ADAPTIVE COMPENSATION OF BIASED SINUSOIDAL DISTURBANCES WITH UNKNOWN FREQUENCY

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Abstract: The problem of designing an output feedback compensator for any biased sinusoidal disturbance is considered. In this paper, we will develop the approach presented in (Marino *et al.*, 2003). In (Marino *et al.*, 2003) a compensator of order (2*n*+6) is proposed, which solves the posed problem by using the adaptive observers developed in (Marino and Tomei, 1995; Marino *et al.*, 2001). This problem is solved by a ($n+4$)-order compensator.

Keywords: adaptive algorithms, disturbance rejection, linear systems, decision feedback, output regulation, observers, single-input/single-output systems.

1. INTRODUCTION AND PROBLEM STATEMENT

The problem of complete rejection of external inaccessible disturbances plays an important role in the modern control theory. In this paper we consider an adaptive compensation problem of a biased sinusoidal disturbance $\overline{w}(t)=\overline{\sigma}_0+\overline{\sigma}\sin(\omega t+\overline{\phi})$ for any unknown constant value of ω, $\overline{\phi}$, $\overline{\sigma}$, $\overline{\sigma}_0$. The known approaches to the given problem can be classified depending on a priori required information about the disturbance. If the frequency ω is known, the posed problem has a classical solution (Davison, 1976; Francis and Wonham, 1975; Jonson, 1971) by modelling the disturbance as a linear exosystem and by using an observer which provides an asymptotic estimate of the disturbance so that it can be cancelled. An interesting result for a linear discrete-time control system affected by an additive sinusoidal disturbance with known frequencies but unknown amplitudes and phases was presented in the paper (Lindquist and Yakubovich, 1997). The main result of this paper concerns the existence and design of a realizable, robust optimal regulator, which is universal in the sense that it does not depend on the unknown amplitudes and phases and is optimal for all choices of such parameters.

If the frequency ω is unknown, the posed problem has been studied in a series of papers (Bodson *et al.*, 1994; Bodson and Douglas, 1997; Hsu *et al.*, 1997; Hsu *et al.*, 1999; Marino *et al.*, 2003; Mojiri and Bakhshai, 2004; Savaresi, 1997), in the case of an unbiased sinusoidal disturbance. In particular in (Bodson and Douglas, 1997), two schemes (direct one and indirect one) are presented and analyzed: while the direct scheme is used for local initial conditions of the frequency estimate, the indirect one (Hsu *et al.*, 1997; Hsu *et al.*, 1999), can be used for larger initial conditions; on the other hand only the direct scheme guarantees exact disturbance compensation. In paper (Savaresi, 1997) a new class of filters (of fourth order) was presented. These filters enable one to estimate harmonic signals with enhanced tracking capability.

In this paper, we will develop the approach presented in (Marino *et al.*, 2003). In (Marino *et al.*, 2003) a compensator of order (2*n*+6) is proposed, which solves the posed problem by using the adaptive observers developed in (Marino and Tomei, 1995; Marino *et al.*, 2001).

Consider the linear single-input, single-output observable system (Marino *et al.*, 2003)

$$\dot{z} = \begin{bmatrix} -a_{n-1} & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -a_1 & 0 & \dots & 1 \\ -a_0 & 0 & \dots & 0 \end{bmatrix} z + \begin{bmatrix} b_{n-1} \\ b_{n-2} \\ \vdots \\ b_0 \end{bmatrix} (u + \overline{w}) =$$

$$= Fz + g(u + \overline{w}), \qquad (1)$$

$$y = [1 \quad 0 \quad \dots \quad 0]z = hz \qquad (2)$$

in which $x \in \Re^n$ is a state, $u \in \Re$ is a control, $\overline{w} \in \Re$ is a modeled disturbance; the output $y \in \Re$, which is the only measured variable, is required to be regulated to zero. The disturbance input $\overline{w}$ is modeled as

$$\overline{w}(t) = \overline{\sigma}_0 + \overline{\sigma}\sin(\omega t + \overline{\phi}), \qquad (3)$$

that is a biased sinusoid of unknown constant magnitude $\overline{\sigma} > 0$, unknown frequency $\omega > 0$, unknown phase $\overline{\phi}$ and unknown bias $\overline{\sigma}_0$.

Consider the following assumptions (Marino *et al.*, 2003).

Assumption 1. All coefficients a_i, b_i $0 \le i \le n-1$ are known.
Assumption 2. The polynomial $a(p) = p^n + a_{n-1}p^{n-1} + \dots + a_1 p + a_0$ has all its roots with negative real part.
Assumption 3. The polynomial $b(p) = b_{n-1}p^{n-1} + \dots + b_1 p + b_0$ has no roots on the imaginary axis.

We perform a series of modeling changes. Consider an input-state-output model (1), (2) in the input-output form

$$y(t) = \frac{b(p)}{a(p)}(u(t) + \overline{w}(t)). \qquad (4)$$

Multiplying both parts of the equation (4) by $(p+\beta)$, we obtain

$$(p+\beta)y = \frac{(p+\beta)b(p)}{a(p)}\left[u + \overline{w}\right] = \overline{u} + w, \qquad (5)$$

where $\beta > 0$ and

$$\overline{u} = \frac{(p+\beta)b(p)}{a(p)}u, \qquad (6)$$

$$w = \frac{(p+\beta)b(p)}{a(p)}\overline{w}. \qquad (7)$$

Model (5) can be written as

$$\dot{y} = -\beta y + \overline{u} + w. \qquad (8)$$

It is easy to see that signal $w(t)$ is biased sinusoidal disturbance and we can rewrite

$$w(t) = \sigma_0 + \sigma\sin(\omega t + \phi).$$

Now we will define the purpose of control as the solution of the problem of an algorithm design which at any initial conditions of the plant ensures the following:

$$\lim_{t\to\infty}|y(t)| = 0. \qquad (9)$$

We shall derive the solution of the problem in two stages. First, assuming, that the signal $w(t)$ is measured (see the equation (7)), we will construct an observer of the disturbance $w(t)$. Further, using results of the first stage, we will solve a complex problem of rejection of external inaccessible disturbance $w(t) = \sigma_0 + \sigma\sin(\omega t + \phi)$ for model (8).

2. DESIGN OF THE DISTURBANCE OBSERVER

In this section, assuming that the signal $w(t)$ is measured, we will construct the observer that ensures the following purpose:

$$\lim_{t\to\infty}|w(t) - \hat{w}(t)| = 0, \qquad (10)$$

where $\hat{w}(t)$ is the estimation of the signal $w(t)$.

It is easy to show that the biased sinusoidal disturbance $w(t) = \sigma_0 + \sigma\sin(\omega t + \phi)$ is modeled by the third-order linear exosystem

$$\begin{cases} \dot{x}_1^* = x_2^*, \\ \dot{x}_2^* = x_3^*, \\ \dot{x}_3^* = -\theta x_2^*, \\ w(t) = x_1^* \end{cases} \qquad (11)$$

or

$$\dot{x}^* = A_0 x^* - d\theta x_2^* = Ax^*, \qquad (12)$$

where vector $x^*(t) = [x_1^* \quad x_2^* \quad x_3^*]^T$, $A_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $d = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\theta & 0 \end{bmatrix}$ and the parameter $\theta = \omega^2$ is unknown.

Note that construction an observer based on measurements of the variable $w(t) = x_1^*$ for the system (11), (12) is quite complicated. The difficulties are caused by immeasurability of the state variable vector $x^* = [x_1^* \quad x_2^* \quad x_3^*]$, and the fact that the parameter $\theta = \omega^2$ is undefined. We will transform model (11), (12) into one convenient for the synthesis of the observer. We introduce a matrix coordinate transformation

$$T = k_1 I + k_2 A + k_3 A^2, \qquad (13)$$

coefficients k_1, k_2 also k_3 are strictly positive.

It is known (Bobtsov and Lyamin, 2000; Bobtsov *et al.*, 2002), that for non-singular matrix $T = k_1 I + k_2 A + k_3 A^2$ there is the coordinate transformation $x^* = Tx$ which turns the system (11) into an equivalent model of the following kind

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \dot{x}_3 = -\theta x_2, \end{cases} \qquad (14)$$

$$w(t) = k_1 x_1 + k_2 x_2 + k_3 x_3. \qquad (15)$$

Thus, model (14), (15) can be used as generator of the biased sinusoidal disturbance $w(t) = \sigma_0 + \sigma \sin(\omega t + \phi)$. Now we will show, that with any strictly positive values of k_1, k_2, k_3 and θ the matrix of coordinate transformation T is non-singular. To do it, we will insert the corresponding components into the equation (13) and we will find a determinant of the matrix T

$$T = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_1 & 0 \\ 0 & 0 & k_1 \end{bmatrix} + \begin{bmatrix} 0 & k_2 & 0 \\ 0 & 0 & k_2 \\ 0 & -\theta k_2 & 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 & 0 & k_3 \\ 0 & -\theta k_3 & 0 \\ 0 & 0 & -\theta k_3 \end{bmatrix} = \begin{bmatrix} k_1 & k_2 & k_3 \\ 0 & k_1 - \theta k_3 & k_2 \\ 0 & -\theta k_2 & k_1 - \theta k_3 \end{bmatrix},$$

$$\det T = k_1 \det \begin{bmatrix} k_1 - \theta k_3 & k_2 \\ -\theta k_2 & k_1 - \theta k_3 \end{bmatrix} =$$

$$= k_1 (k_1 - \theta k_3)^2 + k_1 k_2^2 \theta .$$

It is obvious that $\det T \neq 0$ with any strictly positive k_1, k_2, k_3 and θ.

Now we will start synthesizing the observer of the biased sinusoidal disturbance $w(t)$. We will present the estimation algorithm as

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2, \\ \dot{\hat{x}}_2 = \hat{x}_3, \\ \dot{\hat{x}}_3 = -\hat{\theta}\, \hat{x}_2 + u_x, \end{cases} \qquad (16)$$

$$\hat{w}(t) = k_1 \hat{x}_1 + k_2 \hat{x}_2 + k_3 \hat{x}_3, \qquad (17)$$

or

$$\dot{\hat{x}} = A_0 \hat{x} - d\hat{\theta}\, \hat{x}_2 + du_x, \qquad (18)$$

$$\hat{w} = k^T \hat{x} \qquad (19)$$

where the matrix $A_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $d = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, u_x is the control that solves the problem of observance, $k^T = [k_1 \quad k_2 \quad k_3]$, $\hat{\theta}(t)$ is an estimation of the unknown parameter θ.

To calculate the control u_x we will introduce vector

$$\tilde{x} = x - \hat{x}, \qquad (20)$$

and the parametric error

$$\tilde{\theta} = \theta - \hat{\theta}. \qquad (21)$$

Differentiating the equation (20), we obtain

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}} = A_0 x - d\theta x_2 - A_0 \hat{x} + d\hat{\theta}\, \hat{x}_2 - du_x =$$

$$= A_0 (x - \hat{x}) + d[-(\tilde{\theta} + \hat{\theta})(\tilde{x}_2 + \hat{x}_2) + \hat{\theta}\, \hat{x}_2] - du_x =$$

$$= A_0 \tilde{x} - d\theta \tilde{x}_2 - d\tilde{\theta}\, \hat{x}_2 - du_x. \qquad (22)$$

Let us choose $u_x = \tilde{w} = k^T x - k^T \hat{x} = k^T \tilde{x}$, then the equation (22) will become

$$\dot{\tilde{x}} = A_0 \tilde{x} - d\theta \tilde{x}_2 - d\tilde{\theta}\, \hat{x}_2 - dk^T \tilde{x} =$$

$$= A_c \tilde{x} - d\tilde{\theta}\, \hat{x}_2, \qquad (23)$$

where the closed-loop system matrix is $A_c = A_0 - d\theta - dk^T$.

Let us assume that an algorithm of the estimation of unknown parameter θ is the following:

$$\dot{\hat{\theta}} = -k_a \hat{x}_2 \tilde{w}, \qquad (24)$$

where $k_a > 0$.

Then differentiating the parametric error (21), we obtain

$$\dot{\tilde{\theta}} = \dot{\theta} - \dot{\hat{\theta}} = k_a \hat{x}_2 \tilde{w}. \qquad (25)$$

As it is known from the classical theory of adaptive systems (Fomin *et al.*, 1981; Landau, 1979; Narendra and Annaswamy, 1989), the equilibrium $\tilde{x} = 0$ of the system (23), (25) is asymptotically stable if the transfer function $W(p) = k^T (pI - A_c)^{-1} d$ is strictly positive real, i.e.

- A_c is a Hurwitz matrix;
- $\mathrm{Re}\, W(j\omega) > 0$ for all $-\infty < \omega < \infty$;
- $\lim_{\omega \to \infty} \mathrm{Re}\, \omega^2 W(j\omega) > 0$.

Thus, for condition (10) it is necessary that the equilibrium position $\tilde{x} = 0$ is asymptotically stable, which in turn is feasible with strictly positive real of the transfer function $W(p) = k^T (pI - A_c)^{-1} d$.

Proposition. Let coefficients $k_1 = \alpha_0^3$, $k_2 = 3\alpha_0^2$ and $k_3 = 3\alpha_0$, where number $\alpha_0 > 0$. Then the transfer function $W(p) = k^T (pI - A_c)^{-1} d$ is strictly positive real.

Proof. First we will show that with $\alpha_0 > 0$ the following $A_c = A_0 - d\theta - dk^T$ is a Hurwitz matrix. To do it, we will consider Hurwitz' stability criterion, which for a third-order system looks like this:

$$k_3 k_2 + k_3 \theta \ge k_1 . \qquad (26)$$

Substituting into the inequality (26) parameters $k_1 = \alpha_0^3$, $k_2 = 3\alpha_0^2$ and $k_3 = 3\alpha_0$, we obtain

$$9\alpha_0^3 + 3\alpha_0 \theta \ge \alpha_0^3 \text{ for all } \alpha_0 > 0 \text{ and } \theta > 0 ,$$

whence it follows that $A_c = A_0 - d\theta - dk^T$ is a Hurwitz matrix.

Now we will show that frequency inequalities $\mathrm{Re}\, W(j\omega) > 0$ are true for all $-\infty < \omega < \infty$ and $\lim_{\omega \to \infty} \mathrm{Re}\, \omega^2 W(j\omega) > 0$. For this purpose we will consider the transfer function

$$W(p) = k^T (pI - A_c)^{-1} d = \\ = \frac{k_3 p^2 + k_2 p + k_1}{p^3 + k_3 p^2 + (k_2 + \theta) p + k_1}$$

and substituting $p = j\omega$ and allocating a real part, we obtain

$$\mathrm{Re}\, W(j\omega) = \frac{(k_1 - k_3\omega^2)^2 + (k_2 + \theta - \omega^2) k_2 \omega^2}{(k_1 - k_3\omega^2)^2 + (k_2 + \theta - \omega^2)^2 \omega^2} .$$

Substituting $k_1 = \alpha_0^3$, $k_2 = 3\alpha_0^2$ and $k_3 = 3\alpha_0$ into the last equation, we obtain

$$\mathrm{Re}\, W(j\omega) = \frac{(k_1 - k_3\omega^2)^2 + (k_2 + \theta - \omega^2) k_2 \omega^2}{(k_1 - k_3\omega^2)^2 + (k_2 + \theta - \omega^2)^2 \omega^2} =$$

$$= \frac{(\alpha_0^3 - 3\alpha_0\omega^2)^2 + (3\alpha_0^2 + \theta - \omega^2) 3\alpha_0^2 \omega^2}{(\alpha_0^3 - 3\alpha_0\omega^2)^2 + (3\alpha_0^2 + \theta - \omega^2)^2 \omega^2} =$$

$$= \frac{\alpha_0^6 + 3\alpha_0^4\omega^2 + 3\alpha_0^2\omega^2\theta + 6\alpha_0^2\omega^4}{(\alpha_0^3 - 3\alpha_0\omega^2)^2 + (3\alpha_0^2 + \theta - \omega^2)^2 \omega^2} .$$

Thus the condition $\mathrm{Re}\, W(j\omega) > 0$ is true for all $-\infty < \omega < \infty$, $\alpha_0 > 0$ and $\theta > 0$. It is now easy to show that $\lim_{\omega \to \infty} \mathrm{Re}\, \omega^2 W(j\omega) > 0$ for all $\alpha_0 > 0$ and $\theta > 0$, which in turn, with $A_c = A_0 - d\theta - dk^T$ being a Hurwitz matrix, with $\mathrm{Re}\, W(j\omega) > 0$ for all $-\infty < \omega < \infty$ and $\lim_{\omega \to \infty} \mathrm{Re}\, \omega^2 W(j\omega) > 0$, ensures the strictly positive real of the transfer function $W(p) = k^T (pI - A_c)^{-1} d$.

3. COMPENSATION OF DISTURBANCE

Let us consider the modified equation of the plant of type (8)

$$\dot{y} = -\beta y + \bar{u} + w . \qquad (27)$$

From the equation (27) it is easy to see that $y(t)$ tends to zero if the control $\bar{u} = -w$.

But the control $\bar{u} = -w$ is an ideal situation; in reality we deal with $\bar{u} = -\hat{w}$ and with the unknown $\tilde{w}$. Because the observer (16), (17), (24) makes an estimation of the disturbance $\hat{w}(t) \to w(t)$ with $t \to \infty$ it is reasonable to choose the compensatory control as follows

$$\bar{u} = -\hat{w}$$

and from the equation (6)

$$u = -\frac{a(p)}{(p+\beta) b(p)} \hat{w} .$$

Then the equation (27) will become

$$\dot{y} = -\beta y + \tilde{w} . \qquad (28)$$

It is easy to see that for

$$\lim_{t \to \infty} \tilde{w}(t) = 0$$

we obtain

$$\lim_{t \to \infty} |y(t)| = 0 .$$

However, due the conditions of the problem statement the disturbance $w(t)$ is not measured, and, hence, $\tilde{w}(t)$ is not known, the observer (16), (17), (24) can not be achieved. We will build an achievable observance scheme. In the equation (28) we determine

$$\tilde{w} = \beta y + \dot{y}. \qquad (29)$$

Substituting the equation (29) into (18), we obtain

$$\dot{\hat{x}} = A_0\hat{x} - d\hat{\theta}\hat{x}_2 + (d\dot{y} + d\beta y). \qquad (30)$$

Let $\xi = \hat{x} - dy$, then

$$\begin{cases} \dot{\xi} = \dot{\hat{x}} - d\dot{y} = A_0\hat{x} - d\hat{\theta}\hat{x}_2 + d\beta y, \\ \hat{x} = \xi + dy. \end{cases} \qquad (31)$$

Substituting the equation (29) in (24), we obtain

$$\dot{\hat{\theta}} = -k_a\hat{x}_2\tilde{w} = -k_a\hat{x}_2(\dot{y} + \beta y) = \\ = -k_a\hat{x}_2\dot{y} - k_a\hat{x}_2\beta y. \qquad (32)$$

Introduce a new variable

$$\eta = \hat{\theta} + k_a\hat{x}_2 y. \qquad (33)$$

Differentiating the equation (33), we obtain

$$\dot{\eta} = \dot{\hat{\theta}} + k_a\dot{\hat{x}}_2 y + k_a\hat{x}_2\dot{y} = \\ = -k_a\hat{x}_2\dot{y} - k_a\hat{x}_2\beta y + k_a\dot{\hat{x}}_2 y + k_a\hat{x}_2\dot{y}, \qquad (34)$$

where $\dot{\hat{x}}_2 = \hat{x}_3$ is measured.

From the expression (34) we have an achievable scheme for estimation of parameter $\hat{\theta}$

$$\begin{cases} \dot{\eta} = -k_a\hat{x}_2\beta y + k_a\hat{x}_3 y, \\ \hat{\theta} = \eta - k_a\hat{x}_2 y. \end{cases} \qquad (35)$$

Thus the achievable observance scheme is presented in the equations (31), (35).

4. EXAMPLE

Let us consider the following input-output form

$$y(t) = \frac{b(p)}{a(p)}(u(t) + \overline{w}(t)), \qquad (36)$$

where $a(p) = p^2 + 2p + 1$ and $b(p) = p + 2$. Choose the control according to the section 3

$$u = -\frac{(p^2 + 2p + 1)}{(p+3)(p+2)}\hat{w},$$

where $\beta = 3$ and the estimation algorithm in the following

$$\begin{cases} \dot{\xi}_1 = \hat{x}_2, \\ \dot{\xi}_2 = \hat{x}_3, \\ \dot{\xi}_3 = -\hat{\theta}\hat{x}_2 + 3y, \end{cases} \qquad (37)$$

$$\hat{w}(t) = k_1\hat{x}_1 + k_2\hat{x}_2 + k_3\hat{x}_3, \qquad (38)$$

$$\begin{cases} \hat{x}_1 = \xi_1, \\ \hat{x}_2 = \xi_2, \\ \hat{x}_3 = \xi_3 + y, \end{cases} \qquad (39)$$

and scheme for estimation of parameter $\hat{\theta}$

$$\begin{cases} \dot{\eta} = -\hat{x}_2 y + \hat{x}_3 y, \\ \hat{\theta} = \eta - \hat{x}_2 y. \end{cases} \qquad (40)$$

The results of a computer simulation for variables $y(t)$, $\hat{\theta}(t)$, $u(t)$ and $\overline{w}(t)$ for the case $k_a = 1$, $\hat{\theta}(0) = 0$ and the disturbance $\overline{w}(t) = 3 + 5\sin(2t+1)$ are presented in Fig. 1, 2 and 3. We can see that presented control provides $\lim_{t\to\infty}|y(t)| = 0$.

5. CONCLUSION

A $(n+4)$-order compensator has been designed to reject a biased sinusoidal disturbance with unknown bias, phase, amplitude and frequency in a known linear, asymptotically stable system. This approach developed result (Marino *et al.*, 2003) in the following way

- the range of values of frequency ω is unknown (in work (Marino *et al.*, 2003) the lower bound of ω is known);
- the structure of the regulator is simple compared with that in (Marino *et al.*, 2003);
- the order of the regulator is $n+4$, which is lower than its counterpart in (Marino *et al.*, 2003), the order of the regulator in (Marino *et al.*, 2003) is $2n+6$.

6. ACKNOWLEDGEMENT

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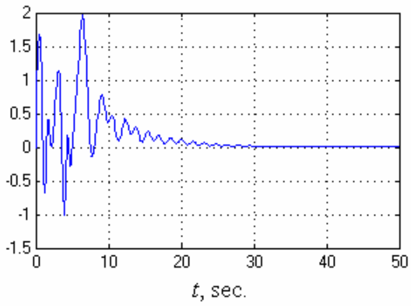

Fig. 1. Transients in control system (36) – (40) for variable $y(t)$.

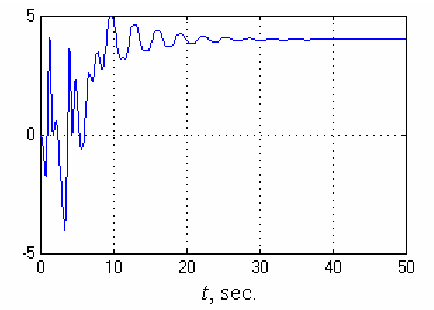

Fig. 2. Transients in control system (36) – (40) for variable $\hat{\theta}(t)$.

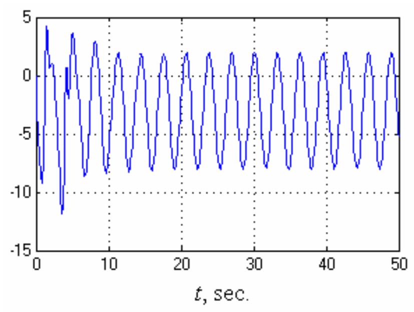

Fig. 3. Transients in control system (36) – (40) for variable $u(t)$.

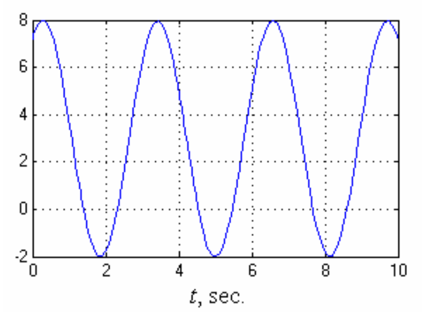

Fig. 4. Transients in control system (36) – (40) for variable $\overline{w}(t)$.

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