STRONG STABILIZATION FOR A CLASS OF NONLINEAR CONTROL SYSTEMS WITH STATE DELAYS

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Abstract: This paper deals with the strong stabilization problem of a class of nonlinear time-varying control systems with state delays. Under appropriate growth conditions on the nonlinear perturbation, new sufficient conditions for the strong stabilizability are established based on the global null-controllability of the nominal linear system. These conditions are presented in terms of the solution of a standard Riccati differential equation. A constructive procedure for finding feedback stabilizing controls is also given.

Keywords: Robust control, state estimation, abstract systems, integral quadratic constraint.

1 INTRODUCTION

Consider a nonlinear time-varying control system with state delays of the form

$$\begin{aligned} \dot{x}(t) &= f(t,x(t),x(t-h),u(t)), \quad t \geq 0, \\ x(t) &= \phi(t),\ t \in [-h,0], \end{aligned} \tag{1}$$

where $h \geq 0, x(t) \in X-$ the state, $u(t) \in U-$ the control, $f(t,x,y,u) : [0,\infty) \times X \times X \times U \to X, \phi(t) : [-h,0] \to X-$ is a given function.

The topic of Lyapunov stability of control systems described by a system of differential equations is an interesting research area in the past decades. An integral part of the stability analysis of differential equations is the existence of inherent time delays. Time delays are frequently encountered in many physical and chemical processes as well as in the models of hereditary systems, Lotka-Volterra systems, control of the growth of global economy, control of epidemics, etc. Therefore, stability problems of time-delay control systems have been the subject of numerous investigations, see; e.g. Ahmed 1990, Chukwu 1992, Niumsup et al. 2000, Phat 2002, Sun et al. 1996).

The standard stability problem is to find a control function $u(t) = h(x(t))$ in order to keep the zero solution of the closed-loop system

$$\dot{x}(t) = f(t,x(t),x(t-h),h(x(t)))$$

exponentially stable in the Lyapunov sense, i.e., the solution $x(t,\phi)$ of the closed-loop system satisfies the condition

$$\exists N > 0, \delta > 0 : \quad \|x(t,\phi)\| \leq Ne^{-\delta t}\|\phi\|, \quad \forall t \geq 0,$$

where $\|\phi\| = \sup_{s\in[-h,0]} \|\phi(s)\|$. In this case one says that the system is stabilizable by the feedback control $u(t) = h(x(t))$ and this control is called a stabilizing feedback control of the system. The positive number $\delta > 0$ depending on the stabilizing control is commonly called a Lyapunov stability exponent. In the literature on control theory of dynamical systems the stabilizability is one of the important qualitative properties and the investigation of the stabilizability has attracted the attention of many researchers, see; e.g. Curtain 1995, Phat 1996, 1996, 2002, Son 1999, Zabczyk 1992. In practice various stabilizability concepts have been defined to improve the efficiency of the stability of control systems. One of the extended stability properties of control systems is the concept of the strong (or complete) stabilizability, originally introduced by Wonham 1967, which plays an important role in many mechanical and control engineering problems (see, Ahmed 1990, Zabczyk 1992). This property relates to a strong exponential stability of the control system, namely, control system (1) is strongly stabilizable if for every given number $\delta > 0$, there exists a feedback control function $u(t) = h(x(t))$ such that the solution $x(t,\phi)$ of the closed-loop system satisfies the condition

$$\exists N > 0: \quad \|x(t,\phi)\| \leq N e^{-\delta t}\|\phi\|, \quad \forall t \geq 0. \tag{2}$$

This means that for any given positive number $\delta > 0$, the system zero-input response of the closed-loop system decays faster than $e^{-\delta t}$. In other words, for any given in advance Lyapunov stability exponent $\delta > 0$, the system can be $\delta-$exponentially stabilizable. Such definition may arise because of controlling of the speed of the real models in many mechanical and physical control systems (see Benssousan et al. 1992, Chukwu 1992). First results on the strong stabilizability of linear time-invariant control systems in finite-dimensional spaces can be found in Wonham 1967, where by studying the spectrum of the system matrices or by solving a modified algebraic Riccati equation it was proved that the global-null controllability (see Kalman 1960) implies the strong stabilizability. Further extensions on the relationship between the strong stabilizability and controllability of infinite-dimensional time-invariant control systems are given in Megan 1975, Phat et al. 2000, Slemrod 1974. However, the strong stabilizability and control design problems for time-varying control systems have not been examined fully in the literature, which are more complicated and given results are lacking. The difficulties increase to the same extent as passing from undelayed to delayed time-varying control systems as well as from linear to nonlinear time-varying delay systems. The aim of this paper is to study the strong stabilizability problem for the following time-varying control delay system

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + A_1(t)x(t-h) + B(t)u(t) \\ &\quad + f(t,x(t),x(t-h),u(t)), \quad t \geq 0, \\ x(t) &= \phi(t),\ t \in [-h,0], \end{aligned} \tag{3}$$

where $A(t), A_1(t) : X \to X, B(t) : U \to X-$ are linear matrix/operator functions and the given nonlinear perturbation term $f(t,x,y,u) : [0,\infty) \times X \times X \times U \to X$ could result from errors in modelling the general linear system (1), adding parameters, or uncertainties and disturbances which exist in any realistic systems. A common approach is to treat the stability of the nominal linear control system. Then, when the nonlinearities satisfy some appropriate growth conditions, one can use the Lyapunov direct method to design a stabilizing feedback control. Based on the global null-controllability assumption of the nominal linear time-varying control system, sufficient conditions for the strong stabilizability are established by solving a standard Riccati differential equation. These conditions depending on the size of the delay do not involve any spectrum of the evolution operator/matrix, and hence are easy to verify and construct. For a systematic exposition of the results, we start with the case of finite-dimensional control systems. Then, the results are directed to infinite-dimensional control systems by extending the relationship between the global null-controllability and the existence of the solution of a Riccati operator equation. A constructive algorithm to find feedback stabilizing controls via the controllability and the solution of curtain Riccati equations is also given. The stability conditions obtained in this paper are even new in the context of linear time-varying control systems, and they can be considered as further extensions of Ikeda et al. 1972, Megan 1975, Phat Linh 2002, Slemrod 1974, Wonham 1967 to nonlinear and time-delayed systems.

2 FINITE-DIMENSIONA SYSTEMS

The following standard notation is adapted throughout this paper. R^+ denotes the set of all real non-negative numbers; R^n denotes n finite-dimensional Euclidean space, with the Euclidean norm $\|.\|$ and the scalar product of two vectors $x^T y$; ^{T} denotes the transpose of the vector/matrix; $R^{n\times m}$ denotes the set of all $(n \times m)$-matrices; A matrix A is symmetric if $A = A^T$; A matrix A is called non-negative definite ($A \geq 0$) if $x^T A x \geq 0$, for all $x \in R^n$;A is positive definite ($A > 0$) if $x^T A x > 0$ for all $x \neq 0$; $M(R^n_+)$ denotes the set of all symmetric non-negative definite matrix functions in $R^{n\times n}$ continuous in $t \in R^+$; X, U denote infinite-dimensional real Hilbert spaces with inner product $\langle .,. \rangle$; $L(X)$ (respectively, $L(U,X)$) denotes the Banach space of all linear bounded operators mapping X into X (respectively, U into X); $L_2([0,t],X)$ denotes the set of all L_2-integrable and X-valued functions on $[0,t]$; $C([0,t],X)$ denotes the set of all $X-$valued continuous function on $[0,t]$; $D(A)$ and A^* denotes the domain and the adjoint of the operator A, respectively; clM denotes the closure of a set M; I denotes the identity operator; An operator $Q \in L(X)$ is called non-negative definite ($Q \geq 0$) if $< Qx, x > \geq 0$, for all $x \in X$; $Q \in L(X)$ is called self-adjoint if $Q = Q^*$; $LO([0,+\infty), X^+)$ denotes the set of all linear bounded set-adjoint non-negative definite operator-valued functions in X continuous in $t \in [0,+\infty)$. Consider the control delay system (3) in finite-dimensional spaces: $X = R^n, U = R^m, n \geq m, A(t) \in R^{n\times n}$, $A_1(t) \in R^{n\times n}$, $B(t) \in R^{n\times m}, \phi(s) \in C([-h,0],R^n)$. Throughout this section we consider the class of admissible controls $u(t) \in L_2([0,T],R^m)$ for every $T > 0$. Furthermore, to guarantee the existence of the solution of the control system, the following conditions will be made throughout this section:

A.1. $A(.)x,\ A_1(.)y,\ B(.)u,\ f(.,x,y,u)$ are continuous function on R^+ for all $x \in R^n, y \in R^n, u \in R^m$.

A.2. There are non-negative continuous functions

$a(t), a_1(t), b(t) : R^+ \to R^+$ such that

$$\|f(t,x,y,u)\| \le a(t)\|x\| + a_1(t)\|y\| + b(t)\|u\|,$$

for all $(t,x,y,u) \in R^+ \times X \times X \times U$.

Definition 2.1. Let $\delta > 0$ be a positive number. Control system (3) is said to be $\delta-$ stabilizable if there is a feedback control $u = h(x)$ such that the solution of the closed-loop system satisfies the condition (2).

Definition 2.2. Control system (3) is said to be strongly stabilizable if it is $\delta-$stabilizable for every $\delta > 0$.

In order to study the strong stabilizability problem, it is important to introduce the global null-controllability definition given by Kalman 1960. Consider the nominal linear time-varying control system $[A(t), B(t)]$ of system (3):

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \quad t \in R^+. \tag{4}$$

Definition 2.3. Linear control system (4) is globally null-controllable (GNC) in finite time if for every state $x \in R^n$, there exist a finite time $T > 0$ and an admissible control $u(t) \in L_2([0,T], R^m)$ such that

$$U(T,0)x + \int_0^T U(T,s)B(s)u(s)ds = 0,$$

where $U(t,s)$ is the fundamental matrix of the linear system $\dot{x}(t) = A(t)x(t)$.

The following well-known controllability criteria will be used later.

Proposition 2.1. (Klamka 1991) *Linear time-varying control system (4) is GNC in finite time if and only if one of the following conditions holds:*

(i) $\exists T > 0:$ *The matrix* $\int_0^T U(T,s)B(s)B^T(s)U^T(T,s)ds$ *is positive definite*

(ii) $\exists t_0 > 0 :$ *rank* $[M_0(t_0), M_1(t_0), ..., M_{n-1}(t_0)] = n$, *where* $M_0(t) = B(t)$ *and*

$$M_{k+1}(t) = -A(t)M_k(t) + \frac{d}{dt}M_k(t),$$

for $k = 0,1,2,...,n-1$, *and* $A(t), B(t)$ *are assumed to be analytical functions on* $[0,\infty)$.

In the sequel, the solution to the stabilizability problem involves a Riccati differential equation (RDE) of the form

$$\dot{P} + A^T P + PA - PBB^T P + Q = 0, \quad P(0) = P_0 \tag{5}$$

where $P(t)$ is an unknown matrix function. Before proceeding to the main result, a sufficient condition for the existence of non-negative positive solution of the RDE (5) is provided in the following proposition.

Proposition 2.2. (Kalman 1960) *Assume that linear control system* $[A(t), B(t)]$ *is GNC, then for every no-negative positive definite bounded function* $Q(t) \ge 0$ *and for every initial matrix* $P_0 \ge 0$, *the RDE (5) has a solution* $P(t) \in M(R^n_+)$, *which is a bounded function on* $[0,\infty)$.

For every $\delta > 0$, we denote $\tilde{A}(t) = A(t) + \delta I$, and consider the following RDE

$$\dot{P} + \tilde{A}^T P + P\tilde{A} - PBB^T P + I = 0. \tag{6}$$

Let us set $b = \sup\{t \in R^+ b(t), \quad B = \sup_{t\in R^+} \|B(t)\|$, and

$$p = \sup_{t\in R^+} \|P(t)\|, \quad a_1 = \sup_{t\in R^+} a_1(t), \quad A_1 = \sup_{t\in R^+} \|A_1(t)\|.$$

The following theorem gives sufficient conditions for $\delta-$stabilizability of the nonlinear control delay system (3).

Theorem 2.1. *Assume that the conditions A.1, A.2 hold and linear control system* $[A(t), B(t)]$ *is GNC in finite time. Nonlinear control delay system (3) is* $\delta-$ *stabilizable if*

$$0 < b < \frac{1}{2Bp^2}, \tag{7}$$

$$a_1 + A_1 < \frac{\sqrt{1 - 2p^2 bB}}{2pe^{\delta h}}, \tag{8}$$

$$\sup_{t\in R^+} a(t) < \frac{1}{4p} - \frac{1}{2}pbB - p(a_1 + A_1)^2, \tag{9}$$

and the stabilizing feedback control is given by

$$u(t) = -\frac{1}{2}B^T(t)P(t)x(t), \tag{10}$$

where $P(t) \in M(R^n_+)$ *is the solution of the RDE (6) with any initial condition* $P_0 \ge 0$.

Note that if $A_1(t) = 0, f(t,x,y,u) = 0$, i.e., $a(t) = a_1(t) = b(t) = 0$, the conditions (7)- (9) automatically hold and then Theorem 2.1 can be applied to the linear control system $[A(t), B(t)]$ in finite-dimensional spaces as follows.

Corollary 2.1. *The finite-dimensional linear control system* $[A(t), B(t)]$ *is strongly stabilizable if it is GNC in finite time.*

Remark 2.1. Corollary 2.1 extends a result of Wonham 1967 to time-varying case and it improves a result of Ikeda et al. 1972, where the controllability assumption was assumed to be more strict: the uniform global controllability.

From the proof of Theorem 2.1, the following procedure of finding stabilizing feedback control can be applied:

Step 1. Verify the GNC of linear control system $[A(t), B(t)]$ by Proposition 2.1.

Step 2. For given $\delta > 0$, find the solution $P(t) \in M(R^n_+)$ of RDE (6).

Step 3. Compute the numbers p, b, B, A_1, a_1 and check the conditions (7)-(9).

Step 4. The stabilizing feedback control $u(t)$ is given by (10).

Example 2.1. Consider the nonlinear control delay system (3) in R^2, where $h = \frac{1}{8}, \delta = 2$ and

$$A(t) = \begin{pmatrix} \frac{1}{20}e^{-4t}\sin^2 t - 5e^{4t} & 0 \\ 0 & \frac{1}{20}e^{-4t}\cos^2 t - 5e^{4t} \end{pmatrix},$$

$$A_1(t) = \begin{pmatrix} e^{-\frac{1}{2}t}\sin t & 0 \\ 0 & e^{-\frac{1}{2}t}\cos t \end{pmatrix}, B(t) = \begin{pmatrix} \sin t & 0 \\ 0 & \cos t \end{pmatrix},$$

$$f(t, x, x(t-h), u) = x\sin^2 t + e^{-\frac{1}{2}t}x(t-h) + e^{-\frac{9}{2}t}u.$$

We have $a(t) = \sin^2 t, \quad a_1(t) = e^{-\frac{1}{2}t}, \quad b(t) = e^{-\frac{9}{2}t}$. We can easily verify the GNC of the linear control system $[A(t), B(t)]$ by Proposition 3.1 (ii), rank $[M_0(t_2), M_1(t_2)] = 2$, with $t_0 = \frac{\pi}{2}$. On the other hand, for $\delta = 2$, and for the defined matrices $\tilde{A}(t), B(t)$,upon some computations we can find that the solution $P(t)$ of the RDE (6) is given by

$$P(t) = \begin{pmatrix} \frac{1}{10}e^{-4t} & 0 \\ 0 & \frac{1}{10}e^{-4t} \end{pmatrix}.$$

Thus, computing the numbers b, B, p, a_1, A_1, we verify the conditions (7)-(9). The system is then $2-$stabilizable with the feedback control

$$u(t) = -\frac{1}{2}\begin{pmatrix} \frac{1}{10}e^{-4t}\sin t & 0 \\ 0 & \frac{1}{10}e^{-4t}\cos t \end{pmatrix}.$$

3 INFINITE-DIMENSIONALSYSTEMS

We now consider the system (3) in infinite-dimensional spaces: $x \in X, u \in U; X, U-$ are real Hilbert spaces, for every $t \in R^+, A(t): X \to X, t \in R^+$ is a linear operator, $A_1(t) \in L(X), B(t) \in L(U, X), f(t, x, y, u): R^+ \times X \times X \times U \to X$. Throughout this section we consider the class of admissible controls $u(t) \in L_2([0, T], U)$ for every $T > 0$. As in [2, 7], for guarantying the existence of the solution of infinite-dimensional control system (3), throughout this section we assume that

B.1. The operator functions $A(.)x, A_1(.)x \in L(X), B(.)u \in L(U, X), f(., x, y, u), t \in R^+$ are continuous on $[0, \infty)$ for every $x \in X, y \in X, u \in U$.

B.2. The linear operator function $A(t): X \to X$, $\mathrm{cl}(D(A(t))) = X$, generates an evolution semigroup operator $U(t, s)$ (see Pazy 1983).

B.3. The nonlinear function $f(t, x, y, u)$ satisfies the condition: there exist non-negative continuous functions $a(t), a_1(t), b(t): R^+ \to R^+$ such that

$$\|f(t, x, y, u)\| \leq a(t)\|x\| + a_1(t)\|y\| + b(t)\|u\|,$$

for all $(t, x, y, u) \in R^+ \times X \times X \times U$. In this case, the mild solution of the nonlinear system (1) in Hilbert space is given by

$$x(t, \phi) = U(t, 0)\phi(t) + \int_0^t U(t, \tau)\big[A_1 x(\tau - h) + B(\tau)u(\tau) + f(\tau, x(\tau), x(\tau - h), u(\tau))\big]d\tau.$$

Before proceeding further, we state the following well-known infinite-dimensional controllability criterion, which will be used later.

Proposition 3.1. (Conti 1982) *Infinite-dimensional linear control system $[A(t), B(t)]$ is GNC iff exist $T > 0, c > 0$ such that*

$$\int_0^T \|B^*(s)U^*(T, s)x^*\|^2 ds \geq c\|U^*(T, 0)x^*\|^2,$$

for all $x^* \in X^*$.

Associated with the infinite-dimensional linear control system $[A(t), B(t)]$, we consider a Riccati operator equation (ROE) described formally by the form

$$\dot{P} + A^*P + PA - PBB^*P + Q = 0. \quad (11)$$

Since $A(t), t \in R^+$ is an unbounded operator, the solution of ROE will be defined as follows.

Definition 3.1. The solution of ROE (13) is a linear operator function $P(t) \in L(X)$ satisfying the following two conditions: (i) The scalar function $\langle P(\cdot)x, y\rangle$ is continuously differentiabe on $[0, \infty)$ for every $x, y \in D(A(.))$. (ii) For all $x, y \in D(A(t)), t \in R^+$:

$$\frac{d}{dt}\langle Px, y\rangle + \langle Px, Ay\rangle + \langle PAx, y\rangle - \langle PBB^*Px, y\rangle + \langle Qx, y\rangle = 0.$$

The existence problem of the solution of ROE (13) in infinite-dimensional case was studied (see; e.g. Boyd 1994, Bittanti et al. 1991, Ginson 1983, Lion 1971, Ootstveen et al. 1998). We first state the following sufficient condition guaranteed the existence of a bounded solution $P(t)$ of ROE (13), which is given in Prato et al. 1990 as follows.

Proposition 3.2. *Let $Q(t) \in LO([0, \infty), X^+)$ be a bounded operator function. If linear control system $[A(t), B(t)]$ is $Q(t)-$stabilizable in the sense that for every initial state x_0,* *there is an admissible control $u(t) \in L_2([0,+\infty),U)$ such that the cost function*

$$J(u) = \int_0^\infty [\|u(t)\|^2 + \langle Q(t)x(t,x_0), x(t,x_0)\rangle]dt, \qquad (12)$$

exists and is finite, then the ROE (13) with any initial condition $P_0 \geq 0$ has the solution $P(t) \in LO([0,\infty),X^+)$, which is also a bounded on R^+ function.

The following proposition will play a key role in the derivation of the existence of the solution of ROE (13) from the global null-controllability of the system $[A(t),B(t)]$.

Proposition 3.3. *If linear control system $[A(t),B(t)]$ is GNC in finite time, then for any bounded operator function $Q(t) \in LO([0,\infty),X^+)$, the ROE (13) with $P_0 \geq 0$ has a bounded solution $P(t) \in LO([0,\infty),X^+)$.*

Theorem 3.1. *Assume the conditions B.1- B.3. Assume that linear control system $[A(t),B(t)]$ is GNC in finite time. The infinite-dimensional nonlinear control delay system (3) is $\delta-$stabilizable if the following conditions hold:*

$$0 < b < \frac{1}{2p^2B}, \quad a_1 + A_1 < \frac{\sqrt{1-2p^2bB}}{2pe^{\delta h}} \qquad (13)$$

$$\sup_{t\in R^+} a(t) < \frac{1}{4p} - \frac{1}{2}pbB - p(a_1+A_1)^2. \qquad (14)$$

The stabilizing feedback control is given by

$$u(t) = -\frac{1}{2}B^T(t)P(t)x(t), \qquad (15)$$

where $P(t)$ is the solution of the ROE (15) with any initial condition $P_0 \geq 0$.

Remark 3.1. It is worth noticing that Theorem 3.1 improves a result of Phat Linh 2003, where the growth condition on the nonlinear perturbation $f(.)$ without state delays was strictly assumed that:

$$\|f(t,x,y,u)\| \leq a(t)\|x\| + b(t),$$

for all $(t,x,y,u) \in R^+ \times X \times X \times U$.

Note that if $f(t,x,y,u) = 0$, i.e. $a = b = a_1 = 0$, we have the following obvious consequence.

Corollary 3.1. *Assume that the infinite-dimensional linear control system $[A(t),B(t)]$ is GNC in finite time. The linear control delay system*

$$\dot{x}(t) = A(t)x(t) + A_1(t)x(t-h) + B(t)u(t),$$

is $\delta-$ stabilizable if $0 < A_1 < \frac{1}{2pe^{\delta h}}$.

In the case if $A_1(.) = 0, f(t,x,y,u) = 0$, the conditions (16), (17) automatically hold and then we have the following subsequence for the strong stabilizability of linear control system, which extends the result of Megan 1975, Slemrod 1974 to the time-varying case.

Corollary 3.2. *The infinite-dimensional linear control system*

$$\dot{x}(t) = A(t)x(t) + B(t)u(t),$$

is strongly stabilizable if the system is GNC in finite time.

Example 3.1. Consider system (3) in the Hilbert spaces l_2, where

$$A(t) : (x_1,x_2,...) \in l_2 \longrightarrow (\frac{1}{8}e^{-4t} - 2e^{4t})(x_1,x_2,...) \in l_2,$$

$$A_1(t) : (x_1,x_2,...) \in l_2 \longrightarrow e^{-\frac{1}{2t}}(x_1,x_2,...) \in l_2,$$

$$B(t) : (u_1,u_2,...) \in l_2 \longrightarrow e^{-2t}(u_1,u_2,...) \in l_2,$$

$$f(t,x,y,u) = \frac{1}{3}x\sin^2 t + \frac{1}{3}e^{-\frac{1}{2}t}y + \frac{4}{5}e^{-\frac{9}{2}t}u, \quad \forall t \geq 0.$$

We have

$$a(t) = \frac{1}{3}\sin^2 t, \quad a_1(t) = \frac{1}{3}e^{-\frac{1}{2}t}, \quad b(t) = \frac{4}{5}e^{-\frac{9}{2}t}.$$

To verify the GNC of the system $[A(t),B(t)]$ we first find the evolution operator $U(t,s)$. Upon some computations we find that $U(t,\tau) = [u_{ij}]$, where

$$\begin{aligned} u_{11}(t,\tau) &= e^{-\frac{1}{32}(e^{-4t}-e^{-4\tau})-\frac{1}{2}(e^{4t}-e^{4\tau})}. \\ u_{22}(t,\tau) &= e^{-\frac{1}{32}(e^{-4t}-e^{-4\tau})-\frac{1}{2}(e^{4t}-e^{4\tau})}. \\ u_{nn}(t,\tau) &= e^{-\frac{1}{32}(e^{-4t}-e^{-4\tau})-\frac{1}{2}(e^{4t}-e^{4\tau})}. \end{aligned}$$

Therefore, defining $\|U^*(T,0)x^*\|^2$, and $\|B^*(\tau)U^*(T,\tau)x^*\|^2$, and applying Proposition 3.1, where $c = 0.08, T = 1$, we can verify the GNC of the system $[A(t),B(t)]$. On the other hand, we have $\tilde{A}(t)x = (\frac{1}{8}e^{-4t} - 2e^{4t} + 2)x$, the ROE

$$\dot{P}(t) + \tilde{A}^*(t)P(t) + P(t)\tilde{A}(t) - P(t)\tilde{B}(t)\tilde{B}^*(t)P(t) + I = 0,$$

has the solution

$$P(t) = \begin{pmatrix} \frac{1}{4}e^{-4t} & 0 \\ 0 & \frac{1}{4}e^{-4t} \end{pmatrix}.$$

and all the conditions (16),(17) are satisfied with

$$b = 4/5, \quad p = 1/4, \quad a_1 = 1/3, \quad A_1 = 1.$$

By Theorem 3.1, the system is $2-$ stabilizable.

4 CONCLUSIONS

In this paper, based on the controllability of the nominal linear control system, sufficient conditions depending on the size of the delay for the strong stabilizability have been established by solving a standard Riccati matrix/operator equation. A constructive procedure for finding the stabilizing feedback control and illustrative examples of the results are given. It is worth mentioning that the results presented in this paper do not involve multiple delays as well as the constraints on both the state and control of the system. These issues will be the subject of the future investigations.

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