RBF-ARX MODEL-BASED ROBUST MPC FOR NONLINEAR SYSTEMS

Hui Peng*, Weihua Gui*, Runmin Zou*, Rafi Youssef*,
Zi-Jiang Yang, and Hideo Shioya †**

** School of Information Science & Engineering, Central South University, Changsha, Hunan, 410083, China. E-mail: huipeng@mail.csu.edu.cn*
*** Department of Electrical and Electronic Systems Engineering, Kyushu University, Fukuoka, 812-8581, Japan.*
† Bailey Japan Co. Ltd., 511 Baraki, Nirayama-cho, Tagata-gun, Shizuoka, 410-2193, Japan.

Abstract: An integrated modeling and robust model predictive control (MPC) approach is proposed for a class of nonlinear systems. First, the nonlinear system is identified off-line by a RBF-ARX model possessing linear ARX model structure and state-dependent Gaussian RBF neural network type coefficients. On the basis of the RBF-ARX model, a combination of a local linearization model and a polytopic uncertain linear parameter-varying (LPV) model are built to approximate the present and the future system's nonlinear behavior respectively. Subsequently, based on the approximate models, a min-max robust MPC algorithm with input constraint is designed for the nonlinear systems. The closed loop stability of the MPC strategy is guaranteed by the use of parameter-dependent Lyapunov function and the feasibility of the linear matrix inequalities (LMIs). Simulation study to a NOx decomposition process illustrates the effectiveness of the modeling and robust MPC approaches proposed in this paper.

Keywords: Nonlinear systems, model predictive control, radial basis function networks, ARX model, robustness, stability, linear matrix inequalities.

1. INTRODUCTION

There have been many reports on the research and application of nonlinear model predictive control (NMPC), in which the control schemes (*e.g.* Mahfouf and Linkens, 1998, Sentoni, *et al.,* 1996) were based on the direct use of nonlinear models, but they involved the on-line solution of a higher order nonlinear optimization problem with constraints, which is usually computationally expensive and may be even unable to guarantee to get a feasible solution in real time control. Another kind of approach used a piecewise linearization technique (*e.g.* Bloemen, *et al.,* 2001, Prasad, *et al.,* 1998) or a polytoipc uncertain LPV (linear parameter varying) model (*e.g.* Kothare, *et al.,* 1996, Lu and Arkun, 2002, Cuzzola, *et al.,* 2002) to describe the nonlinear behavior of a system, so that the model was linearized in each sampling interval. This resulted in the solution of one or more quadratic programming problems or LMIs (linear matrix inequalities) at each such interval, as in case of linear MPC. Nevertheless, the identification experiment of many linear models which are valid only in each small region is not an easy work in practice. Besides, the NMPC designs based on an on-line estimated affine model representing a nonlinear plant, such as the neural network predictor (Liu, *et al.,* 1998) and the quasi-linear autoregressive model (Lakhdari, *et al.,* 1995) were also reported. However, fast and accurate online estimation of a complicated model providing a good fit to a nonlinear process may be difficult in real application.

For a class of smooth nonlinear processes whose operating-point changes with time and its dynamic behavior may be represented by a linear model at each operating-point, the RBF-ARX model and its parameter optimization method (Peng, *et al.,* 2003a) was proposed in order to effectively characterize such nonlinear systems. The RBF-ARX model is a kind of hybrid pseudo-linear time-varying model which is composed of Gaussian radial basis function (RBF) neural networks and linear ARX model structure. The offline identified RBF-ARX model-based nonlinear MPC has been investigated both in simulation and in real industrial

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application (Peng, *et al.,* 2002, 2003b, c, 2004) where the satisfactory nonlinear modeling accuracy and significant effectiveness and feasibility of the proposed NMPC were verified. Furthermore, some stability conclusions on the offline identified RBF-ARX model-based nonlinear MPC were also given in Peng, *et al.* (2003b, c).

This paper presents an integrated modeling and robust MPC approach for nonlinear systems on the basis of the RBF-ARX modeling technique. First, the nonlinear system is identified off-line by a RBF-ARX model. From the identified RBF-ARX model, a local linearization model is obtained to represent the current behavior of the nonlinear system being controlled, and a polytopic uncertain LPV model is built to approximate the future system's nonlinear behavior. Subsequently, based on two approximate models, a min-max robust MPC algorithm with constraint is designed for the nonlinear systems. The closed loop stability of the MPC strategy is guaranteed by the use of parameter-dependent Lyapunov function and the feasibility of the linear matrix inequalities. Simulation study to a NOx decomposition process illustrates the effectiveness of the integrated modeling and robust MPC approach proposed in this paper.

2. NONLINEAR SYSTEM APPROXIMATION BY RBF-ARX MODEL

Consider a SISO nonlinear system

$$y(t+1) = f\left(\mathbf{W}(t)\right) \tag{1}$$

$$\mathbf{W}(t) = \left[y(t),\cdots, y(t-n_y+1), u(t),\cdots,u(t-n_u+1)\right]^{\mathrm{T}} \tag{2}$$

where $y(t)$ is the output, $u(t)$ is the input. If the function $f(\bullet)$ in (1) is continuously differentiable at an arbitrary equilibrium point, nonlinear system (1) may be then approximated by the following RBF-ARX model (Peng, *et al.,* 2003a, 2004)

$$\begin{cases} y(t+1) = \pi_0\left(\mathbf{W}(t)\right) + \sum_{i=0}^{n_y-1} \pi_{y,i}\left(\mathbf{W}(t)\right) y(t-i) \\ \qquad\qquad + \sum_{i=0}^{n_u-1} \pi_{u,i}\left(\mathbf{W}(t)\right) u(t-i) + \xi(t+1) \\ \pi_0\left(\mathbf{W}(t)\right) = c_0^0 + \sum_{k=1}^{m} c_k^0 \exp\left[-\lambda_{y,k}^2 \left\|\mathbf{W}(t)-\mathbf{Z}_k^y\right\|_2^2\right] \\ \pi_{j,i}\left(\mathbf{W}(t)\right) = c_{i,0}^j + \sum_{k=1}^{m} c_{i,k}^j \exp\left[-\lambda_{j,k}^2 \left\|\mathbf{W}(t)-\mathbf{Z}_k^j\right\|_2^2\right] \\ \mathbf{Z}_k^j = \left[z_{k,1}^j \quad z_{k,2}^j \quad \cdots \quad z_{k,n_w}^j \right]^{\mathrm{T}},\ j=y,u \end{cases} \tag{3}$$

where n_y, n_u, m, and n_w are the orders; c_k^0 s and $c_{i,k}^j$ s are the scalar weighting coefficients; $\mathbf{Z}_k^j$ s are the centers of Gaussian RBF neural networks; $\lambda_{j,k}$ s are the scaling parameters; $\|\cdot\|_2$ denotes the vector 2-norm, and $\xi(t)$ denotes noise usually regarded as a Gaussian white noise independent of the observations. The signal $\mathbf{W}(t)$ in (3) is a process variable causing the operating-point of the system to change with time. $\mathbf{W}(t)$ has direct or indirect relation with input or output of the system, in some cases probably being just the input or/and output itself. For example, in a nonlinear thermal power plant, $\mathbf{W}(t)$ may be the load demand of the plant.

RBF-ARX model (3) is constructed as a global model for MPC design, and it is identified off-line from observation data in order to avoid the potential problem caused by the failure of on-line parameter estimation during real time control. It is easy to see that the local linearization of model (3) is a linear ARX model at each operating-point by fixing $\mathbf{W}(t)$ at time t in (3). It is natural and appealing to interpret model (3) as a locally linear ARX model in which the evolution of the process at time t is governed by a set of AR coefficients $\{\pi_{y,i}\ ,\ \pi_{u,i}\}$, and a local mean π_0, all of which depend on the 'operating-point' of the process at time t. Model (3) deals with a nonlinear process by splitting the state space up into a large number of small segments, and regarding the process as locally linear within each segment. Because of the satisfactory properties of RBF networks in function approximation as well as in learning local variation, the use of the operating-point dependent functional coefficients makes the RBF-ARX model capable of effectively representing the dynamics of the system at each operating-point. The RBF-ARX model incorporates the advantages of the state-dependent ARX model in nonlinear dynamics description and the RBF networks in function approximation. In general, the model does not need many RBF centres compared with a single RBF network model, because the complexity of the model is dispersed into the lags of the autoregressive parts of the model (Peng, *et al.,* 2003a, 2004).

It is easy to rewrite model (3) as the following polynomial form

$$y(t+1) = \sum_{i=0}^{n-1} a_{i,t} y(t-i) + \sum_{i=0}^{n-1} b_{i,t} u(t-i) + \pi_{0,t} + \xi(t+1) \tag{4}$$

Where $n = \max\{n_y, n_u\}$, $\pi_{0,t} = \pi_0\left(\mathbf{W}(t)\right)$

$$\begin{cases} a_{i,t} = \begin{cases} c_{i,0}^y + \sum_{k=1}^{m} c_{i,k}^y \exp\left[-\lambda_{y,k}^2 \left\|\mathbf{W}(t)-\mathbf{Z}_k^y\right\|_2^2\right],\ i \le n_y \\ 0\ ,\ i > n_y \end{cases} \\ b_{i,t} = \begin{cases} c_{i,0}^u + \sum_{k=1}^{m} c_{i,k}^u \exp\left[-\lambda_{u,k}^2 \left\|\mathbf{W}(t)-\mathbf{Z}_k^u\right\|_2^2\right],\ i \le n_u \\ 0\ ,\ i > n_u \end{cases} \end{cases} \tag{5}$$

Let y_r be the desired output, and define the deviation variables below

$$\begin{aligned} \bar{y}(t+i) &= y(t+i) - y_r,\quad i = 1,-1,-2,\cdots \\ \bar{u}(t+j) &= u(t+j) - u(t+j-1),\quad j = 0,-1,-2,\cdots \end{aligned} \tag{6}$$

From (4) and (6), we may obtain the output prediction $\bar{y}(t+1\,|\,t)$ given by

$$\bar{y}(t+1\,|\,t) = \sum_{i=0}^{n-1} a_{i,t}\bar{y}(t-i) + \sum_{i=0}^{n-1} b_{i,t}\bar{u}(t-i) + \psi_t \tag{7}$$

$$\psi_t = -y_r + \sum_{i=0}^{n-1} a_{i,t} y_r + \sum_{i=0}^{n-1} b_{i,t} u(t-i-1) + \pi_{0,t} \tag{8}$$

The absolute value of ψ_t defined in (8), *i.e.* $|\psi_t|$, may be regarded as an index of describing whether the system goes into steady-state, because $|\psi_t|$ should be zero if the input $\{u(t)\}$ is perfect and the output $\{y(t)\}$ is stabilized on y_r under steady-state. In terms of the meaning of ψ_t, and from (7) one may use the following linear time-varying model (9) to approximate the future nonlinearities of the system and to make $|\psi_{t+j|t}|$ be zero by designing a set of 'perfect' inputs $\{\bar{u}(t|t), \bar{u}(t+1|t), \bar{u}(t+2|t), \cdots\}$:

$$\begin{aligned}\bar{y}(t+j+1|t) &= \sum_{i=0}^{n-1} a_{i,t+j}\bar{y}(t+j-i|t) \\ &\quad + \sum_{i=0}^{n-1} b_{i,t+j}\bar{u}(t+j-i|t), \quad j \geq 1\end{aligned} \tag{9}$$

where $\begin{cases}\bar{y}(t+j-i|t) = \bar{y}(t+j-i), & j \leq i \\ \bar{u}(t+j-i|t) = \bar{u}(t+j-i), & j \leq i\end{cases}$

From (5), one can see that the coefficients $a_{i,t+j}$ and $b_{i,t+j}$ in model (9) usually could not be obtained at time t, but their varying zones are known from (5). From the polynomial model (7) and (9), a state-space model can be built by defining a state vector given below:

$$\begin{cases}\mathbf{X}(t+j|t) = \left[x_{1,t+j|t} \;\; x_{2,t+j|t} \;\; \cdots \;\; x_{n,t+j|t}\right]^{\mathrm{T}} \\ x_{1,t+j|t} = \bar{y}(t+j|t) \\ x_{k,t+j|t} = \sum_{i=1}^{n+1-k} a_{i+k-1,t+j-1}\bar{y}(t+j-i|t) \\ \qquad + \sum_{i=1}^{n+1-k} b_{i+k-1,t+j-1}\bar{u}(t+j-i|t) \\ k = 2,3,\cdots,n; \;\; j = 0,1,2,\cdots\end{cases} \tag{10}$$

The state-space model corresponding to model (7) and (9) can be then respectively given by

$$\begin{cases}\mathbf{X}(t+1|t) = \mathbf{A}_t\mathbf{X}(t|t) + \mathbf{B}_t\bar{u}(t|t) + \mathbf{\Xi}(t) \\ \mathbf{A}_t = \begin{bmatrix} a_{1,t} & 1 & 0 & \cdots & 0 \\ a_{2,t} & 0 & 1 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ a_{n-1,t} & 0 & 0 & \cdots & 1 \\ a_{n,t} & 0 & 0 & \cdots & 0 \end{bmatrix}, \; \mathbf{B}_t = \begin{bmatrix} b_{1,t} \\ b_{2,t} \\ \vdots \\ b_{n,t} \end{bmatrix} \\ \mathbf{\Xi}(t) = [\psi_t \;\; 0 \;\; \cdots \;\; 0]^{\mathrm{T}}\end{cases} \tag{11}$$

And

$$\begin{cases}\mathbf{X}(t+j+1|t) = \mathbf{A}_{t+j|t}\mathbf{X}(t+j|t) + \mathbf{B}_{t+j|t}\bar{u}(t+j|t) \\ \mathbf{A}_{t+j|t} = \begin{bmatrix} a_{1,t+j|t} & 1 & 0 & \cdots & 0 \\ a_{2,t+j|t} & 0 & 1 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ a_{n-1,t+j|t} & 0 & 0 & \cdots & 1 \\ a_{n,t+j|t} & 0 & 0 & \cdots & 0 \end{bmatrix}, \; \mathbf{B}_{t+j|t} = \begin{bmatrix} b_{1,t+j|t} \\ b_{2,t+j|t} \\ \vdots \\ b_{n,t+j|t} \end{bmatrix} \\ j \geq 1\end{cases} \tag{12}$$

Notice that the state $\mathbf{X}(t|t)$, $\mathbf{\Xi}(t)$ and the state-matrices $[\mathbf{A}_t, \mathbf{B}_t]$ in (11) can be obtained by (10), (8), (5) and (11) on the basis of the measured input/output data and the offline identified RBF-ARX model (4). Furthermore, according to RBF-ARX model (4-5) and model (12), it can be shown that the future state-matrices $[\mathbf{A}_{t+j|t}, \mathbf{B}_{t+j|t}] (j \geq 1)$ in (12) belong to two convex polytopic sets respectively, which are given by

$$\mathbf{\Omega}_A := \left\{\begin{aligned} &\mathbf{A}_{t+j|t} : \mathbf{A}_{t+j|t} = \sum_{i=1}^{L_m} \alpha_i(t+j|t)\mathbf{A}_i, \\ &\sum_{i=1}^{L_m} \alpha_i(t+j|t) = 1, \; \alpha_i(t+j|t) \geq 0 \end{aligned}\right\} \tag{13}$$

$$\mathbf{\Omega}_B := \left\{\begin{aligned} &\mathbf{B}_{t+j|t} : \mathbf{B}_{t+j|t} = \sum_{i=1}^{L_m} \beta_i(t+j|t)\mathbf{B}_i, \\ &\sum_{i=1}^{L_m} \beta_i(t+j|t) = 1, \; \beta_i(t+j|t) \geq 0 \end{aligned}\right\} \tag{14}$$

where

$$L_m = L_{m-1} + m; \; m = 1,2,\cdots; \; L_0 = 1$$

$$\mathbf{A}_1 = \begin{bmatrix} c_{1,0}^y & 1 & 0 & \cdots & 0 \\ c_{2,0}^y & 0 & 1 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ c_{n-1,0}^y & 0 & 0 & \cdots & 1 \\ c_{n,0}^y & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\mathbf{A}_2 = \begin{bmatrix} c_{1,0}^y + c_{1,1}^y & 1 & 0 & \cdots & 0 \\ c_{2,0}^y + c_{2,1}^y & 0 & 1 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ c_{n-1,0}^y + c_{n-1,1}^y & 0 & 0 & \cdots & 1 \\ c_{n,0}^y + c_{n,1}^y & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\mathbf{A}_3 = \begin{bmatrix} c_{1,0}^y + c_{1,2}^y & 1 & 0 & \cdots & 0 \\ c_{2,0}^y + c_{2,2}^y & 0 & 1 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ c_{n-1,0}^y + c_{n-1,2}^y & 0 & 0 & \cdots & 1 \\ c_{n,0}^y + c_{n,2}^y & 0 & 0 & \cdots & 0 \end{bmatrix}$$

… …

$$\mathbf{A}_{L_m} = \begin{bmatrix} \sum_{i=0}^{m} c_{1,i}^y & 1 & 0 & \cdots & 0 \\ \sum_{i=0}^{m} c_{2,i}^y & 0 & 1 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \sum_{i=0}^{m} c_{n-1,i}^y & 0 & 0 & \cdots & 1 \\ \sum_{i=0}^{m} c_{n,i}^y & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\mathbf{B}_1 = \left[c_{1,0}^u \;\; c_{2,0}^u \;\; \cdots \;\; c_{n,0}^u\right]^{\mathrm{T}}$$

$$\mathbf{B}_2 = \left[c_{1,0}^u + c_{1,1}^u \;\; c_{2,0}^u + c_{2,1}^u \;\; \cdots \;\; c_{n,0}^u + c_{n,1}^u\right]^{\mathrm{T}}$$

$$\mathbf{B}_3 = \left[c_{1,0}^u + c_{1,2}^u \;\; c_{2,0}^u + c_{2,2}^u \;\; \cdots \;\; c_{n,0}^u + c_{n,2}^u\right]^{\mathrm{T}}$$

… …

$$\mathbf{B}_{L_m} = \left[\sum_{i=0}^{m} c_{1,i}^{u} \quad \sum_{i=0}^{m} c_{2,i}^{u} \quad \cdots \quad \sum_{i=0}^{m} c_{n,i}^{u} \right]^{\mathrm{T}}$$

In summary, a local linear model (11) is obtained to describe the current nonlinear dynamics, and the future nonlinear behavior is assumed to vary within a region constructed by an uncertain linear parameter time-varying model (12) whose dynamic matrix $\mathbf{A}_{t+j|t}$ belongs to $\mathbf{\Omega}_A$ in (13) and $\mathbf{B}_{t+j|t}$ is in the convex polytope $\mathbf{\Omega}_B$ in (14). In the next section, we give the formulations of the min-max MPC algorithm based on these models.

3. MIN-MAX MPC ALGORITHM

MPC consists of a step-by-step optimization technique. At each step new measurements are obtained and a cost function depending on the predicted future states of the plant is minimized. Consider the following robust performance objective:

$$\min_{\bar{u}(t|t),\bar{u}(t+j|t),j=1,2,\cdots} \; \max_{\mathbf{A}_{t+j|t}\in\mathbf{\Omega}_A,\mathbf{B}_{t+j|t}\in\mathbf{\Omega}_B} J_\infty(t)$$

$$\text{s.t. (11), (12), } -\underline{u} \le \bar{u}(t+i\,|\,t) \le \bar{u}\,,\ \ i \ge 0 \qquad (15)$$

with

$$J_\infty(t) = \mathbf{X}(t\,|\,t)^{\mathrm{T}}\mathbf{W}\mathbf{X}(t\,|\,t) + \bar{u}(t\,|\,t)^{\mathrm{T}}\mathbf{R}\bar{u}(t\,|\,t) + J_\infty^1(t) \qquad (16)$$

$$J_\infty^1(t) = \sum_{j=1}^{\infty}\left\{\mathbf{X}(t+j\,|\,t)^{\mathrm{T}}\mathbf{W}\mathbf{X}(t+j\,|\,t) + \bar{u}(t+j\,|\,t)^{\mathrm{T}}\mathbf{R}\bar{u}(t+j\,|\,t)\right\} \qquad (17)$$

where $\mathbf{W} \ge \mathbf{0}$ and $\mathbf{R} > \mathbf{0}$ are suitable weighting matrices. The control input $\bar{u}(t\,|\,t)$ in (16) is a free control move and is the first computed move which gets implemented on the plant. The rest of the future control moves are given by a state feedback:

$$\bar{u}(t+j\,|\,t) = F(t)\mathbf{X}(t+j\,|\,t),\ j \ge 1 \qquad (18)$$

We provide an LMI synthesis procedure for the infinite-horizon MPC problem (15). This LMI formulation is achieved by defining a quadratic function:

$$V(j,t) = \mathbf{X}(t+j\,|\,t)^{\mathrm{T}}\mathbf{P}(j,t)\mathbf{X}(t+j\,|\,t),\ j \ge 1 \qquad (19)$$

Where $\forall t \ge 0,\ \forall j \ge 1,\ \mathbf{P}(j,t) > \mathbf{0}$. If the objective function (15) is well defined, we can state that $V(\infty,t) = 0$ because $\mathbf{X}(\infty\,|\,t) = \mathbf{0}$. In view of this, we impose a bound on the cost function $J_\infty^1(t)$ by the following design requirement:

$$\begin{aligned} &V(j+1,t) - V(j,t) \le \\ &\quad -\left\{\mathbf{X}(t+j\,|\,t)^{\mathrm{T}}\mathbf{W}\mathbf{X}(t+j\,|\,t) + \bar{u}(t+j\,|\,t)^{\mathrm{T}}\mathbf{R}\bar{u}(t+j\,|\,t)\right\}, \\ &\quad \forall \mathbf{A}_{t+j|t} \in \mathbf{\Omega}_A,\ \ \forall \mathbf{B}_{t+j|t} \in \mathbf{\Omega}_B,\ \ j \ge 1 \end{aligned} \qquad (20)$$

By summing (20) for $j = 1 \text{ to } \infty$, we obtain the following constraint:

$$\max_{\mathbf{A}_{t+j|t}\in\mathbf{\Omega}_A,\mathbf{B}_{t+j|t}\in\mathbf{\Omega}_B} J_\infty^1(t) < V(1,t) \qquad (21)$$

Therefore, problem (15) is equivalent to the following one:

$$\begin{aligned} \min_{\bar{u}(t|t),\bar{u}(t+j|t),j=1,2,\cdots} \ & \mathbf{X}(t\,|\,t)^{\mathrm{T}}\mathbf{W}\mathbf{X}(t\,|\,t) + \bar{u}(t\,|\,t)^{\mathrm{T}}\mathbf{R}\bar{u}(t\,|\,t) \\ & + \mathbf{X}(t+1\,|\,t)^{\mathrm{T}}\mathbf{P}(1,t)\mathbf{X}(t+1\,|\,t) \end{aligned}$$

$$\text{s.t. (11), (12), (20), } -\underline{u} \le \bar{u}(t+i\,|\,t) \le \bar{u}\,,\ \ i \ge 0 \qquad (21)$$

If there exist the Lyapunov matrices $\mathbf{P}_{ik}\,(i,k = 1,2,\cdots L_m)$ which is used to build a parameter-dependent Lyapunov natrix:

$$\mathbf{P}(j,t) = \sum_{i=1}^{L_m}\alpha_i(t+j\,|\,t)\sum_{k=1}^{L_m}\beta_k(t+j\,|\,t)\mathbf{P}_{ik} \qquad (22)$$

Problem (21) can then be solved as showed in Theorem 1.

Theorem 1. *The optimization problem (21) with the control law (18) can be solved by the following semi-definite programming:*

$$\min_{\gamma,\bar{u}(t|t),\mathbf{Y},\mathbf{G},\mathbf{Q}_{ik},\mathbf{Z}} \gamma \qquad (23)$$

Subject to

$$\begin{bmatrix} 1 & * & * & * \\ \mathbf{A}_t\mathbf{X}(t\,|\,t) + \mathbf{B}_t\bar{u}(t\,|\,t) + \mathbf{\Xi}(t) & \mathbf{Q}_{ik} & * & * \\ \mathbf{W}^{1/2}\mathbf{X}(t\,|\,t) & \mathbf{0} & \gamma\mathbf{I} & * \\ \mathbf{R}^{1/2}\bar{u}(t\,|\,t) & \mathbf{0} & \mathbf{0} & \gamma\mathbf{I} \end{bmatrix} \ge \mathbf{0} \qquad (24)$$

$$\begin{bmatrix} \mathbf{G} + \mathbf{G}^T - \mathbf{Q}_{ik} & * & * & * \\ \mathbf{A}_i\mathbf{G} + \mathbf{B}_k\mathbf{Y} & \mathbf{Q}_{ef} & * & * \\ \mathbf{W}^{1/2}\mathbf{G} & \mathbf{0} & \gamma\mathbf{I} & * \\ \mathbf{R}^{1/2}\mathbf{Y} & \mathbf{0} & \mathbf{0} & \gamma\mathbf{I} \end{bmatrix} \ge \mathbf{0} \qquad (25)$$

$$i,k,e,f = 1,2,\cdots,L_m$$

$$-\underline{u} \le \bar{u}(t\,|\,t) \le \bar{u} \qquad (26)$$

$$\begin{bmatrix} \mathbf{Z} & \mathbf{Y} \\ \mathbf{Y}^{\mathrm{T}} & \mathbf{G} + \mathbf{G}^T - \mathbf{Q}_{ik} \end{bmatrix} \ge \mathbf{0},\ \ \mathbf{Z}_{jj} \le \left[\min\{\underline{u},\ \bar{u}\}\right]^2 \qquad (27)$$

where the symbol $*$ *induces a symmetric structure,* $\mathbf{P}_{ik} = \gamma\mathbf{Q}_{ik}^{-1} > 0$ *,* $\mathbf{Z}$ *is a symmetric matrix and the feedback gain is given by* $F(t) = \mathbf{Y}\mathbf{G}^{-1}$.

Proof: Introducing (12), (18) and (19) into (20), one can get

$$\mathbf{X}(t+j\,|\,t)^{\mathrm{T}}\left\{\begin{aligned} &\left[\mathbf{A}_{t+j|t} + \mathbf{B}_{t+j|t}F(t)\right]^{\mathrm{T}}\mathbf{P}(j+1,t) \\ &\times\left[\mathbf{A}_{t+j|t} + \mathbf{B}_{t+j|t}F(t)\right] \\ &-\mathbf{P}(j,t) + F(t)^{\mathrm{T}}\mathbf{R}F(t) + \mathbf{W} \end{aligned}\right\}\mathbf{X}(t+j\,|\,t) \le 0 \qquad (28)$$

That is satisfied for all $j \ge 1$ if

$$\begin{aligned} &\left[\mathbf{A}_{t+j|t} + \mathbf{B}_{t+j|t}F(t)\right]^{\mathrm{T}}\mathbf{P}(j+1,t)\left[\mathbf{A}_{t+j|t} + \mathbf{B}_{t+j|t}F(t)\right] \\ &\qquad - \mathbf{P}(j,t) + F(t)^{\mathrm{T}}\mathbf{R}F(t) + \mathbf{W} \le 0 \end{aligned} \qquad (29)$$

It is easy to be confirmed that inequality (29) is satisfied if and only if there exist $L_m \times L_m$ symmetric positive matrices $\mathbf{P}_{ik}\,(i,k=1,2,\cdots L_m)$ such that the parameter-dependent Lyapunov matrix $\mathbf{P}(j,t)$ is obtained by (22), and

$$\left[\mathbf{A}_i+\mathbf{B}_k F(t)\right]^{\mathrm{T}}\mathbf{P}_{ef}\left[\mathbf{A}_i+\mathbf{B}_k F(t)\right]-\mathbf{P}_{ik}+F(t)^{\mathrm{T}}\mathbf{R}F(t)+\mathbf{W}\le 0 \tag{30}$$

Furthermore, define $F(t)=\mathbf{Y}\mathbf{G}^{-1}$, $\mathbf{Q}_{ik}=\gamma\mathbf{P}_{ik}^{-1}$, and $\mathbf{Q}_{ef}=\gamma\mathbf{P}_{ef}^{-1}\ \left(i,k,e,f=1,2,\cdots L_m\right)$. By using Schur complement, inequality (30) can be expressed as LMIs (25). Minimization problem (21) is now equivalent to

$$\min_{\gamma,\bar{u}(t|t),\mathbf{Y},\mathbf{G},\mathbf{Q}_{ik}} \gamma$$

Subject to (11), (25), $-\underline{u}\le\bar{u}(t+i\,|\,t)\le\bar{u}$, $i\ge 0$, and

$$\begin{aligned}&\mathbf{X}(t\,|\,t)^{\mathrm{T}}\mathbf{W}\mathbf{X}(t\,|\,t)+\bar{u}(t\,|\,t)^{\mathrm{T}}\mathbf{R}\bar{u}(t\,|\,t)\\&\quad+\mathbf{X}(t+1\,|\,t)^{\mathrm{T}}\mathbf{P}(1,t)\mathbf{X}(t+1\,|\,t)\le\gamma\end{aligned} \tag{31}$$

Introducing (11) into (31), and according to (22), the resulted inequality is equivalent to the following one:

$$\begin{aligned}&\left[\mathbf{A}_t\mathbf{X}(t\,|\,t)+\mathbf{B}_t\bar{u}(t\,|\,t)+\boldsymbol{\Xi}(t)\right]^{\mathrm{T}}\mathbf{P}_{ik}\\&\quad\times\left[\mathbf{A}_t\mathbf{X}(t\,|\,t)+\mathbf{B}_t\bar{u}(t\,|\,t)+\boldsymbol{\Xi}(t)\right]\\&\quad+\mathbf{X}(t\,|\,t)^{\mathrm{T}}\mathbf{W}\mathbf{X}(t\,|\,t)+\bar{u}(t\,|\,t)^{\mathrm{T}}\mathbf{R}\bar{u}(t\,|\,t)\le\gamma\end{aligned} \tag{32}$$

By using Schur complement and noting that $\mathbf{P}_{ik}=\gamma\mathbf{Q}_{ik}^{-1}\ \left(i,k=1,2,\cdots L_m\right)$, inequality (32) can be expressed as LMIs (24). Furthermore, referring to Cuzzola, *et al*. (2002) it is clear that the input constraint $-\underline{u}\le\bar{u}(t+i\,|\,t)\le\bar{u}$, $i\ge 1$ is met if LMIs (27) are satisfied. Therefore, one can conclude that minimization problem (21) is finally equivalent to $\min_{\gamma,\bar{u}(t|t),\mathbf{Y},\mathbf{G},\mathbf{Q}_{ik},\mathbf{Z}}\gamma$, subject to (24,25,26,27). □

On the basis of the conclusion showed in Theorem 1, the following result analogous to Lemma 2 and Theorem 3 of Kothare, *et al*. (1996) may be obtained:

Theorem 2. *Assume that the LMI optimization problem (23) admits a solution at time t.*

1) *Then it admits a solution for all future instants.*
2) *The feasible receding horizon control law obtained from Theorem 1 robustly asymptotically stabilizes the closed-loop system represented by (11) and (12).* □

4. SIMULATION STUDY

In general coal (or oil or gas)-fired electric power generation plants, the concentration of nitrogen oxide (NO_x) emission in flue gas from boiler is usually very high, and the flue gas with high concentration NO_x can not be directly discharged into the air. To protect the environment, the NO_x concentration in exhaust gas from stack must be controlled within a limit set by environmental regulations. The most commonly used technology is the selective catalytic reduction method, as in (Dolanc, *et al.,* 2001), which uses ammonia (NH3) to decompose NO_x. The purpose of the NO_x decomposition (de-NO_x) process control in thermal power plants is to control the NO_x within a limit set, while at the same time reducing expensive ammonia (NH3) gas consumption as much as possible. Conventional PID controller usually has difficulty in achieving a good trade-off between control performance and ammonia consumption. Fig.1 shows the structure diagram of a de-NO_x process in a thermal power plant. The dynamic behavior of this process is nonlinear and non-stationary, and depends on the time-varying load demand $w(t)$ of the power plant (Matsumura, *et al.,* 1997). At different load level, the process's dynamic behavior (between the de-NO_x device outlet NO_x $y(t)$ and the total NH3 flow-set $u(t)+v_2(t)$ in Fig.1) is also different, and may be represented by a linear model. In Fig.1, a PID feedback and feedforward controller, whose design approach is similar to (Dolanc, *et al.,* 2001), has already existed in the system, and its parameters were well tuned for entire load-varying region. The predictive controller in Fig.1 is added to improve control performance.

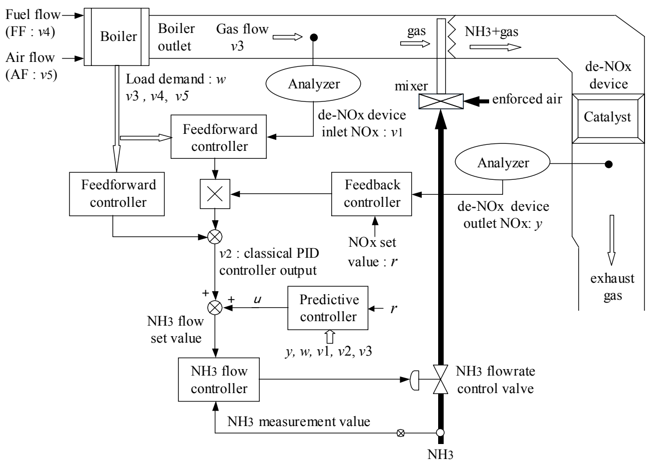

Fig. 1. Diagram of a de-NO_x device and its control system; $y(t)$ is the de-NO_x device outlet NO_x; $u(t)$ is the output of the predictive controller; $w(t)$ is the load demand; $v_1(t)$ is the de-NO_x device inlet NO_x; $v_2(t)$ is the existing PID controller output.

In this simulation study, we use RBF-ARX model (3) to describe the dynamics of the de-NO_x process in which the variables of describing operating-point state in RBF-ARX model (3) is the load demand sequence $\mathbf{W}(t-1)$, and $y(t)$ is the de-NO_x device outlet NO_x and $u(t)$ is the total NH3 flow-set. Fig.2 shows the estimated result of model (3) for the training-data taken from a real de-NOx process being controlled by the PID controller alone where the structured nonlinear parameter optimization method (SNPOM) (Peng, *et al.,* 2003a) is utilized for estimating model (3). Fig.3 gives the simulation results showing the control performance comparison between the RBF-ARX model-based robust NMPC (where n_y=4, n_u=5, $m=1$, $n_w=2$, $\mathbf{R}$=0.2$\mathbf{I}$) proposed in Section 3 and the PID control alone. From

Fig.3 it is clear that the control performance of the NMPC proposed is far better than that of the PID control.

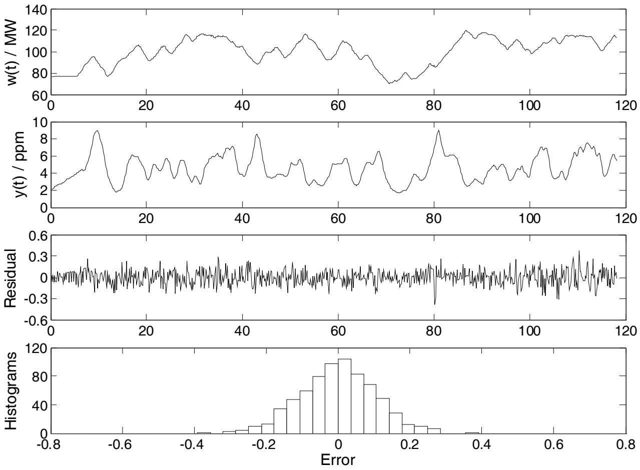

Fig. 2. Residual of RBF-ARX model (3) estimated off-line using the SNPOM.

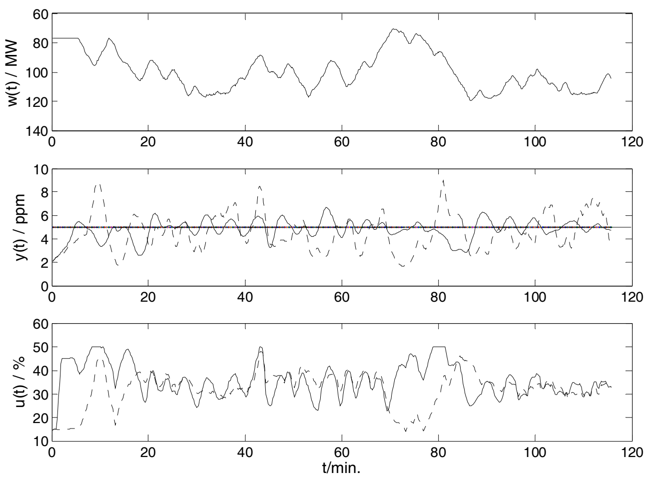

Fig. 3. Comparison of control performance between PID alone (Dotted line) and the RBF-ARX model based robust NMPC (Solid line); $y(t)$ set at 5 ppm.

5. CONCLUSIONS

For a class of nonlinear systems, a combination of a local linearization model and a polytopic uncertain linear parameter-varying model was proposed on the basis of the RBF-ARX model to approximate the present and the future system's nonlinear behavior respectively. The proposed models-based min-max robust predictive control algorithm with constraints was designed for the nonlinear systems. The closed loop stability could be guaranteed by the use of parameter-dependent Lyapunov function and the feasibility of the linear matrix inequalities. Simulation study showed the effectiveness of the modeling and MPC approaches proposed.

REFERENCES

Bloemen, H.H.J., T.J.J. Van Den Boom and H.B. Verbruggen (2001). Model-based predictive control for Hammerstein-Wiener systems. *Int. J. of Control,* **74**, 482-485.

Cuzzola, F.A., J.C. Geromel and M. Morari (2002). An improved approach for constrained robust model predictive control. *Automatica*, **38**, 1183- 1189.

Dolanc, G., S. Strmcnik and J. Petrovcic (2001). NOx selective catalytic reductive control based on simple models. *J. of Process Control*, **11**, 35-51.

Kothare, M.V., V. Balakrishnan and M. Morari (1996). Robust constrained model predictive control using linear matrix inequalities. *Automatica*, **32**, 1361-1379.

Lakhdari, Z., M. Mokhtari, Y. Lécluse and J. Provost (1995). Adaptive predictive control of a class of nonlinear systems – A case study. *in IFAC Proceedings: Adaptive Systems in Control and Signal Processing, Budapest, Hungary,* 209-214.

Liu, G.P., V. Kadirkamanathan and S.A. Billings (1998). Predictive control for non-linear systems using neural networks. *Int. J. of Control,* **71**, 1119- 1132.

Lu, Y. and Y. Arkun (2002). A scheduling quasi-min-max model predictive control algorithm for nonlinear systems. *J. of Process Control*, **12**, 589-604.

Mahfouf, M. and D.A. Linkens (1998). Non-linear generalized predictive control (NLGPC) applied to muscle relaxant anaesthesia. *Int. J. of Control,* **71**, 239-257.

Matsumura, S., T. Iwahara, K. Ogata, S. Fujii and M. Suzuki (1997). Improvement of de-NOx device control performance using software sensor. *Proceedings of the 11th IFAC Symposium on System Identification*, Fukuoka, Japan, pp. 1507- 1512.

Peng, H., T. Ozaki, V. Haggan-Ozaki and Y. Toyoda (2002). A nonlinear exponential ARX model- based multivariable generalized predictive control strategy for thermal power plants. *IEEE Trans. on Control Systems Technology*, **10**, 256-262.

Peng, H., T. Ozaki, V. Haggan-Ozaki and Y. Toyoda (2003a). A parameter optimization method for the radial basis function type models. *IEEE Trans. on Neural Networks,* **14**, 432-438.

Peng, H., T. Ozaki, K. Nakano, V. Haggan-Ozaki and Y. Toyoda (2003b). Stability analysis of the RBF-ARX model based nonlinear predictive control. *Proceedings of the European Control Conference (ECC2003),* Cambridge, UK.

Peng, H., T. Ozaki, M. Mori, H. Shioya and V. Haggan-Ozaki (2003c). Modeling and control of nonlinear nitrogen oxide decomposition process. *Proceedings of IEEE 42st International Conference on Decision and Control (CDC'03)*, Hawaii, USA, 4770-4775.

Peng, H., T. Ozaki, Y. Toyoda, H. Shioya, K. Nakano, V. Haggan-Ozaki and M. Mori (2004). RBF-ARX model based nonlinear system modeling and predictive control with application to a NO_x decomposition process. *Control Engineering Practice*, **12**, 191-203.

Prasad, G., E. Swidenbank and B.W. Hogg (1998). A local model networks based multivariable long-range predictive control strategy for thermal power plants. *Automatica,* **34**, 1185-1204.

Sentoni, G., O. Agamennoni, A. Desages and J. Romagnoli (1996). Approximate models for nonlinear process control. *AIChE Journal,* **42**, 2240-2250.