HYBRID CONTROL OF SYSTEMS WITH INPUT DELAY

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Abstract: In this paper the concept of multiple models and concept of switching controllers is used. The analog part of the system is described by finite set of continuous-time models with input delays. The continuous-time models include unmodeled dynamics in the form of affine matrix family. Using suitable transformation the models with input delay is converted in the delay free models. Then for multiple models, for fixed subsystem, using LMI tool, the robust LQ controller with prescribed degree of stability is derived.. The switching rule, between robust LQ controllers, is based on the selection of the best performance of the closed loop subsystems. Finally is proved that original hybrid system is stable.

Key words: Multiple models, Input delay, LMI tool, switching rule, stability

1. INTRODUCTION

Hybrid systems are digital real-time systems which are embeded in analog envirouments. Analog part of the hybrid system is described with differential equations (Goodwin et al., 2001) and discrete part of the hybrid systems is a event driven dynamics which can be described using concept from discrete event systems. (Cassandras and Lafortune, 1999).

From the clasical control theory point of view hybrid systems can be considered as a switching control between analog feedback loops (Liberzon, 2003; Savkin and Evans, 2002).

Recently, the new approach for adaptive control is introduced (Narendra and Xiang , 2000). In the case of large parameters errors the clasic adaptive control results in a slow convergence with large transient errors. To overcome such problems the concept of multiple models is proposed. Similar approach is taken, also, in (Wang et al., 1999) where mapping of hybrid state to hybrid control is based on system performance.

In this paper we will adapt the multi model approach for control of systems with input delay. Time delay is a difficult element in process control systems (Shinskey, 1988). Mathematical formalism for description of time-delay systems belongs to the class of functional differential equations which are infinite dimensional. The excellent overview of some recent results for control of time-delay system is given in (Richard, 2003). Some new results for stability of time delay systems are presentid in (Gu et al., 2003). Design of feedback control law for time-delay systems, based on dynamic programming, is presented in (Yanushevsky, 1978).

In this paper we will use predictor-like techniques for systems with input delay. Such techniques is considered in (Kwon and Pearson, 1980; Arstein 1982). Using suitable transformation the original problem can be described in the form of delay free system.

For description of input delay system we will use multiple model concept. Parts of the input signal have different delays. In the model, also, unmodeled dynamics, in the form of affine matrix family, is present. Such kind of uncertainty in the context of limited data rate in control systems with networks (Filipovic, 2004) is considered. In this paper the robust hybrid LQ controller with prescribed degree of stability, for systems with input delay, using LMI tool, is proposed. It is formally proved that such hybrid closed-loop system is stable.

2. SYSTEM DESCRIPTION BY MULTIPLE MODELS

We will assume that the process model is a member of admissible process models

$$\mathcal{F} = \bigcup_{p \in \mathcal{P}} \mathcal{F}_p \tag{1}$$

Where $\mathcal{P}$ is matrix index set which represents the range of parametric uncertainty so that for each fixed $p \in \mathcal{P}$ the subfamily $\mathcal{F}_p$ accaunts for unmodeled dynamics. Usually $\mathcal{P}$ is compact subset of finite-dimensional normed vector space (Hespanaha et al., 2001).

The process with input delay will be described in the next form

$$\dot{x}(t) = A_p(q)x(t) + \sum_{i=0}^{r} B_{pi} u(t - h_i) \ , \ \ h_0 = 0 \tag{2}$$

$$A_p(q) = A_{p0} + \sum_{i=1}^{l} q_{pi} A_{pi} \quad , \ \ q_p \in Q_1 \subset R^l$$

$$B_{pi} = \left[0 \cdots 0 \ \ B_{pi1}^{(i)} \ \ 0 \cdots 0 \right]$$

$$u^T(t) = \left[u^{0^T}(t) u^{(1)^T}(t) \cdots u^{(r)^T}(t) \right]$$

$$p = 1,2,..,s$$

where $x \in R^n$, $u \in R^m$ and $A_p(q)$, and B_{pi} are nxn and nxm matrices respectively. The $h_i(i = 1,2,..,r)$ is the input delay. The dimensions of vectors $u^{(i)}(t)(i = 1,2,..,r)$ correspond to dimensions of matrices $B_{pi1}^{(i)}(i = 1,2,..,r)$.

Relation (2) describes the continuous part of the system. The event driven part can be described in the next form

$$p^+(t) = \varphi_1(p(t), \sigma(t)) \tag{3}$$

where $p(t)$ is a discrete event variable, $\sigma(t)$ is a discrete input and $\varphi_1(\cdot,\cdot)$ is a function which describes behaviour of $p(t)$. It is important to note that

$$p^+(t) = p(t_{n+1}) \ , \ \mathrm{p}(t) = p(t_n) \ , \ t_n < t_{n+1} \tag{4}$$

Specific form of switching sequence will be described in the next part of the paper.

3. THE SWITCHING CONTROLLERS

For complex processes the regulation problem can be solve by family of controllers (Hespanaha et al., 2001)

$$\{ C_q : q \in \mathcal{D} \} \tag{5}$$

where $\mathcal{D}$ is index set. It is supposed that the family is sufficiently rich so that every admissible proces model can be stabilized by controller C_q for some index $q \in \mathcal{D}$. In this paper will be considered the case (1(

$$\mathcal{F} = \mathcal{D} \tag{6}$$

Now will formulate transformation, for system (2), which put the system (2) in the finite dimensional form. It means that transformed system will be described by ordinary differential equation. That will be described in the next lemma.

Lemma 1. System (2), by next transformation

$$y(t) = x(t) + \int_{t-h_0}^{t} e^{-A_p(q)(s-t)} e^{-A_p(q)h_0} B_{p0} u(s) ds + \int_{t-h_1}^{t} e^{-A_p(q)(s-t)} e^{-A_p(q)h_1} B_{p1} u(s) ds + \cdots + \int_{t-h_r}^{t} e^{-A_p(q)(s-t)} e^{-A_p(q)h_r} B_{pr} u(s) ds$$

obtains the form

$$\dot{y}(t) = A_p(q) y(t) + \sum_{i=0}^{r} e^{-A_p(q)h_i} B_{pi} u(t)$$

$$p = 1,2,..,s \ , \ h_0 = 0$$ ▪

Proof: Owing the space is omited.

Remark 1: For $r = 1$ and $h_0 = 0$ transformation is proposed in (Kwon and Pearson, 1980). This approach is known as a predictor-like techniques.

We naw will introduce the optimal LQ controller with the prescribed degree of stability α for fixed p and

$$\sum_{i=1}^{l} q_{pi} A_{pi} = 0 \ \ , \ \ q_{pi} \in Q_1 \subset R^l \tag{9}$$

In that case relation (8) has the form

$$\dot{y}(t) = A_{p0}y(t) + \sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}u(t) \tag{10}$$

The index of performance is (Anderson and Moore, 1969)

$$J = \int_{t_0}^{\infty} e^{2\alpha t}\left(y^T(t)Qy(t) + u^T(t)Ru(t)\right)dt \quad , \quad \alpha > 0 \tag{11}$$

where Q and R are positive definite matrices. The optimal controller is

$$u(t) = -R^{-1}\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right)^T P_p y(t) \tag{12}$$

whereby matrix P_p is a solution of next algebraic Riccati eqution

$$P_p\left(A_{p0} + \alpha I\right) + \left(A_{p0}^T + \alpha I\right)P_p -$$
$$- P_p\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right)R^{-1}P_p + Q = 0 \tag{13}$$

It is well known fact that for LQ controller (11)-(13) the Lyapunov function has the form (Polyak and Scherbakov, 2002)

$$V(x) = y^T(t)P_p y(t) \tag{14}$$

In the presence of unmodeled dynamics the transformed closed-loop system has the next form

$$\dot{y}(t) = A_p(q)y(t) + \sum_{i=0}^{r} e^{-A_p(q)h_i} B_{pi}u(t) \tag{15}$$

$$u(t) = -R^{-1}\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right)^T P_p y(t) \tag{16}$$

$$p = 1,2,..,s \quad , \quad h_0 = 0$$

The next goal is to finde matrix P_p without the help of Riccati equation (13). Instead, we will use the LMI tool. Such approach can reduces a very wide variety of problems in control theory to a few standard convex optimization problems. The result will be formulated in the form of theorem.

Now we will, for fixed p, formulate theorem.

Theorem 1. Let us suppose that the closed-loop system (15, (16) is satisfied

1° For fixed $p-th$ subsystem, couple

$$\left[A_{p0} + \alpha \ , \sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right]$$

is controlable

2° Matrices Q and R are positive definite

3° $Y_p(\gamma_p)$ is the solution of the next LMI

$$\begin{bmatrix} C_p & D_p \\ E_p & F_p \end{bmatrix} \le 0$$
$$C_p = \left(A_p(q) + \alpha I\right)Y_p + Y_p\left(A_p(q) + \alpha I\right)^T +$$
$$+ \gamma_p^{-2}\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right)^T R^{-1}\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right)$$
$$D_p = \gamma_p^{1/2}Y_pQ^{1/2} \quad , \quad E_p = \gamma_p^{1/2}Q^{1/2}Y_p \quad ,$$
$$F_p = -I \ , \ Y_p > 0 \ , \ q \in Q_1 \subset R^l$$

4° For $\forall \gamma_p > 0$

$$\gamma_p^* = \arg\min_{\gamma_p} \varphi(\gamma_p), \ \varphi(\gamma_p) = \gamma_p^{-1}x_0^T Y_p(\gamma_p)x_0$$

Then for feedback law

$$u(t) = -R^{-1}\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right)^T \left(Y_p(\gamma_p^*)\right)^{-1} y(t)$$

upper bound for index of performance is

$$J_p \le \varphi(\gamma_p^*)$$

for $\forall q \in Q_1 \subset R^l$ and fixed p ▪

Proof: Let us introduce next transformation

$$\dot{\hat{y}}(t) = e^{\alpha t}x(t) \quad , \quad \hat{u}(t) = e^{\alpha t}u(t) \tag{17}$$

Now from relations (15) and (16) we have

$$\dot{\hat{y}}(t) = \tilde{A}_p(q)\hat{y}(t) \ , \ \tilde{A}_p(q) = A_p(q) + \alpha I -$$
$$-\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right)R^{-1}\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right)^T P_p \tag{18}$$

Using relation (14) one can conclude that $\dot{V} < 0$ will be satisfied if

$$\tilde{A}_p^T(q)P_p + P_p\tilde{A}_p(q) < 0 \tag{19}$$

From (18) and (19) we have

$$\left(A_p(q) + \alpha I\right)^T P_p + P_p\left(A_p(q) + \alpha I\right) -$$
$$- 2P_p\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right)R^{-1}\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right)^T P_p < 0 \tag{20}$$

Let us multiplay last inequality from left and right side with the matrix $Y_p = P_p^{-1}$, for $Y_p > 0$, and introduce parameter $\gamma_p > 0$. We have

$$\left(A_p(q)+\alpha I\right)^T Y_p + Y_p\left(A_p(q)+\alpha I\right) - 2\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right) R^{-1}\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right)^T + \gamma_p \cdot \left[\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right) R^{-1}\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right)^T + Y_p Q Y_p\right] \le 0$$

$$Y_p > 0 \tag{21}$$

Now we will multiplay last inequality from left and right side with the matrix Y_p^{-1}. Follows

$$\tilde{A}_p^T(q)\left(\frac{1}{\gamma_p}Y_p^{-1}\right) + \left(\frac{1}{\gamma_p}Y_p^{-1}\right)\tilde{A}_p(q) \le -Q + Y_p^{-1}\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right) R^{-1}\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right)^T Y_p^{-1} \tag{22}$$

Using result from (Andreev, 1976, p.141) one can get

$$\tilde{A}_p^T(q) Z_p + Z_p \tilde{A}_p(q) = -\left[Q + Y_p^{-1}\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right) R^{-1}\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right)^T Y_p^{-1}\right]$$

$$J_p = y_0^T Z_p y_0 \tag{23}$$

If we substract relation (23) from relation (22) we have

$$\tilde{A}_p^T(q)\left(\frac{1}{\gamma_p}Y_p^{-1} - Z_p\right) + \left(\frac{1}{\gamma_p}Y_p^{-1} - Z_p\right)\tilde{A}_p^T(q) \le 0 \tag{24}$$

Using result from (Dullerud and Paganini, 2000, p.140) from last relation follows

$$\frac{Y_p^{-1}}{\gamma_p} - Z_p \ge 0 \quad \text{and} \quad \frac{Y_p^{-1}}{\gamma_p} \ge Z_p \tag{25}$$

From (23) and (25) one can get

$$J_p \le \frac{1}{\gamma_p} y_0^T Y_p^{-1} y_0 \tag{26}$$

The inequality (21) can be rewritten in the next form

$$\left(A_p(q)+\alpha I\right)^T Y_p + Y_p\left(A_p(q)+\alpha I\right) + \left(\gamma_p - 2\right) \cdot \left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right) R^{-1}\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right)^T + \left(\gamma_p^{1/2} Y_p Q^{1/2}\right)\left(\gamma_p^{1/2} Q^{1/2} Y_p\right) \le 0 \quad , \quad Y_p > 0 \tag{27}$$

Using result from (Dullerud and Paganini, 2000, p.46) we can, from (27), get LMI formulated in theorem. Using LMI from theorem one can find solution $Y_p(\gamma_p)$. After that we can construct function

$$\varphi(\gamma_p) = \gamma_p^{-1} y_0^T Y_p(\gamma) y_0 \tag{28}$$

From last relation we can find

$$\gamma_p^* = \arg\min_{\gamma_p} \varphi(\gamma_p) \tag{29}$$

Now feedback law has the form

$$u(t) = -R^{-1}\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right)^T \left(Y_p(\gamma_p^*)\right)^{-1} y(t) \tag{30}$$

and upper bound for index of performance is

$$J_p \le \varphi(\gamma_p^*) \tag{31}$$

Theorem is proved ▪

4. ROBUST STABILITY OF HYBRID CONTROL SYSTEMS

Using relation (15) and fact that $h_0 = 0$, system can be described in the form

$$\dot{y}(t) = A_{p0} y(t) + B_{p0} u(t) + \Delta(y(t), p(t), u(t), q)$$

$$\Delta(y(t), p(t), u(t), q) = \sum_{i=1}^{l} q_{pi} A_p y(t) + \sum_{i=1}^{r} e^{-A_p(q)h_i} B_{pi} u(t) \tag{33}$$

where $q_p \in Q_1 \subset R^l$ and $p = 1,2,..,s$.

A) The analog feedback

$$u(t) = -R^{-1}\left(\sum_{i=0}^{r} e^{-A_{p0}h_i} B_{pi}\right)^T \left(Y_p(\gamma_p^*)\right)^{-1} y(t) \tag{34}$$

$p = 1,2,..,s$

whereby $Y_p(\gamma_p^*)$ is the solution of LMI which is defined in Theorem 1.

B) The discrete feedback is

$$p_1 = \min\left(\varphi\left(\gamma_p^*\right)\right) \ , \quad p = 1,2,..,s$$
$$\gamma_p^* = \arg\min_{\gamma_p} \varphi\left(\gamma_p\right)$$
$$\varphi\left(\gamma_p\right) = \gamma_p^{-1} y^T Y_p\left(\gamma_p\right) y^T(t) \tag{35}$$

Now we will formulate theorem in which will be proved robust stability of the original system (2).

Theorem 2. Let us suppose that for dynamic hybrid system (32)-(35) is valid

1°

$$\left\| A_{p0} + \alpha - B_{p0} R^{-1} \left(\sum_{i=0}^{r} e^{-A_{p0} h_i} B_{pi} \right)^T \left(Y_p\left(\gamma_p^*\right)\right)^{-1} \hat{y}(t) \right\| \leq$$
$$\leq k_{p1} \varphi\left(\gamma_p^*\right) + c$$
$$\left\| A_{p0} + \alpha - B_{p0} R^{-1} \left(\sum_{i=0}^{r} e^{-A_{p0} h_i} B_{pi} \right)^T \left(Y_p\left(\gamma_p^*\right)\right)^{-1} \hat{y}(t) \right\| \geq 1$$

$$k_{p1} > 0 \ \ , \ c > 0 \ \ , \ p = 1,2,..,s$$

2° $\left\|\hat{y}(t)\right\| \leq k_{2p} \varphi\left(\gamma_p^*\right)$

$k_{2p} > 0 \ , \ \left\|\hat{y}(t)\right\| \geq 1 \ , \ p = 1,2,..,s$

3° $r_\Delta \in [0, \infty)$

where

$$r_\Delta = \sup_{q \in Q_1} \left\| \Delta\hat{y}(t), p(t), \hat{u}(t), q \right\|$$

Then the completely controllable system (2) is stabilized by the control law

$$u(t) = -K\left(x(t) + \sum_{i=1}^{r} \int_{t-h_i}^{t} e^{-A_p(q)(s-t)} e^{-A_p(q)h_i} B_{pi} u(s) ds \right)$$

where

$$K = R^{-1} \left(\sum_{i=0}^{r} e^{-A_{p0} h_i} B_{pi} \right)^T \left(Y_p\left(\gamma_p^*\right)\right)^{-1}$$

is a feedback gain matrix which stabilizes the completely contrllable ordinary system

$$\left[A_{p0} + \alpha \ \ , \ \sum_{i=0}^{r} e^{-A_{p0} h_i} B_{pi} \right] \quad ▪$$

Proof: From relation (32) for any $\tau \in [t, t+1]$ we have (36)

$$\hat{y}(t) = \hat{y}(\tau) -$$
$$-\int_t^\tau \left[A_{p0} + \alpha - B_{p0} R^{-1} \left(\sum_{i=0}^{r} e^{-A_{p0} h_i} B_{pi} \right)^T \left(Y_p\left(\gamma_p^*\right)\right)^{-1} \right] \cdot$$
$$\cdot \hat{y}(\theta) d\theta + \int_t^\tau \Delta\left(\hat{y}(\theta), p(\theta), u(\theta), q\right) d\theta$$

From last relation we have in (37)

$$\left\|\hat{y}(t)\right\| \leq \left\|\hat{y}(\tau)\right\| +$$
$$-\int_t^\tau \left\| \left[A_{p0} + \alpha I - B_{p0} R^{-1} \left(\sum_{i=0}^{r} e^{-A_{p0} h_i} B_{pi} \right)^T \left(Y_p\left(\gamma_p^*\right)\right)^{-1} \right] \cdot \right.$$
$$\left. \cdot \hat{y}(\theta) \right\| d\theta + \int_t^\tau \left\| \Delta\left(\hat{y}(\theta), p(\theta), u(\theta), q\right) \right\| d\theta$$

Let us introduce next sets

$$\Omega_1 = \left\{ \tau \in [t, t+1] : \right.$$
$$\left\| \left[\left[A_{p0} + \alpha I - B_{p0} R^{-1} \left(\sum_{i=0}^{r} e^{-A_{p0} h_i} B_{pi} \right)^T \right] \right. \right. \tag{38}$$
$$\left. \left. \cdot \left(Y_p\left(\gamma_p^*\right)\right)^{-1} \right] \hat{y}(t) \right\| \leq 1 \right\}$$

$$\Omega_2 = \tau \in [t, t+1] - \Omega_1 \tag{39}$$

Now from (37) and condition 1° of theorem follows

$$\left\|\hat{y}(t)\right\| \leq \left\|\hat{y}(\tau)\right\| + 1 + c + k_{p1} \varphi\left(\gamma_p^*\right) + r_\Delta \tag{40}$$

Using last inequality and condition 2° of theorem we have

$$\left\|\hat{y}(t)\right\| \leq \int_t^{t+1} \left(\left\|\hat{y}(\theta)\right\| + 1 + c + k_{p1} \varphi\left(\gamma_p^*\right) + r_\Delta \right) d\theta \leq$$
$$\leq \left(k_{p2} + k_{p1}\right) \varphi\left(\gamma_p^*\right) + 2 + c + r_\Delta \tag{41}$$

Since the right-hand side is independent of t we have

$$\left\|\hat{y}(t)\right\|_\infty \leq \left(k_{p1} k_{p2}\right) \varphi\left(\gamma_p^*\right) + 2 + c + r_\Delta \tag{42}$$

From relation (17) follows

$$\left\|y(t)\right\|_\infty \leq \left\|\hat{y}(t)\right\|_\infty < \infty \tag{43}$$
$$p = 1,2,..,s$$

Transformation relation has the compact form

$$y(t) = x(t) + \sum_{i=1}^{r} \int_{t-h_i}^{t} e^{-A_p(q)(s-t)} e^{-A_p(q)h_i} B_{pi} u(s) ds \quad (44)$$

From last relation we have

$$\|x(t)\| \le \|y(t)\| + \sum_{i=1}^{r} h_i \max_{-h_i \le \theta \le 0} \left\| e^{A_p(q)\theta} \right\| \cdot \cdot \|B_{pi}\| \|K\| \|y(t)\| \quad (45)$$

Using relation (52) and (54) one can conclude

$$\|x(t)\|_{\infty} < \infty \quad p = 1,2,..,s \quad (46)$$

Theorem is proved ▪

Remark 2: In this remark we will comment assumptions 1° and 2° of Theorem2. It is well known fact that optimally designed controllers via Riccati equations always guarantee stability. Such fact suggests that, if system performance indices are approprietelly selected, optimality of performance or boundedness of performance, will provide stability and robusness. Such idea is used in (Wang et al., 1999).In this paper we prove that index of performanse for every closed-loop subsystem has a finite upper bound. Using that fact we generalize concept of performance dominant condition from (Wang et al., 1999). Generalization has the form as in first two conditions in Theorem2.

6. CONCLUSION

In this paper the problem of design of hybrid LQ contrroller for systems with input time-delay is considered. The problem is solved using suitable transformation which convert original time-delay systems in the form of delay free systems. All ressults of the theorems are, however, for original systems. In the paper the concept of multiple models and switching controllers are used. Switching strategy is determined using index of performance. For hybrid systems the asymptotic sability in the sense of $\infty -$ norm is proved.

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