$; $a_6 = 0$.

In this paper, we apply the open-loop combination method (Liu, 2001) to design the proper integrated controller. First, design the individual controller by MSS controller design methods (Liu and Mills, 2000) and then integrate the individual controllers to meet the total specifications.

For the speed control loop, we need to satisfy the specifications f_1 and f_2. Using the MSS controller design method (Liu and Mills, 2000), we design the sample controller [Eq.(11)] to satisfy one specification at one time (Goodwin, *et al*., 2001),

$$J_u^1 = 5 + \frac{1}{s} \quad J_u^2 = 10 + \frac{5}{s} \tag{11}$$

then $f_1(H_1) = 0.027597$; $f_2(H_1) = 0.005576$
$f_1(H_2) = 0.035498$; $f_2(H_2) = 0.001256$

From the MSS controller design method (Liu and Mills, 2000), we need to solve the inequality [Eq.(12)]

$$\begin{bmatrix} f_1(H_1) & f_1(H_2) \\ f_2(H_1) & f_2(H_2) \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \le \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{12}$$
$$l_i \ge 0,\ l_1 + l_2 = 1$$

Using the linear programming optimization routine in Matlab, $l_1 = 0.708204$, $l_2 = 0.291796$ were found. Then the final MSS controller [Eq.(13)] was derived (Liu and Mills, 2000):

$$J_u = \frac{6.459(s+7.941)(s+0.5185)(s+0.2023)}{s(s+8.713)(s+0.473)} \tag{13}$$

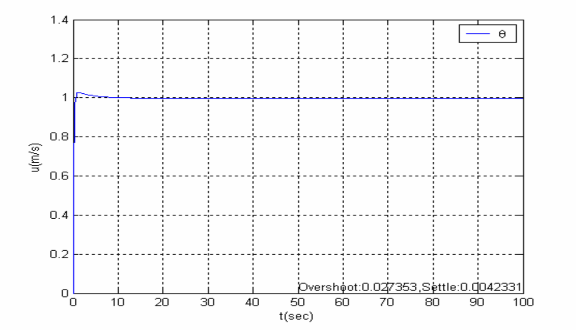


Fig.3. Speed control loop simulation

Figure 3 is the simulation results of the controller; it satisfies the two specifications i.e.
$f_1 = 0.027353$; $f_2 = 0.0042331$.

For pitch attitude control loop, we also use the same methods. We design two sample controllers [Eq.(14)] to satisfy one specification at a time (Goodwin, *et al*., 2001),

$$J_q^1 = -15 - \frac{5}{s} \quad J_q^2 = -5 - \frac{10}{s} \tag{14}$$

Then we can get the specification matrix, i.e.

$$\begin{bmatrix} f_3(H_1) & f_3(H_2) \\ f_4(H_1) & f_4(H_2) \end{bmatrix} = \begin{bmatrix} 0.183986 & 0.388989 \\ 0.021383 & 0.010465 \end{bmatrix} \tag{15}$$

Since

$$\begin{bmatrix} f_3(H_1) & f_3(H_2) \\ f_4(H_1) & f_4(H_2) \end{bmatrix} \begin{bmatrix} 0.710093 \\ 0.289907 \end{bmatrix} = \begin{bmatrix} 0.222820 \\ 0.018313 \end{bmatrix} \le \begin{bmatrix} a_3 \\ a_4 \end{bmatrix},$$

we have $l_1 = 0.710093$, $l_2 = 0.289907$. Then the MSS controller (16) is given (Liu and Mills, 2000): :

$$J_q = \frac{-12.1009\ (s+0.5005)\ (s^2 + 0.3134s + 0.1235)}{s\,(s^2 + 0.2449s + 0.1224)} \tag{16}$$

The simulation results are shown in Figure 4. It shows that the control objectives can be satisfied successfully.

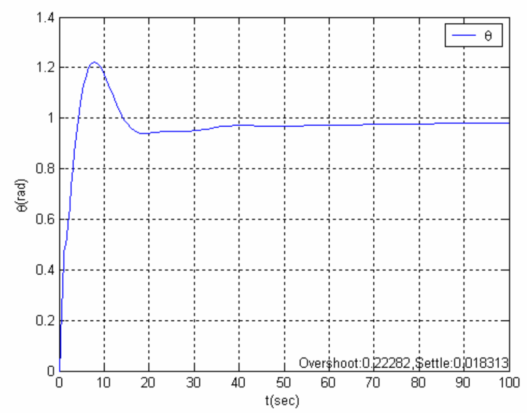


Fig.4. Pitch control loop

From the above simulation results, it is obvious that the MSS controller satisfies the required objectives of the respective loop.

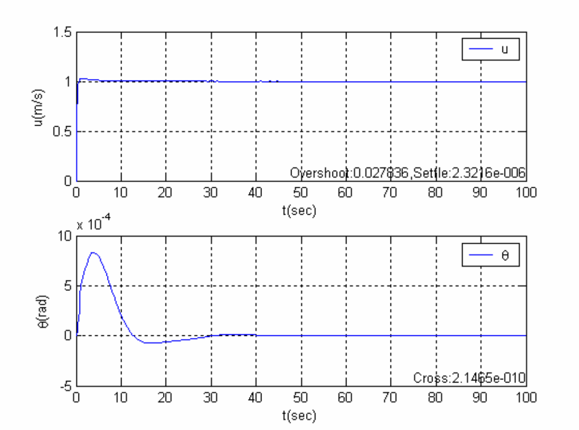
Now we integrate the individual loops (Liu, 2001) through open-loop combination method to form the integrated control system where the total specifications are also evaluated.

According to the open-loop combination method (Liu, 2001), the integrated controllers

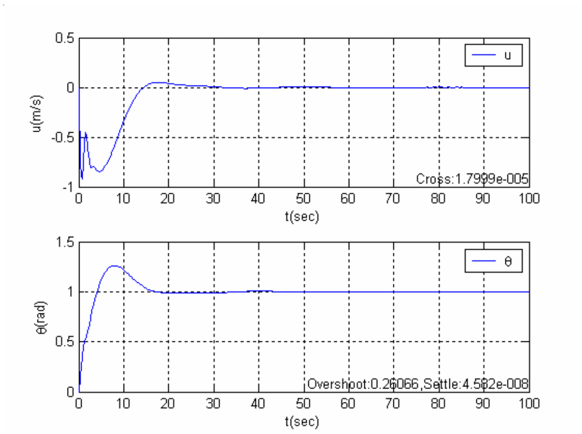
$$K_q = l\,J_q; \quad K_u = \bar{l}\,J_u \tag{17}$$

where l and $\bar{l}$ are constant coefficients. We manage to select proper coefficients to achieve our specifications. Comparing different simulation results with different coefficients, $l = \bar{l} = 1$ are found to be the proper coefficients for controller [Eq.(17)]. The simulation results of the integrated pitch/speed autopilot are presented in Figure 5.

It can be found from Figure 5(a) that the performance under the integrated control system is f_1 =0.027836, f_2 =2.3216 $\times 10^{-6}$, respectively. It means that overshoot of the integrated system becomes larger than individual loop but it still at the satisfactory level. At the same time, the settling time value becomes smaller than before probably due to the influence of the other channel. The cross effect is very small, $f_6 = 2.1465\times10^{-10}$. Similar conclusions may be drawn from Figure 5(b), in which the following performance is obtained: f_3 = 0.26066, $f_4 = 4.582\times10^{-8}$, the cross effect $f_5 = 1.799\times10^{-5}$.



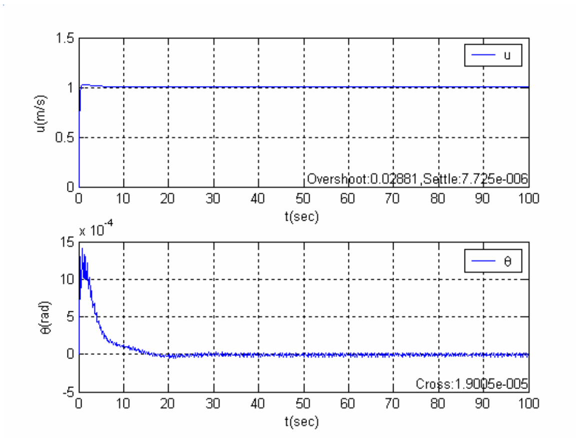
(a) u and q of u_c



(b) u and q of q_c

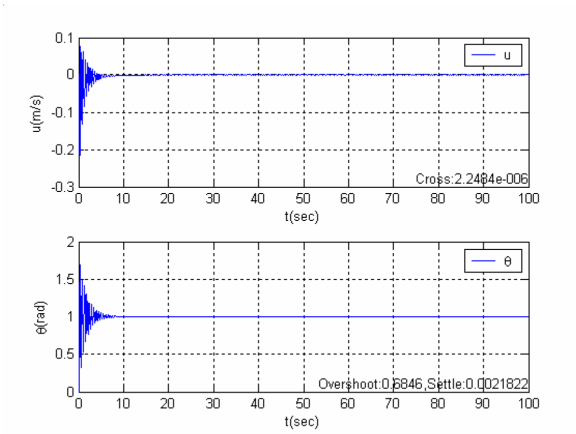
Fig.5. Longitudinal integrated control of F-16 linear model

It is obvious that the MSS controller can be applied to the linear system of the F-16 fighter. In order to evaluate the effectiveness of the MSS controller in a practical aircraft longitudinal control channels, we perform the nonlinear simulation of F-16 fighter flying at the altitude of 5000 metres with Mach number value 0.6. The results are shown in Figure 6.



(a) u and q of u_c

(f_1 = 0.02881, $f_2 = 7.725\times10^{-6}$; f_6 = 1.9005×10^{-5})



(b) u and q of q_c

(f_3 = 0.6846, $f_4 = 2.1882\times10^{-3}$, $f_5 = 2.2484\times10^{-6}$)

Fig.6. Longitudinal integrated control of 6DoF F-16 nonlinear model

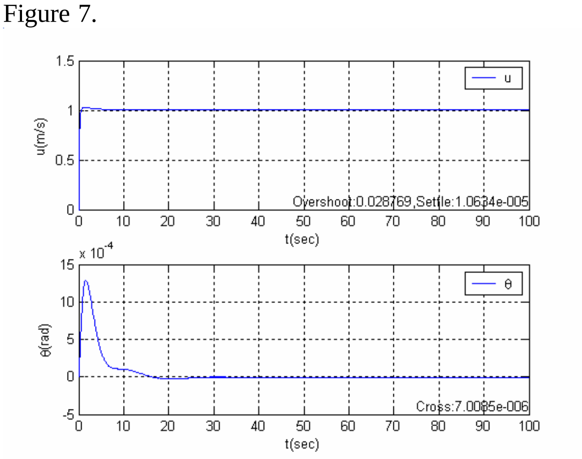
The simulation results of the nonlinear F-16 fighter show that the MSS controller is not satisfactory since some performance specifications cannot be satisfied. So we need to fine-tune the MSS controller to achieve an acceptable control effect. Note that the derivative control will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response (Goodwin, *et al.*, 2001). Now we add a derivative control to the integrated controller K_q [Eq.(17)] to become

$$K_q = l J_q + t s \tag{18}$$

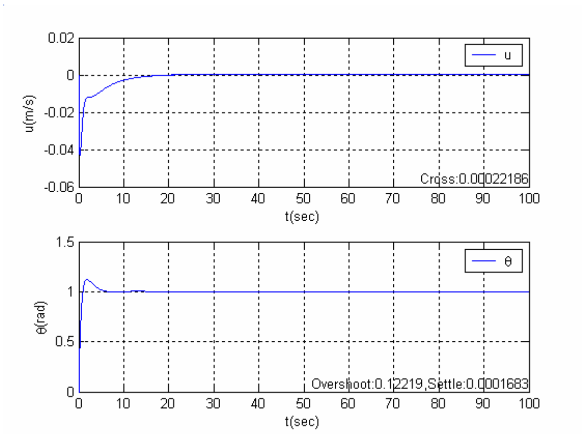
where t is the derivative control gain. Using the trial-and-error technique, we select the gain $t = -5$. Then the final controller [Eq.(19)] is given as:

$$\begin{aligned} K_u &= \frac{6.459(s+7.941)(s+0.5185)(s+0.2023)}{s(s+8.713)(s+0.473)} \\ K_q &= \frac{-5(s+1.568)(s+0.7765)(s^2+0.3207s+0.1229)}{s(s^2+0.2449s+0.1224)} \end{aligned} \tag{19}$$

The simulation results of the nonlinear F-16 fighter with the final integrated controller are shown in Figure 7.



(a) u and q of u_c



(b) u and q of q_c

Fig.7. Longitudinal integrated control of 6DoF F-16 nonlinear model with the final controller

It can be seen that the resulting fine-tune control system satisfies the design specifications simultaneously in spite of the differences between the results of the linear and the nonlinear F-16 fighter models.

Remark: Since there are differences between the linear model of the system and its original nonlinear model, the initial linear controller applied to the nonlinear system may not satisfy the performance specifications that were found achievable in the linear model. So we need to adjust the linear controller by adding a derivative control when applied to the nonlinear model.

In fact, in this paper, we can also carry out the fine-tuning at the MSS controller design stage of the individual loop. When we have derived a MSS controller, we apply it to the nonlinear model of the individual loop. If the MSS controller is found to be unsatisfactory, the MSS controller should be adjusted (or fine-tuned) until an acceptable one is obtained. Using this approach, we can obtain the similar controller and simulation results as reported earlier in the paper. The simulation results demonstrate that the MSS design method (Liu and Mills, 2000) with controller fine-tuning can be applied to the nonlinear F-16 fighter longitudinal control system.

5. CONCLUSIONS

In this paper, we apply the MSS controller design method (Liu and Mills, 2000) to the nonlinear F-16 fighter simulation. Fine-tuning is performed by adding a derivative control action to the MSS loop controller. The simulation results verify the effectiveness of the above design method in nonlinear F-16 longitudinal control system. In the future work, we will continue to study the robustness of MSS controller design method and its application in the more complicated aircraft autopilots.

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