ADAPTIVE CONTROL OF CHAOS IN A CONGESTION CONTROL MODEL

Kai Jiang, Xiao Fan Wang [1] , Yugeng Xi, Xiang Li

Department of Automation, Shanghai Jiao Tong University
1954 Huashan Road, 200030, Shanghai, PR China

Abstract: Random Early Detection (RED) is the most promising Active Queue Management (AQM) mechanism. Its interaction with Transmission Control Protocol (TCP) can be modeled as a discrete-time dynamical system, which may exhibit complex bifurcation and chaos behavior. An improved RED algorithm is proposed by using a real-time, adaptive, model-independent (RTAMI) technique. Bifurcation analysis and numerical simulation results show that the new algorithm can control the bifurcation and chaotic dynamic of a TCP-RED map. The situation of the presence of UDP traffic is also studied.

Keywords: Congestion Control, Random Early Detection, Adaptive Control, Chaos Control

1. INTRODUCTION

With the unprecedented increase of the Internet, network congestion has become a crucial problem. Not properly adjusted network can cause severe congestion and significantly performance decrease, for example throughput and packet drop rate. One of congestion control approaches is to regulate congestion level at each bottleneck router through an Active Queue Management (AQM) mechanism. Random Early Detection (RED) algorithm is the most promising AQM mechanism (Floyd and Jacobson, 1997). It attempts to control average queue length and drop data packets earlier in order to notify the sources about the incipient congestion. However, it has been shown that the performance of RED is quite sensitive to the traffic loads and parameter settings (Diot *et al.*, 2001; May *et al.*, 1999). Not properly tuned RED algorithm may result in heavy oscillation of average queue length and severe delay variation, and thus induce network instability. Recently, researchers have proposed many modifications to the original RED, e.g. Adaptive RED (ARED) (Floyd *et al.*, 2001), and Fuzzy RED (Trinh and Molnár, 2004).

Although the concept of RED algorithm is very simple, the interaction of RED gateway with TCP connections has shown complicated dynamic behavior. Recently, we have performed time series analysis of TCP-RED and have demonstrated that under certain circumstances the dynamic behavior of aggregate traffic is chaotic (Jiang *et al.*, 2003; Jiang *et al.*, 2004*b*). Ranjan *et al.* have used a nonlinear discrete-time map to approximately model TCP-RED congestion control system (Ranjan *et al.*, 2002). They have found that the TCP-RED map could exhibit complicated bifurcation and chaos phenomena in the queue dynamics. The nonlinear dynamics of TCP-RED motivate researchers to control congestion by utilizing the existing bifurcation and chaos control techniques. For example, La *et al.* have proposed a washout filter to postpone the occurrence of bifurcation and increase the stability of TCP-RED (La *et al.*, 2002*a*). We have also investigated time-

[1] Corresponding author: xfwang@sjtu.edu.cn Fax: 86-21-62932344

delayed feedback approach to controlling chaos in TCP-RED and improving the performance of TCP-RED system (Jiang *et al.*, 2004*a*; Jiang *et al.*, 2004*c*; Chen *et al.*, 2004).

In this paper, we propose an improved RED algorithm to alleviate the instability of RED, based on a real-time, adaptive, model-independent (RTAMI) technique (Christini and Collins, 1997). Bifurcation analysis and numerical simulation results show that RED-RTAMI can successfully control bifurcation and chaos in the TCP-RED model. Furthermore, we investigate the situation of the presence of User Datagram Protocol (UDP) traffic.

The rest of the paper is organized as follows. In Section 2, we introduce the RED algorithm and model TCP-RED congestion control system as a discrete-time dynamical system. Section 3 describes the RED-RTAMI algorithm and gives the proving of its linear stability and bifurcation analysis. Section 4 provides the numerical simulation results to controlling chaos in TCP-RED map. Section 5 considers the situation of the presence of UDP traffic. Finally, section 6 concludes the paper.

2. DISCRETE-TIME MODELLING OF TCP-RED AND RED-DFC ALGORITHM

2.1 The RED Algorithm

We briefly introduce the "gentle" version of the RED algorithm (Floyd, 2000). The essence of RED is to probabilistically drop (or marking) packets earlier that can notify the sources about the incipient congestion. The estimated average queue length in RED is updated at the sampling time t_k of packet arrival according to

$$\bar{q}_{e,k} = A(\bar{q}_{e,k-1}, q_k) = (1 - w_q)\bar{q}_{e,k-1} + w_q q_k \quad (1)$$

where $\bar{q}_{e,k}$ and q_k are the estimate of average queue length and instantaneous queue length, respectively. $w_q \in (0, 1)$ is weight parameter. The average queue estimate in Eq. (1) is an exponentially weighted moving average of instantaneous queue length. Each arriving packet is dropped with probability p_k defined as following:

$$p_k = H(\bar{q}_{e,k}) = \begin{cases} 0, & 0 \le \bar{q}_{e,k} < q_{\min} \\ \dfrac{\bar{q}_{e,k} - q_{\min}}{q_{\max} - q_{\min}} p_{\max}, & q_{\min} \le \bar{q}_{e,k} < q_{\max} \\ p_{\max} + \dfrac{1 - p_{\max}}{q_{\max}}(\bar{q}_{e,k} - q_{\max}), & q_{\max} \le \bar{q}_{e,k} \le 2q_{\max} \\ 1, & 2q_{\max} \le \bar{q}_{e,k} \le B \end{cases} \quad (2)$$

where the nonnegative $q_{\min}$ and $q_{\max}$ are the lower and higher threshold values, and B is the buffer size. The control parameters of RED mechanism are w_q, $q_{\min}$, $q_{\max}$, and $p_{\max}$.

2.2 A Discrete-Time Model of TCP-RED

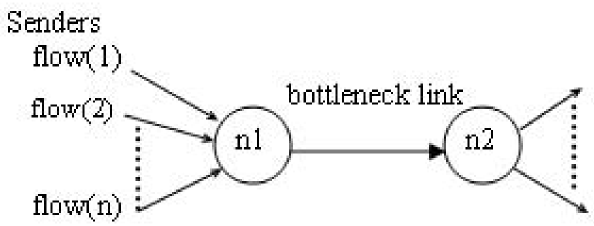

Fig. 1. Topology of the network

Now we introduce a discrete time model of TCP-RED (Ranjan *et al.*, 2002). We consider a simple network with a bottleneck link that implements RED algorithm and is shared by n TCP flows, as shown in Fig. 1. All TCP connections are assumed to be TCP Reno and long-lived connections. The capacity of bottleneck link is denoted by C Mbps, and the buffer sizes at the link by B. The round-trip propagation delay (without queuing delay) of each TCP flow is given by R_0 ms. We denote the average packet size by M.

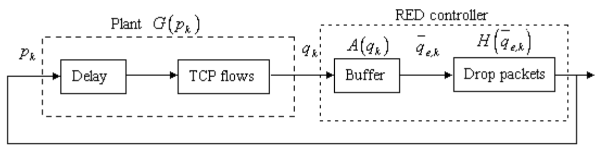

Fig. 2. TCP-RED as a discrete-time feedback control system.

The TCP-RED congestion control system can be viewed as a discrete-time feedback control system (Fig. 2). At time t_k, according to the instantaneous queue length q_k, RED algorithm estimates average queue length $\bar{q}_{e,k}$ via a low-pass filter A (1). The average queue length $\bar{q}_{e,k}$ is used to compute the feedback signal p_k (2). After a RTT (round-trip time) feedback delay, TCP end-users detect the drop probability p_k and adjust their packets sending rates. The end-users' behavior results in a new queue length $q_{e,k+1}$ and the throughput of the connections, which is a plant function G of drop probability p_k. Therefore, the TCP-RED congestion control system can be modeled by the following discrete-time model:

$$\begin{aligned} \bar{q}_{e,k+1} &= A\left(\bar{q}_{e,k}, G\left(H\left(\bar{q}_{e,k}\right)\right)\right) \\ &= \left(1 - w_q\right)\bar{q}_{e,k} + w_q G\left(H\left(\bar{q}_{e,k}\right)\right) \equiv f\left(\bar{q}_{e,k}\right) \end{aligned} \quad (3)$$

An explicit expression of the plant function G can be approximately given as follows (Firoiu and Borden, 2000):

$$G(p_k) = \begin{cases} 0, & p_k \geq p_1 \\ \frac{C}{M}\left(T_R^{-1}\left(p_k, \frac{C}{n}\right) - R_0\right), & \\ & p_2 \leq p_k < p_1 \\ B, & p_k < p_2 \end{cases} \quad (4)$$

where

$$p_1 = T_{p_k}^{-1}\left(\frac{C}{n}, R_0\right), \quad p_2 = T_{p_k}^{-1}\left(\frac{C}{n}, R_0 + \frac{BM}{C}\right)$$

$T$is the throughput of a TCP flow (in bits/sec) and M is maximum segment size or packet size (in bits). T_R^{-1} and $T_{p_k}^{-1}$ are the inverse of $T(p_k, R)$ in R and p_k, respectively. p_1 is the maximum dropping probability for which the system is fully utilized. If $p_k > p_1$, the TCP sources will have too small sending rates to keep the link fully utilized. p_2 is the minimum dropping probability for which no packet overflows the buffer. If $p_k < p_2$, the TCP sources will have too large sending rates to keep all packets in the buffer.

The throughput of each TCP flow can be descried as a complex steady state model (Firoiu and Borden, 2000). Here we adopt a simple version throughput function described as follows:

$$T(p_k, R) = \frac{MK}{R\sqrt{p_k}} \quad (5)$$

where K is a constant and $1 \leq K \leq \sqrt{8/3}$ and R is round trip time. Then the plant function G can be computed as:

$$G(p_k) = \begin{cases} 0, & p_k \geq p_1 \\ \frac{nK}{\sqrt{p_k}} - \frac{R_0 C}{M}, & p_2 \leq p_k < p_1 \\ B, & p_k < p_2 \end{cases} \quad (6)$$

where $p_1 = \left(\frac{nMK}{R_0 C}\right)^2$, $p_2 = \left(\frac{nMK}{BM + R_0 C}\right)^2$.

Assuming that $p_1 < p_{\max}$. The TCP-RED map (3) can be described as (Chen *et al.*, 2004):

$$\begin{aligned} \bar{q}_{e,k+1} &= f(\bar{q}_{e,k}) \\ &= \begin{cases} (1-w_q)\bar{q}_{e,k}, & \bar{q}_{e,k} > b_1 \\ \tilde{f}(\bar{q}_{e,k}), & b_2 < \bar{q}_{e,k} \leq b_1 \\ (1-w_q)\bar{q}_{e,k} + w_q B, & \bar{q}_{e,k} \leq b_2 \end{cases} \end{aligned} \quad (7)$$

where

$$\begin{aligned} \tilde{f}(\bar{q}_{e,k}) &= (1-w_q)\bar{q}_{e,k} \\ &+ w_q\left(\frac{nK}{\sqrt{\frac{\bar{q}_{e,k}-q_{\min}}{q_{\max}-q_{\min}} p_{\max}}} - \frac{R_0 C}{M}\right), \end{aligned}$$

$$b_1 = \frac{p_1(q_{\max} - q_{\min})}{p_{\max}} + q_{\min},$$

$$b_2 = \frac{p_2(q_{\max} - q_{\min})}{p_{\max}} + q_{\min}.$$

The TCP-RED map (7) has a fixed point $\bar{q}_e^*$ lies in the desired interval $[b_2, b_1]$, which is a real solution of the following equation:

$$(\bar{q}_e^* - q_{\min})\left(\bar{q}_e^* + \frac{R_0 C}{M}\right)^2 = \frac{(nK)^2}{p_{\max}}(q_{\max} - q_{\min}) \quad (8)$$

The eigenvalue of the system at the fixed point is

$$\begin{aligned} a &\equiv \frac{\partial f(\bar{q}_e^*, p_{\max})}{\partial \bar{q}_e} \\ &= 1 - w - \frac{wnK}{2(\bar{q}_e^* - q_{\min})^{1.5}}\sqrt{\frac{q_{\max} - q_{\min}}{p_{\max}}} < 1 \end{aligned} \quad (9)$$

Furthermore,

$$\begin{aligned} b &\equiv \frac{\partial f(\bar{q}_e^*, p_{\max})}{\partial p_{\max}} \\ &= -\frac{wnK}{2} p_{\max}^{1.5}\sqrt{\frac{q_{\max} - q_{\min}}{\bar{q}_e^* - q_{\min}}} < 0 \end{aligned} \quad (10)$$

A necessary and sufficient condition for the linear stability of the fixed point $\bar{q}_e^*$ of the TCP-RED map (7) is $|a| < 1$.

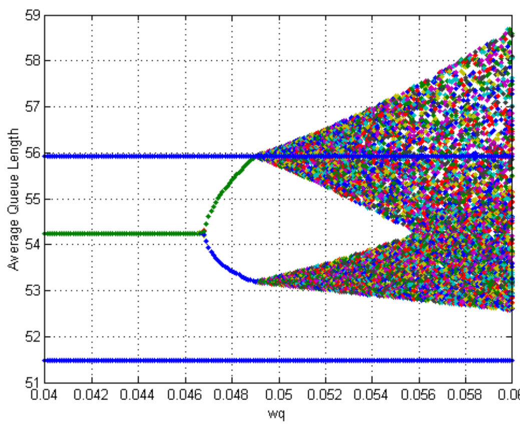

Fig. 3. Bifurcation plot of the TCP-RED map (7) with bifurcation parameter w_q. The upper line represents b_1 and the lower one represents b_2.

Map (7) can exhibit complicated nonlinear dynamics (Chen *et al.*, 2004). As an example, we set the following parameters:

$$\begin{aligned} &B = 300 packets, n = 20, K = \sqrt{8/3}, R_0 = 0.1\,\text{sec} \\ &C = 1500 kbps, M = 0.5 kb, q_{\max} = 100, q_{\min} = 50 \end{aligned}$$

with $p_{\max} = 0.1$ and the initial state $\bar{q}_{e,0} = 60$. Fig. 3 shows the bifurcation plot of the TCP-RED map (7) with bifurcation parameter w_q. The map exhibited a period-two bifurcation at $w_q^* \approx 0.047$. For $w_q < w_q^*$ the average queue length kept at the fixed point $\overline{q}_e^*$. For $w_q^* < w_q < 0.049$, the system exhibits a 'benign' period-two oscillation, which lies in the region (b_2, b_1). Continuing to increase w_q leads to a border collision bifurcation and for $w_q > 0.056$, the system is in the chaotic oscillation region. The chaotic oscillation can result in rapid deterioration of throughput and delay, so it is 'malignant' and should be eliminated.

3. RED-RTAMI: AN IMPROVED RED ALGORITHM

The real-time, adaptive, model-independent control algorithm (RTAMI) proposed by Christini and Collins can be used to stabilize underlying unstable periodic orbits in low-dimensional chaotic and nonchaotic dynamical systems (Christini and Collins, 1997). Its main benefits are that it does not require learning stage of the control algorithm and adapts the control parameter to the changes of system parameters.

We re-write the TCP-RED map as following:

$$\bar{q}_{e,k+1} = f(\bar{q}_{e,k}, p_{\max,k}) \tag{11}$$

Here we use the RTAMI technique to adaptively adjust the parameter $p_{\max}$ in the RED as following:

$$p_{\max,k} = \bar{p}_{\max} + \delta p_{\max,k} = \bar{p}_{\max} + \frac{\overline{q}_{e,k} - \overline{q}_{e,k}^*}{g_k} \tag{12}$$

where $\bar{p}_{\max}$ is the mean parameter value, $\overline{q}_{e,k}^*$ is the current estimate of fixed point $\overline{q}_e^*$ and g_k is the control sensitivity g at index k. The ideal value of g is the sensitivity of $\overline{q}_e^*$ to perturbations: $g_{ideal} = \delta\overline{q}_e^*/\delta p_{\max}$.

The RED-RTAMI algorithm repeatedly estimates $\overline{q}_e^*$ and g. When control is initiated, $g$can be set to an arbitrary value (as long as the sign of g matches that of g_{ideal}). After each measurement of $\overline{q}_{e,k}$, $\overline{q}_e^*$ is estimated using

$$\overline{q}_{e,k}^* = \sum_{i=0}^{N-1} \frac{\overline{q}_{e,k-i}}{N}. \tag{13}$$

At each iteration, after $\overline{q}_e^*$ is estimated via Eq. (13), the RTAMI algorithm evaluates whether the estimate of g should be adapted. The value of g is not adapted if the desired control precision ε has been achieved. Control precision has not been achieved if

$$\left|\overline{q}_{e,k} - \overline{q}_{e,k-1}^*\right| > \varepsilon \tag{14}$$

is satisfied by at least L out of the N previous data points. If the magnitude of g need to be adapted, the following criteria

$$sign\left(\overline{q}_{e,k} - \overline{q}_{e,k-1}\right) = sign\left(\overline{q}_{e,k-1} - \overline{q}_{e,k-2}\right) \tag{15}$$

$$\begin{aligned} &\frac{1}{N}\sum_{i=0}^{N-1} \frac{|\Delta\overline{q}_1| - |\Delta\overline{q}_2|}{|\Delta\overline{q}_1|} < r\%, \\ &\Delta\overline{q}_1 = \overline{q}_{e,k-i-1} - \overline{q}_{e,k-i-2}^*, \\ &\Delta\overline{q}_2 = \overline{q}_{e,k-i} - \overline{q}_{e,k-i-1}^* \end{aligned} \tag{16}$$

are computed. The RED-RTAMI algorithm increases the magnitude of g (i.e., $g_{n+1} = g_n\rho$, where ρ is the adjustment factor) if Eq. (15) is satisfied for at least L out of the N previous data points. The magnitude of g is decreased (i.e., $g_{n+1} = g_n/\rho$) if Eq. (16) is satisfied. If neither Eq. (15) nor Eq. (16) is satisfied, then g is not adapted because $\overline{q}_e$ is properly approaching the estimate of $\overline{q}_e^*$.

Theorem 1: Consider the discrete-time control system described by Eq. (11) and Eq. (12). The fixed point $\overline{q}_e^*$ is asymptotically stable if and only if $|a + b/g_k| < 1$, where $a \equiv \partial f(\overline{q}_e^*, p_{\max})/\partial\overline{q}_e$, $b \equiv \partial f(\overline{q}_e^*, p_{\max})/\partial p_{\max}$.

Proof: Denote $\delta\overline{q}_{e,k} = \overline{q}_{e,k} - \overline{q}_e^*$. Linearizing system (11)-(12) about the fixed point gives

$$\begin{aligned} \delta\overline{q}_{e,k+1} &= a\delta\overline{q}_{e,k} + b\delta p_{\max,k} \\ &= a\delta\overline{q}_{e,k} + \frac{b}{g_k}\left(\overline{q}_{e,k} - \overline{q}_{e,k}^*\right) \\ &\approx a\delta\overline{q}_{e,k} + \frac{b}{g_k}\left(\overline{q}_{e,k} - \overline{q}_e^*\right) \\ &= a\delta\overline{q}_{e,k} + \frac{b}{g_k}\delta\overline{q}_{e,k} = \left(a + \frac{b}{g_k}\right)\delta\overline{q}_{e,k} \end{aligned} \tag{17}$$

The stability of Eq. (17) is determined by the characteristic equation

$$\lambda - (a + b/g_k) = 0. \tag{18}$$

The necessary and sufficient condition for asymptotic stability at the fixed point $(\overline{q}_e^*, p_{\max}^*)$ is $-1 < \lambda < 1$. This condition is satisfied if and only if $|a + b/g_k| < 1$ holds.

Now we study the effectiveness of the RED-RTAMI algorithm (12) via bifurcation analysis. The period-doubling bifurcation (PDB) point is a point at which the eigenvalue of the system becomes -1. We choose w_q as the bifurcation parameter. The PDB point of the original TCP-RED map (7) is

$$w_{RED}^* = 2/(1+\beta) \tag{19}$$

where

$$\beta = \frac{nK}{2\left(\overline{q}_e^* - q_{\min}\right)^{1.5}\sqrt{\frac{p_{\max}}{q_{\max} - q_{\min}}}} > 0$$

On the other hand, the PDB point of the controlled system (11-12) is given by

$$w^*_{RTAMI} = \frac{2}{1+\beta+\frac{1}{g_k p_{\max}}\beta\left(\bar{q}_e^* - q_{\min}\right)} > w^*_{RED} \quad (20)$$

since $\bar{q}_e^* > q_{\min}, p_{\max} > 0$ and $g_k < 0$. This implies that period-doubling bifurcation in the TCP-RED map can be successfully postponed by the RTAMI control algorithm.

4. NUMERICAL SIMULATIONS

In this section we apply RED-RTAMI algorithm to control chaos in the TCP-RED map. The topology of simulated network is shown as Fig. 1. We consider FTP traffic only. System parameters are chosen as follows:

$$\begin{array}{l} B = 300\ packets, n = 20, K = \sqrt{8/3} \\ R_0 = 0.1 \text{ sec}, C = 1500\ kbps, M = 0.5\ kb \\ q_{\min} = 50, q_{\max} = 100, w_q = 0.07, N = 10 \\ L = 3,\ \varepsilon = 0.001,\ r = 1.005,\ \bar{q}_{e,0} = 60 \end{array} \quad (21)$$

with $\bar{p}_{\max} = 0.1$ and the initial value $g_0 = -92$. The RED drop rate $p_{\max,k}$ is in the range between 0.01 and 0.5.

Fig. 4 shows the numerical simulation results when RED-RTAMI was activated at $k = 100$. For $k < 100$ the TCP-RED map was chaotic (Chen *et al.*, 2004). After a short transient process, the controlled system was stabilized on a fixed point. The fixed point is much better than the originally chaotic oscillation. From the viewpoint of network engineering, this behavior can be viewed as 'benign' since it lies in the region between $q_{\min}$ and $q_{\max}$.

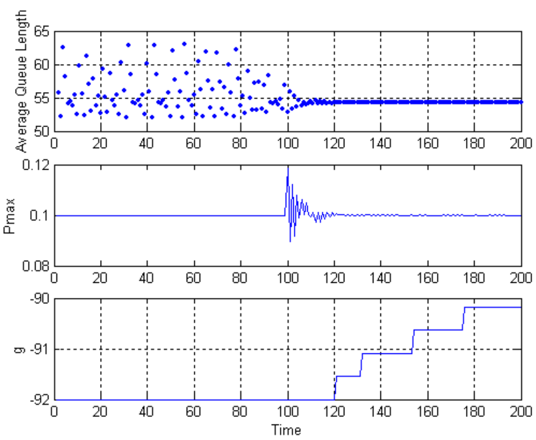

Fig. 4. Control of chaos in the TCP-RED map using RTAMI technique at time $k = 100$.

5. RED-RTAMI FOR UDP TRAFFIC

Besides TCP, the User Datagram Protocol (UDP) is another major protocol over the Internet. Most real-time applications, for example Internet telephone, IP telephone, and real-time stream media, are based on UDP protocol. However, UDP traffic is connectionless and irresponsible to the network congestion. If there is no proper control mechanism to handle UDP's traffic, it will use up most of the bandwidth over the Internet. Thus, UDP traffic often deteriorates the stability of Internet.

In this section we consider the performance of RED-RTAMI algorithm at the situation of the presence of UDP traffic. Given the UDP load C_u, the available capacity for the TCP connections becomes $C - C_u(1-p_k)$ (La *et al.*, 2002*b*) and the plant function G in Eq. (4) is changed into

$$G(p_k) = \begin{cases} 0, & p_k \geq p_3 \\ \dfrac{C - C_u(1-p_k)}{M}\zeta, & p_4 \leq p_k < p_3 \\ B, & p_k < p_4 \end{cases} \quad (22)$$

where $\zeta = T_R^{-1}\left(p_k, \frac{C-C_u(1-p_k)}{n}\right) - R_0$,
$p_3 = T_{p_k}^{-1}\left(\frac{C-C_u(1-p_k)}{n}, R_0\right)$,
$p_4 = T_{p_k}^{-1}\left(\frac{C-C_u(1-p_k)}{n}, R_0 + \frac{BM}{C-C_u(1-p_k)}\right)$.

Then the TCP-UDP-RED map can be expressed as follows:

$$\begin{aligned} \bar{q}_{e,k+1} &= g(\bar{q}_{e,k}) \\ &= \begin{cases} (1-w_q)\bar{q}_{e,k}, & \bar{q}_{e,k} > b_3 \\ \tilde{g}\left(\bar{q}_{e,k}\right),\ b_4 < & \bar{q}_{e,k} \leq b_3 \\ (1-w_q)\bar{q}_{e,k} + w_q B, & \bar{q}_{e,k} \leq b_4 \end{cases} \end{aligned} \quad (23)$$

where

$$\begin{aligned} &\tilde{g}\ \left(\bar{q}_{e,k}\right) = (1-w_q)\bar{q}_{e,k} + \\ &w_q\,(\frac{CnK}{\sqrt{\eta\left(\bar{q}_{e,k}-q_{\min}\right)}\left(C - C_u\left(1-\eta\left(\bar{q}_{e,k}-q_{\min}\right)\right)\right)} \\ &- \frac{R_0 C}{M}), b_3 = \frac{p_3(q_{\max}-q_{\min})}{p_{\max}} + q_{\min}, \\ &b_4 = \frac{p_4\left(q_{\max}-q_{\min}\right)}{p_{\max}} + q_{\min}. \end{aligned}$$

Here, $\eta = \sqrt{\frac{p_{\max}}{q_{\max}-q_{\min}}}$. p_3 and p_4 are the positive, real solutions of the following two equations, respectively.

$$C_u p_3^{3/2} + (C - C_u)\,p_3^{1/2} - \frac{nMK}{R_0} = 0$$

$$C_u p_4^{3/2} + (C - C_u)\,p_4^{1/2} - \frac{nMK}{\frac{BM}{C} + R_0} = 0$$

This TCP-UDP-RED model can also exhibit complex nonlinear dynamics (La *et al.*, 2002*b*). Here, the UDP traffic C_u = 100kbps and other system parameters are chosen as Eq. (21). Fig. 5 shows the numerical results of applying the RED-RTAMI algorithm (14) to control chaotic behavior of the TCP-UDP-RED map. After a short transition, the original chaotic system can also be stabilized at the fixed point.

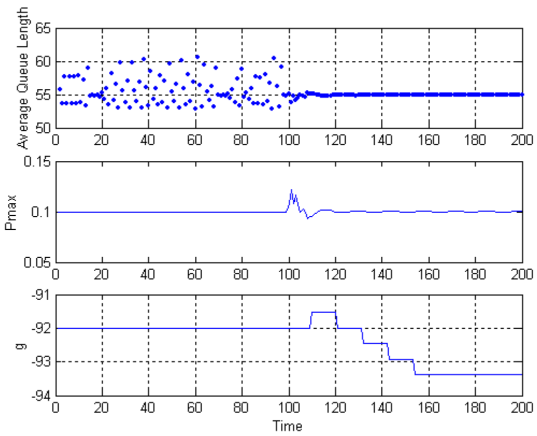

Fig. 5. Control of chaos in the TCP-UDP-RED map using RTAMI technique at time $k = 100$.

6. CONCLUSIONS

In this work, we have investigated RTAMI technique to adaptive control the chaotic dynamics in an Internet congestion control model – TCP-RED map. Bifurcation analysis and numerical simulation results demonstrate that RED-RTAMI can enhance the stability of TCP-RED map. The proposed algorithm can also give similar performance in TCP-UDP-RED map. We will implement RED-RTAMI algorithm in the *ns-2* simulator and further investigate its performance under various network scenarios.

7. ACKNOWLEDGMENTS

This work was supported by the National Science Fund for Distinguished Young Scholars of P. R. China under Grant No. 60225013, the National Science Foundation of P. R. China under Grant No. 70271072 and 90412004.

REFERENCES

Chen, L., X. F. Wang and Z. Z. Han (2004). Controlling bifurcation and chaos in internet congestion control model. *International Journal of Bifurcation and Chaos,* **14**, 1863–1876.

Christini, D. J. and J. J. Collins (1997). Real-time, adaptive, model independent control of low-dimensional chaotic and nonchaotic dynamical systems. *IEEE Transactions on Circuits Systems I,* **44**, 1027–1030.

Diot, C., G. Iannaccone and M. May (2001). Aggregate traffic performance with active queue management and drop from tail. *ACM SIGCOMM Computer Communication Review,* **31**, 4–13.

Firoiu, V. and M. Borden (2000). A study of active queue management for congestion control. In: *Proc. Of IEEE INFOCOM*. Vol. 3. pp. 1435–1445.

Floyd, S. (2000). Recommendation on using the "gentle" variant of red. *Available from: http://www.aciri.org/floyd/red/gentle.html.*

Floyd, S. and V. Jacobson (1997). Random early detection gateways for congestion avoidance. *IEEE Transactions On Networking,* **1**, 397–413.

Floyd, S., R. Gummadi and S. Shenker (2001). Adaptive red: an algorithm for increasing the robustness of red's active queue management. *Available from: http://www.icir.org/∼floyd.*

Jiang, K., X. F. Wang and Y. G. Xi (2003). Chaotic time series analysis of tcp-red congestion control system. In: *Proc. of the 22nd Chinese Control Conference*. pp. 616–620.

Jiang, K., X. F. Wang and Y. G. Xi (2004*a*). Controlling internet congestion with time-delayed state feedback. *Dynamics of Continuous, Discrete and Impulsive Systems Series B: Applications and Algorithms,* **11a**, 128 –135.

Jiang, K., X. F. Wang and Y. G. Xi (2004*b*). Nonlinear analysis of red - a comparative study. *Chaos, Solitons and Fractals,* **21**, 1153–1162.

Jiang, K., X. F. Wang and Y. G. Xi (2004*c*). A robust red algorithm based on time-delayed feedback control. In: *Proc. of 5th Asian Control Conference*. pp. 707–712.

La, R. J., P. Ranjan and E. H. Abed (2002*a*). Feedback control of random early detection. In: *Proc. of 40th Annual Allerton Conference on Communication, Control, and Computing.*

La, R. J., P. Ranjan and E. H. Abed (2002*b*). Nonlinear dynamics of mixed tcp and udp traffic under red. *Available from: http://www.ece.umd.edu/∼hyongla/papers/med2002.pdf.*

May, M., C. Diot, B. Lyles and J. Bolot (1999). Reasons not to deploy red. In: *Proc. of 7th. International Workshop on Quality of Service.*

Ranjan, P., E. H. Abed and R. J. La (2002). Nonlinear instabilities in tcp-red. In: *Proc. of IEEE INFOCOM*. pp. 249–258.

Trinh, T. A. and S. Molnár (2004). A comprehensive performance analysis of random early detection mechanism. *Telecommunication Systems,* **25**, 1,2, 9–31.