THE PARAMETRIZATION OF ALL ROBUST STABILIZING REPETITIVE CONTROLLERS

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Abstract: In this paper, we investigate the parameterization of all robust stabilizing repetitive controllers for single-input/single-output continuous time non-minimum phase systems. The repetitive control system is a type of servo mechanism designed for a periodic reference input. When repetitive control design methods are applied to real systems, the influence of uncertainties in the plant must be considered. In some cases, the uncertainties in the plant make the repetitive control system unstable, even though the controller was designed to stabilize the nominal plant. The stability problem with uncertainty is known as the robust stability problem. Several papers to design robust stabilizing repetitive control systems have been published. However the parametrization of all robust stabilizing repetitive controllers has not been considered. In this paper, we propose the parametrization of all robust stabilizing repetitive controllers for non-minimum phase systems. Finally, a numerical example is illustrated to show the effectiveness of the proposed parametrization.

Keywords: parametrization, robust stability, repetitive controller

1. INTRODUCTION

In this paper, we examine the parametrization of all robust stabilizing repetitive controllers. The repetitive control system is a type of servo mechanism designed for a periodic reference input (Inoue *et al.*, 1980; Hara *et al.*, 1988; Omata *et al.*, 1987).

When repetitive control design methods are applied to real systems, the influence of uncertainties in the plant must be considered. In some cases, the uncertainties in the plant make the repetitive control system unstable, even though the controller was designed to stabilize the nominal plant. The stability problem with uncertainty is known as the robust stability problem (Doyle *et al.*, 1989). The robust stability problem of repetitive control systems was considered by (Hara *et al.*, 1994). The robust stability condition for repetitive control systems was reduced to the μ synthesis problem (Hara *et al.*, 1994), but the μ synthesis problem cannot be solved analytically. That is, in order to solve the μ synthesis problem, we must solve an H_∞ problem iteratively using the $D - K$ iteration method. Furthermore, the convergence of iterative methods to solve the μ synthesis problem is not guaranteed. (Yamada *et al.*, 2003*b*) tackle this problem and propose a design method of robust repetitive control systems without solving the μ synthesis problem. The method by (Yamada *et al.*, 2003*b*) is effective for the minimum phase systems, however the method by (Yamada *et al.*, 2003*b*) is not so effective for the non-minimum phase systems, since the frequency range in which the output follows the periodic reference input is restricted. Therefore, (Yamada *et al.*, 2003*a*) gave a design method for robust repetitive control systems for non-minimum phase system such that the frequency range in which the output follows the periodic reference input is not restricted. However, the parametrization of all robust stabilizing repetitive controllers has not been considered.

In this paper, we propose the parametrization of all robust stabilizing repetitive controllers for non-minimum phase systems. The basic idea of the method is as follows. If the repetitive control system is robustly stable for the plant, then the repetitive controller must satisfy the robust stability condition. This implies that if the repetitive control system is robustly stable, then the repetitive controller is included in the parametrization of all robust stabilizing controllers for the plant. The parametrization of all robust stabilizing controllers for the plant is obtained using H_∞ control theory based on the Riccati equation (Doyle *et al.*, 1989) and the Linear Matrix Inequality (LMI) (Iwasaki and Skelton, 1994; Gahinet and Apkarian, 1994). Robust stabilizing controllers for the plant include a free parameter, which is designed to achieve desirable control characteristics. If the free parameter is chosen to give the control system robust servo characteristics for periodic reference input, then the controller operates as a robust repetitive controller. Conversely a repetitive controller stabilizing the plant robustly, then the free-parameter is written by an appropriate form. Using this idea, we obtain the parametrization of all robust stabilizing repetitive controllers. Finally, a numerical example is shown to illustrate the effectiveness of the proposed parametrization.

Notations

R	the set of real numbers.
R_+	$R \cup \{\infty\}$.
$R(s)$	the set of real rational function with s.
RH_∞	the set of stable proper real rational functions.
H_∞	the set of stable causal functions.
$D^\perp$	orthogonal complement of D, i.e., $\begin{bmatrix} D & D^\perp \end{bmatrix}$ or $\begin{bmatrix} D \\ D^\perp \end{bmatrix}$ is unitary.
A^T	transpose of A.
$A^\dagger$	pseudo inverse of A.
$\rho(\{\cdot\})$	spectral radius of $\{\cdot\}$.
$\bar{\sigma}(\{\cdot\})$	largest singular value of $\{\cdot\}$.
$\\|\{\cdot\}\\|_\infty$	H_∞ norm of $\{\cdot\}$
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	represents the state space description $C(sI-A)^{-1}B+D$

2. PROBLEM FORMULATION

Consider the unity feedback system in

$$\begin{cases} y = G(s)u \\ u = C(s)(r-y) \end{cases}, \tag{1}$$

where $G(s) \in R(s)$ is the plant, $C(s)$ is the controller, $y \in R$ is the output and $r \in R$ is the reference input with period T satisfying

$$r(t+T) = r(t) \quad (\forall t \geq 0). \tag{2}$$

The nominal plant of $G(s)$ is denoted by $G_m(s) \in R(s)$. Both $G(s)$ and $G_m(s)$ are assumed to have no zero or pole on the imaginary axis. In addition, it is assumed that the number of poles of $G(s)$ in the closed right half plane is equal to the number of poles of $G_m(s)$. The relation between the plant $G(s)$ and the nominal plant $G_m(s)$ is written as

$$G(s) = G_m(s)(1+\Delta(s)). \tag{3}$$

The set of $\Delta(s)$ is all rational functions satisfying

$$|\Delta(j\omega)| < |W_T(j\omega)| \quad (\forall \omega \in R_+), \tag{4}$$

where $W_T(s)$ is an asymptotically stable rational function.

The robust stability condition for the plant $G(s)$ with uncertainty $\Delta(s)$ satisfying (4) is given by

$$\|T(s)W_T(s)\|_\infty < 1, \tag{5}$$

where $T(s)$ is the complementary sensitivity function given by

$$T(s) = \frac{G_m(s)C(s)}{1+G_m(s)C(s)}. \tag{6}$$

According to (Inoue *et al.*, 1980; Hara *et al.*, 1988; Omata *et al.*, 1987), in order the output y to follow the reference input r in (1) with small steady state error, the controller $C(s)$ must have the following structure

$$C(s) = \hat{C}(s) + \bar{C}(s)\frac{q(s)e^{-sT}}{1-q(s)e^{-sT}}, \tag{7}$$

where $q(s) \in R(s)$ is a low-pass filter satisfying $q(0) = 1$, $\hat{C}(s) \in R(s)$ and $\bar{C}(s) \in R(s)$. In the following, $q(s)e^{-sT}/(1-q(s)e^{-sT})$ define the internal model for the periodic signal with period T.

The problem considered in this paper is to give the parametrization of all robust stabilizing repetitive controllers $C(s)$ written by the form in (7) such that the repetitive control system in (1) is robustly stable and the output y follows the periodic reference input r with small steady state error even in the presence of uncertainty $\Delta(s)$. That is, we find the parametrization of all robust stabilizing repetitive controllers satisfying (5).

3. THE PARAMETRIZATION OF ALL ROBUST STABILIZING REPETITIVE CONTROLLERS

In this section, we give the parametrization of all robust stabilizing repetitive controllers.

In order to obtain the parametrization of all robust stabilizing repetitive controllers, we must see that the robust stabilizing repetitive controllers hold (5). The problem of obtaining the controller $C(s)$, which is not necessarily a repetitive controller, satisfying (5) is equivalent to the following H_∞ problem. In order to obtain the controller $C(s)$ satisfying (5), we consider the control system shown in Fig. 1. $P(s)$ is selected such that the

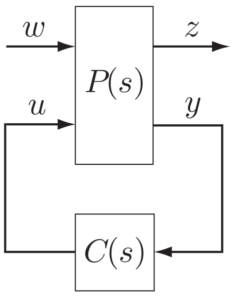

Fig. 1. Block diagram of H_∞ control problem

transfer function from w to z in Fig. 1 is equal to $T(s)W_T(s)$. The state space description of $P(s)$ is, in general,

$$\left\{\begin{array}{rcllr} \dot{x}(t) & = & Ax(t) & +B_1 w(t) & +B_2 u(t) \\ z(t) & = & C_1 x(t) & & +D_{12} u(t), \\ y(t) & = & C_2 x(t) & +D_{21} w(t) & \end{array}\right. \quad (8)$$

where $A \in R^{n\times n}$, $B_1 \in R^n$, $B_2 \in R^n$, $C_1 \in R^{1\times n}$, $C_2 \in R^{1\times n}$, $D_{12} \in R$, $D_{21} \in R$. $P(s)$ is called the generalized plant. $P(s)$ is assumed to satisfy the standard assumption in (Doyle *et al.*, 1989). Under these assumptions, according to (Doyle *et al.*, 1989), following lemma holds true.

Lemma 1. If controllers satisfying (5) exist, both

$$\begin{aligned} & X\left(A - B_2 D_{12}^{\dagger} C_1\right) + \left(A - B_2 D_{12}^{\dagger} C_1\right)^T \\ & + X\left(B_1 B_1^T - B_2\left(D_{12}^T D_{12}\right)^{-1} B_2^T\right) X \\ & + \left(D_{12}^{\perp} C_1^T\right)^T D_{12}^{\perp} C_1^T = 0 \end{aligned} \quad (9)$$

and

$$\begin{aligned} & Y\left(A - B_1 D_{21}^{\dagger} C_2\right)^T + \left(A - B_1 D_{21}^{\dagger} C_2\right) Y \\ & + Y\left(C_1^T C_1 - C_2^T\left(D_{21} D_{21}^T\right)^{-1} C_2\right) Y \\ & + B_1 D_{21}^{\perp}\left(B_1 D_{21}^{\perp}\right)^T = 0 \end{aligned} \quad (10)$$

have solutions $X \geq 0$ and $Y \geq 0$ such that

$$\rho(XY) < 1 \quad (11)$$

and both

$$A - B_2 D_{12}^{\dagger} C_1 + \left(B_1 B_1^T - B_2\left(D_{12}^T D_{12}\right)^{-1} B_2^T\right) X$$

and

$$A - B_1 D_{21}^{\dagger} C_2 + Y\left(C_1^T C_1 - C_2\left(D_{21} D_{21}^T\right)^{-1} C_2\right)$$

have no eigenvalue in the closed right half plane. Using X and Y, the parameterization of all controllers satisfying (5) is given by

$$\begin{aligned} & C(s) \\ & = C_{11}(s) + C_{12}(s)Q(s)(I - C_{22}(s)Q(s))^{-1} C_{21}(s), \end{aligned} \quad (12)$$

where

$$\begin{bmatrix} C_{11}(s) & C_{12}(s) \\ C_{21}(s) & C_{22}(s) \end{bmatrix} = \left[\begin{array}{c|cc} A_c & B_{c1} & B_{c2} \\ \hline C_{c1} & D_{c11} & D_{c12} \\ C_{c2} & D_{c21} & D_{c22} \end{array}\right] \quad (13)$$

$$\begin{aligned} A_c = {} & A + B_1 B_1^T X - B_2\left(D_{12}^{\dagger} C_1 + E_{12}^{-1} B_2^T X\right) \\ & - (I - XY)^{-1}\left(B_1 D_{21}^{\dagger} + Y C_2^T E_{21}^{-1}\right) \\ & \left(C_2 + D_{21} B_1^T X\right) \end{aligned}$$

$$\begin{aligned} B_{c1} &= (I - XY)^{-1}\left(B_1 D_{21}^{\dagger} + Y C_2^T E_{21}^{-1}\right) \\ B_{c2} &= (I - XY)^{-1}\left(B_2 + Y C_1^T D_{12}\right) E_{12}^{-1/2} \end{aligned}$$

$$\begin{aligned} C_{c1} &= -D_{12}^{\dagger} C_1 - E_{12}^{-1} B_2^T X \\ C_{c2} &= -E_{21}^{-1/2}\left(C_2 + D_{21} B_1^T X\right) \end{aligned}$$

$$\begin{aligned} D_{c11} &= 0 \\ D_{c12} &= E_{12}^{-1/2} \\ D_{c21} &= E_{21}^{-1/2} \\ D_{c22} &= 0 \end{aligned}$$

$$\begin{aligned} E_{12} &= D_{12}^T D_{12} \\ E_{21} &= D_{21} D_{21}^T \end{aligned}$$

and the free parameter $Q(s) \in H_\infty$ is any function satisfying $\|Q(s)\|_\infty < 1$ (Doyle *et al.*, 1989).

Using Lemma 1, the parametrization of all robust stabilizing repetitive controllers so that the system in (1) is internally stable is given by following theorem.

Theorem 1. If repetitive controllers satisfying (5) exist, both (9) and (10) have solutions $X \geq 0$ and $Y \geq 0$ satisfying (11) and both

$$A - B_2 D_{12}^{\dagger} C_1 + \left(B_1 B_1^T - B_2 \left(D_{12}^T D_{12} \right)^{-1} B_2^T \right) X$$

and

$$A - B_1 D_{21}^{\dagger} C_2 + Y \left(C_1^T C_1 - C_2 \left(D_{21} D_{21}^T \right)^{-1} C_2 \right)$$

have no eigenvalue in the closed right half plane. Using X and Y, the parametrization of all robust stabilizing repetitive controllers satisfying (5) is given by

$$C(s) = C_{11}(s) + C_{12}(s) Q(s) \left(I - C_{22}(s) Q(s) \right)^{-1} C_{21}(s), \tag{14}$$

where $C_{ij}(s)(i = 1, 2; j = 1, 2)$ are given by (13) and the free parameter $Q(s) \in H_\infty$ is any function satisfying $\|Q(s)\|_\infty < 1$ and written by

$$Q(s) = \frac{Q_{n1}(s) + Q_{n2}(s) e^{-sT}}{Q_{d1}(s) + Q_{d2}(s) e^{-sT}}. \tag{15}$$

Here $Q_{n1}(s) \in RH_\infty$, $Q_{n2}(s) \in RH_\infty$, $Q_{d1}(s) \neq 0 \in RH_\infty$ and $Q_{d2}(s) \in RH_\infty$ are any functions satisfying

$$\left(Q_{d1}(0) + Q_{d2}(0) \right) - \left(Q_{n1}(0) + Q_{n2}(0) \right) C_{22}(0) = 0 \tag{16}$$

and

$$C_{12}(s) C_{21}(s) \left(Q_{n1}(s) Q_{d2}(s) - Q_{n2}(s) Q_{d1}(s) \right) \neq 0. \tag{17}$$

(Proof) First, necessity is shown. That is, if the robust repetitive controller written by (7) stabilizes the control system in (1), then $C(s)$ and $Q(s)$ are written by (14) and (15), respectively. From Lemma 1, the parametrization of all robust stabilizing controllers $C(s)$ for $G(s)$ is written by (14), where $\|Q(s)\|_\infty < 1$. In order to prove the necessity, we will show that if $C(s)$ written by (7) stabilizes the control system in (1), then the free parameter $\|Q(s)\|_\infty < 1$ in (14) is written by (15). Substituting $C(s)$ in (7) into (14), we have

$$Q(s) = \frac{Q_{n1}(s) + Q_{n2}(s) e^{-sT}}{Q_{d1}(s) + Q_{d2}(s) e^{-sT}} \tag{18}$$

where

$$Q_{n1} = \bar{C}_d(s) C_{12d}(s) C_{21d}(s) C_{22d}(s) q_d(s) \cdot \left(\hat{C}_d(s) C_{11n}(s) - \hat{C}_n(s) C_{11d}(s) \right), \tag{19}$$

$$\begin{aligned} Q_{n2}(s) &= C_{12d}(s) C_{21d}(s) C_{22d}(s) q_n(s) (\bar{C}_d(s) \hat{C}_n(s) C_{11d}(s) \\ &-\bar{C}_d(s) \hat{C}_d(s) C_{11n}(s) - \bar{C}_n(s) \hat{C}_d(s) C_{11d}(s)), \end{aligned} \tag{20}$$

$$\begin{aligned} Q_{d1}(s) &= \bar{C}_d(s) q_d(s) (C_{11n}(s) C_{22n}(s) C_{12d}(s) C_{21d}(s) \hat{C}_d(s) \\ &-C_{11d}(s) C_{22n}(s) C_{12d}(s) C_{21d}(s) \hat{C}_n(s) \\ &-C_{11d}(s) C_{22d}(s) C_{12n}(s) C_{21n}(s) \hat{C}_d(s)) \end{aligned} \tag{21}$$

and

$$\begin{aligned} Q_{d2}(s) &= \bar{C}_d(s) \hat{C}_d(s) q_n(s) (C_{11d}(s) C_{22d}(s) C_{12n}(s) C_{21n}(s) \\ &-C_{11n}(s) C_{22n}(s) C_{12d}(s) C_{21d}(s)) \\ &+C_{11d}(s) C_{22n}(s) C_{12d}(s) C_{21d}(s) q_n(s) \\ &\cdot(\bar{C}_d(s) \hat{C}_n(s) - \hat{C}_d(s) \bar{C}_n(s)) \end{aligned} \tag{22}$$

Here, $\hat{C}_n(s) \in RH_\infty$ and $\hat{C}_d(s) \in RH_\infty$ are coprime factors of $\hat{C}(s)$ on RH_∞ satisfying

$$\hat{C}(s) = \hat{C}_n(s) \hat{C}_d^{-1}(s). \tag{23}$$

$\bar{C}_n(s) \in RH_\infty$ and $\bar{C}_d(s) \in RH_\infty$ are coprime factors of $\bar{C}(s)$ on RH_∞ satisfying

$$\bar{C}(s) = \bar{C}_n(s) \bar{C}_d^{-1}(s). \tag{24}$$

$C_{11n}(s) \in RH_\infty$ and $C_{11d}(s) \in RH_\infty$ are coprime factors of $C_{11}(s)$ on RH_∞ satisfying

$$C_{11}(s) = C_{11n}(s) C_{11d}^{-1}(s). \tag{25}$$

$C_{12n}(s) \in RH_\infty$ and $C_{12d}(s) \in RH_\infty$ are coprime factors of $C_{12}(s)$ on RH_∞ satisfying

$$C_{12}(s) = C_{12n}(s) C_{12d}^{-1}(s). \tag{26}$$

$C_{21n}(s) \in RH_\infty$ and $C_{21d}(s) \in RH_\infty$ are coprime factors of $C_{21}(s)$ on RH_∞ satisfying

$$C_{21}(s) = C_{21n}(s) C_{21d}^{-1}(s). \tag{27}$$

$C_{22n}(s) \in RH_\infty$ and $C_{22d}(s) \in RH_\infty$ are coprime factors of $C_{22}(s)$ on RH_∞ satisfying

$$C_{22}(s) = C_{22n}(s) C_{22d}^{-1}(s). \tag{28}$$

From (19)$\sim$(22), all of $Q_{n1}(s)$, $Q_{n2}(s)$, $Q_{d1}(s)$ and $Q_{d2}(s)$ are included in RH_∞. Thus, we have shown that if $C(s)$ written by (7) stabilize the control system in (1), $Q(s)$ in (14) is written by (15). Since $q(0) = 1$, (16) holds true.

Next, sufficiency is shown. That is, if $C(s)$ and $Q(s) \in H_\infty$ is settled by (14) and (15), respectively, then the controller $C(s)$ is written by the form in (7) and $q(0) = 1$ holds true. Substituting (15) into (14), we have

$$C(s) = \hat{C}(s) + \bar{C}(s)\frac{q(s)e^{-sT}}{1 - q(s)e^{-sT}}, \tag{29}$$

From simple manipulation, $\hat{C}(s)$, $\bar{C}(s)$ and $q(s)$ are denoted by

$$\hat{C}(s) = \frac{C_{11}(s)Q_{d1}(s) + (C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s))Q_{n1}(s)}{Q_{d1}(s) - C_{22}(s)Q_{n1}(s)}, \tag{30}$$

$$\bar{C}(s) = \frac{C_{12}(s)C_{21}(s)}{(Q_{d1}(s) - C_{22}(s)Q_{n1}(s))} \cdot \frac{(Q_{n1}(s)Q_{d2}(s) - Q_{n2}(s)Q_{d1}(s))}{(Q_{d2}(s) - C_{22}(s)Q_{n2}(s))} \tag{31}$$

and

$$q(s) = -\frac{Q_{d2}(s) - C_{22}(s)Q_{n2}(s)}{Q_{d1}(s) - C_{22}(s)Q_{n1}(s)}. \tag{32}$$

We find that if $C(s)$ and $Q(s)$ is settled by (14) and (15), respectively, then the controller $C(s)$ is written by the form in (7). From (17), $C_2(s) \neq 0$ holds true. Substituting (16) into (32), we have $q(0) = 1$.

We have thus proved Theorem 1. ∎

Remarks 1. Even if any of the standard assumptions in (Doyle *et al.*, 1989) do not hold, if (A, B_2) is stabilizable and (A, C_2) is detectable, using the result in (Iwasaki and Skelton, 1994), we can obtain the parameterization of all stabilizing controllers $C(s)$ satisfying (5). In this case, the paremetrization of all robust repetitive controller is obtained using the same manner as proof of Theorem 1.

4. NUMERICAL EXAMPLE

In this section, a numerical example is shown to illustrate the effectiveness of the proposed method.

Let us consider to design a robust stabilizing repetitive controllers for the class of the plant $G(s)$ in (3) written by

$$G_m(s) = \frac{2}{s^3 + 2s^2 - 13s + 10} \tag{33}$$

and

$$W_T(s) = \frac{(s+6)(s+900)(s+5000)}{1.35 \times 10^9}. \tag{34}$$

The period T in (2) is $T = 4$[sec]. Solving the robust stability problem using Riccati equation based H_∞ control as Theorem 1, the parametrization of all robust stabilizing controllers $C(s)$ is obtained. In addition, we find that $C_{22}(s)$ is of minimum phase. Since $C_{22}(s)$ is of minimum phase, we settle $Q_{n1}(s)$, $Q_{n2}(s)$, $Q_{d1}(s)$ and $Q_{d2}(s)$ in (15) as

$$Q_{n1}(s) = 0, \tag{35}$$

$$Q_{n2}(s) = \frac{\bar{q}(s)}{C_{22}(s)}, \tag{36}$$

$$Q_{d1}(s) = 1 \tag{37}$$

and

$$Q_{d2} = 0, \tag{38}$$

where $\bar{q}(s)$ is written by

$$\bar{q}(s) = \frac{1}{\left(1 + \frac{s}{455}\right)\left(1 + \frac{s}{550}\right)\left(1 + \frac{s}{500}\right)}. \tag{39}$$

That is, $Q(s)$ in (15) is written by

$$Q(s) = Q_{n2}(s)e^{-sT}. \tag{40}$$

The gain plot of free parameter $Q(s)$, $\bar{q}(s)$ and $1/C_{22}(s)$ are shown in Fig. 2. Here the solid line

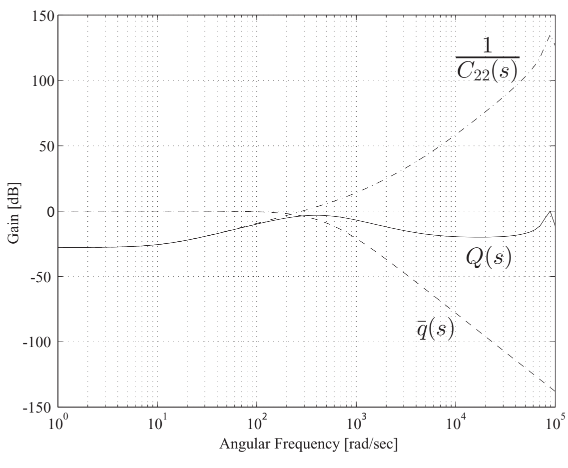

Fig. 2. Gain plot of $Q(s)$, $\bar{q}(s)$ and $1/C_{22}(s)$

shows the gain plot of $Q(s)$, the dashed line shows that of $q(s)$ and the dash-dot line shows that of $1/C_{22}(s)$. From (40) and $Q_{n2}(s) \in RH_\infty$ in (36), Fig. 2 shows that $Q(s)$ satisfies $\|Q(s)\|_\infty < 1$.

Let $\Delta(s)$ be denoted by

$$\Delta(s) = \frac{s+6}{1000}. \tag{41}$$

The gain plot of $1/\Delta(s)$ and $1/W_T(s)$ are shown in Fig. 3. Here, the solid line shows the gain plot

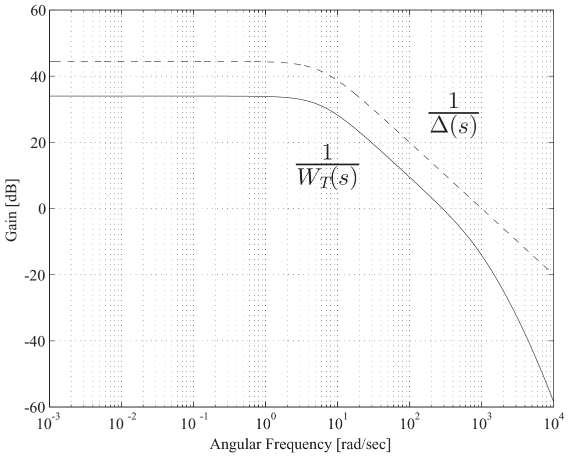

Fig. 3. Gain plot of $1/W_T(s)$ and $1/\Delta(s)$

of $1/W_T(s)$ and the dashed line shows that of $1/\Delta(s)$. Fig. 3 shows that the uncertainty $\Delta(s)$ satisfies (3).

Using the obtained robust stabilizing repetitive controller $C(s)$ in (14), the response for the reference input $r = \sin(\pi/2\, t)$ is shown in Fig. 4. Here,

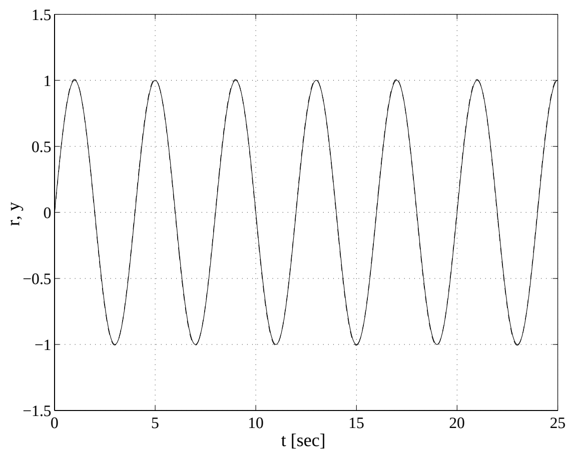

Fig. 4. Response for the reference input $r = \sin(\pi/2\, t)$

the solid line shows the response of the output y and the dotted line shows that of the reference input r. In this figure, the output y follows the reference input r with small steady state error.

In this way, a robust stabilizing repetitive controller is easily designed using Theorem 1.

5. CONCLUSIONS

In this paper, we proposed the parametrization of all robust stabilizing repetitive controllers. A numerical example is shown to illustrate the effectiveness of the obtained parametrization.

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