

ADAPTIVE CONTROL OF TWIN ROTOR MIMO SYSTEM: POLYNOMIAL APPROACH

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Abstract: This paper presents real – time control of the Twin Rotor MIMO System laboratory model. The objective laboratory model is a multivariable nonlinear unstable system of high order. It is not possible to stabilize the system and satisfactorily track reference signals using classical controllers with fixedly set parameters. Two methods based on self – tuning control utilizing adaptive approach are discussed. Both designed methods are based on polynomial approach. In first case an algorithm taking into account internal interactions among input and output variables was used. The second method utilizes the principle of decentralized control with an additional logical supervisor for switching of the recursive identification in particular loops. In both cases stability of closed loop system was ensured and after an adaptation phase the asymptotic tracking of reference signals was achieved. Quality of control achieved by particular methods is compared and discussed. *Copyright © 2005 IFAC*

Key words: Polynomial methods, Nonlinear system, Self-tuning control, Multivariable control, Decentralized control, Real-time control.

1. INTRODUCTION

In this paper, a comparison of two approaches for adaptive control of a multivariable laboratory model (Twin Rotor MIMO System) is presented.

The classical approach to the control of MIMO systems is based on a design of a matrix controller used to control all system outputs at a time. The computation of matrix controller is realized by one central computer. Basic advantage of this approach is possibility of reaching optimal control courses because the controller can use all information known about the controlled system. Disadvantage of usage of the central matrix controller is its demands to computer resources because the number of operations and consumed memory depend on the square of the number of controlled signals. Nowadays this problem is reduced thanks to great progress in the development of computer hardware and leads just to the increased price of the control system. Another

disadvantage lies in influences of its faults to the controlled system. In case of failure of central controller, all the controlled signals are afflicted and thus the reliability of the controller is fundamental. Reaching of required reliability can then be unbearable from the financial point of view, especially in critical applications.

Alternative solution for the control of MIMO systems is usage of decentralized approach. In this case, the system is considered as a set of interconnected subsystems and the output of each subsystem is influenced not only by the input to this subsystem but also by the input to the other subsystems. Each subsystem is controlled by stand - alone controller. Thus, the decentralized control is based on decomposition of the MIMO system to subsystems and design of a controller for each subsystem (Cui and Jacobson, 2002). Another advantage of the decentralized approach is, that setting of controller parameters (in this case a choice of poles of the

characteristic polynomial) is for SISO control loops a lot more easier than for MIMO control loops. On the other hand, the control courses of decentralized control system are suboptimal because the controllers do not use information from the other subsystems. Disadvantageous is also a limited applicability of the decentralized control only for symmetric systems (systems with an equal number of inputs and outputs).

2. TWIN ROTOR MIMO SYSTEM

The real-time laboratory model Twin Rotor MIMO System (producer Feedback Instruments, LTD United Kingdom) is shown in Fig 1. This system provides a high-order, non-linear system with significant cross-coupling. The main parts of the system are the pedestal, the jib connected to pedestal and two propellers at the ends of the jib. The system jib can freely rotate around vertical axes by about 330 degrees (process output $y_1(t)$) and horizontal axis by about 100 degrees (process output $y_2(t)$). The system inputs $u_1(t), u_2(t)$ are the voltages used to drive motors of the propellers and outputs are angular rotations with respect to horizontal and vertical axes.

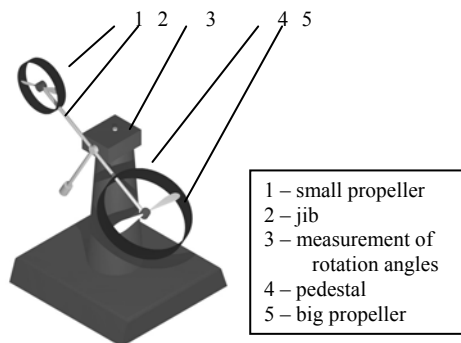


Fig. 1. Twin rotor MIMO model (helicopter)

Despite the strong interactions in the system, decomposition to subsystems is straightforward: the first subsystem consists of the small propeller which drives the angular rotation around vertical axis, the second subsystem consists of the big propeller driving the angular rotation around horizontal axis. Before the control circuit was connected as a closed loop, the experiments obtaining a static characteristic of the systems had been performed. The influence of the first system input to the second system output is small but the course of the second output is a sign of nonlinearity of the system. Another problem of the control of this system is a big hysteresis which is present in the system. The static characteristics of first subsystem, which was measured for increasing and decreasing input signal, are shown in Fig. 2. The great influence of changes of second system input to the first output was confirmed by this measurement.

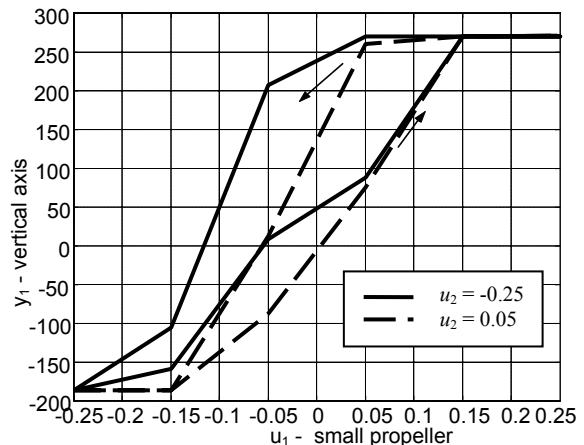


Fig. 2. Static characteristics of the first subsystem showing hysteresis

3. DECENTRALIZED CONTROL USING LOGIC SUPERVISOR

Each output of multivariable controlled system can be affected by each system input. The measure of the affection is determined not only by cross - coupling of the MIMO system but also by the course of the system input signals. When the decentralized approach is used to control such a system then, from the point of view of controller in particular subsystem, the transfer function varies in time even if the MIMO system is linear and stable.

The presence of subsystems interconnections is the main reason for using self-tuning controllers in decentralized approach to ensure required course of controlled variables. Identification algorithms suitable for usage in decentralized control must include weighting of identification data where new data affect estimation of model parameters more than older data. This requirement is a consequence of the time varying influences of the other subsystems to the identified subsystem. The influence of control variable (u_i) to the corresponding controlled variable (y_i) decreases with increasing gain of subsystems interconnections. This could lead to unstable process of recursive parameter estimates. The stability of recursive identification can be increased by ensuring that just one of the controllers connected to the multivariable systems works in adaptive regime in particular time. Recursive identification parts of other controllers are suspended and parameter model estimates are constant for that time. The process of switching on and off the recursive identification is controlled by a new part of a control circuit – the supervisory system. Switching the identification on and off can be described as a process of transferring token among subsystems where only the controller, which currently has a token, can perform recursive identification. The token is move to other subsystem when the selected criterion is fulfilled.

The supervisory system represents a second level of a control and thus a control circuit with supervisory

system has a hierarchical structure of control (Bobál, *et al.* 2004).

The inclusion of supervisory logic into control circuit brings the problem of defining a strategy for switching the identification of individual subsystems on and off i.e. moving the token between subsystems. Three basic approaches can be used in deciding when to suspend identification of particular subsystem and move token to the other one:

- on base of elapsed time,
- on base of values from the currently identified subsystem,
- on base of values from the other subsystems.

It is also possible to combine these approaches. The basic ideas of these approaches are discussed in (Chalupa, 2003).

A logic supervisor has been proposed to utilize and simplify the design of supervisory logic. This approach is suitable for usage in real-time industrial applications. The idea of logical supervisor is based on the following two principles:

- Assigning priorities of individual subsystems
- On-line evaluation of criterions for each subsystem

The logic supervisor was tested in connection with the controllers from the Self-Tuning Controllers Library (Bobál and Chalupa, 2002). The properties of controllers were tested in MATLAB/Simulink environment using control scheme shown in Fig. 3. The model contains continuous TITO system controlled by two self-tuning controllers and logical supervisor which provides identification switching between input-output pairs. Model also includes saturation blocks applied to control values.

The quality of the control process is affected by many parameters e.g. sampling period, algorithm of control law, presence algorithm of logical supervisor, saturation, initial parameter estimates. The on-line identification uses recursive least squares method with adaptive directional forgetting.

The same controller configuration as in case of the MIMO control loop design has been chosen for the control of the Twin Rotor MIMO System. The structure of the SISO control loop is shown in Fig. 4. The transfer function of the closed control loop is given by the relation

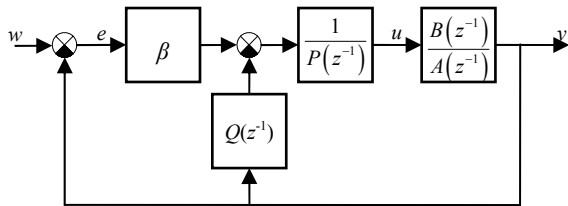


Fig. 4. Control loop with SISO controller

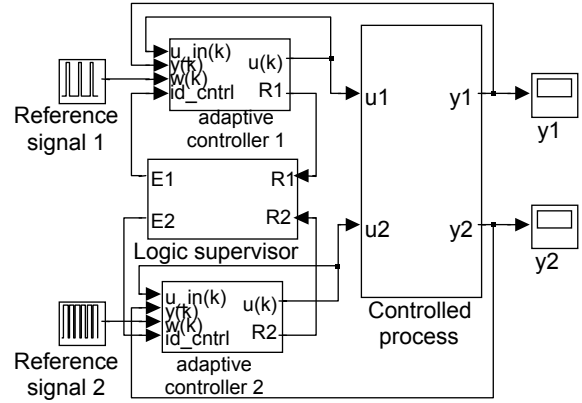


Fig. 3. Simulink control circuit with TITO controlled system

$$G_w(z) = \frac{Y(z)}{W(z)} = \frac{\beta B(z^{-1})}{A(z^{-1})P(z^{-1}) + B(z^{-1})[Q(z^{-1}) + \beta]} \quad (1)$$

where

$$A(z^{-1})P(z^{-1}) + B(z^{-1})[Q(z^{-1}) + \beta] = D(z^{-1}) \quad (2)$$

is the characteristic polynomial of the closed loop.

Polynomials

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2}, B(z^{-1}) = 1 + b_1 z^{-1} + b_2 z^{-2} \quad (3)$$

represent the controlled system and

$$P(z^{-1}) = (1 - z^{-1})(1 + \gamma z^{-1}) \quad (4)$$

$$Q(z^{-1}) = (1 - z^{-1})(q_0 - q_2 z^{-1})$$

are polynomials of the controller. From (4) it is obvious that the controller contains the integrator.

The characteristic polynomial

$$D(z^{-1}) = 1 + d_1 z^{-1} + d_2 z^{-2} \quad (5)$$

is described by the dominant poles for a second order continuous-time model

$$D(s) = s^2 + 2\xi\omega_n s + \omega_n^2 \quad (6)$$

where the dominant poles are given by the desired damping factor ξ and the natural frequency ω_n of the closed-loop. Then the controller algorithm, so called **PP2B-1**, is given by following equation (Bobál and Chalupa, 2002)

$$u(k) = -(q_0 + \beta)y(k) + (q_0 + q_2)y(k-1) - q_2 y(k-2) - (\gamma - 1)u(k-1) + \gamma u(k-2) + \beta w(k) \quad (7)$$

4. MULTIVARIABLE CONTROL

4.1. Mathematical Model of the Twin Rotor MIMO System

The examined twin motor MIMO system is a typical example of a two inputs – two outputs system with significant cross – coupling. Internal structure of the kind of system is shown in Fig. 5

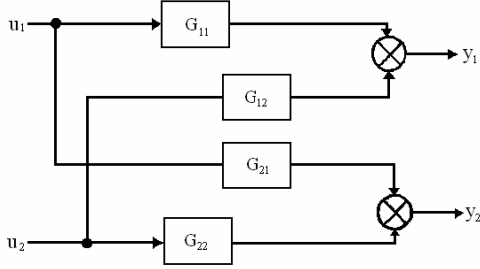


Fig. 5. A two input – two output system

The transfer matrix of the system is defined as

$$Y(z) = G(z)U(z) = \begin{bmatrix} G_{11}(z) & G_{12}(z) \\ G_{21}(z) & G_{22}(z) \end{bmatrix} U(z) \quad (8)$$

where

$$U(z) = [u_1(z), u_2(z)]^T \quad (9)$$

is the vector of manipulated variables and

$$Y(z) = [y_1(z), y_2(z)]^T \quad (10)$$

is the output vector.

It is possible to assume that the dynamic behaviour of the system can be described in the neighbourhood of steady state by the discrete linear model in the form of the matrix fraction (Kučera, 1991)

$$G(z) = A^{-1}(z^{-1})B(z^{-1}) = B_1(z^{-1})A_1^{-1}(z^{-1}) \quad (11)$$

Where polynomial matrices $A \in R_{22}[z^{-1}]$, $B \in R_{22}[z^{-1}]$ are the left coprime factorization of matrix $G(z^{-1})$ and matrices $A_1 \in R_{22}[z^{-1}]$, $B_1 \in R_{22}[z^{-1}]$ are the right coprime factorization of $G(z^{-1})$.

The algorithm was designed for the following model with second order polynomials

$$A(z^{-1}) = \begin{bmatrix} 1 + a_1 z^{-1} + a_2 z^{-2} & a_3 z^{-1} + a_4 z^{-2} \\ a_5 z^{-1} + a_6 z^{-2} & 1 + a_7 z^{-1} + a_8 z^{-2} \end{bmatrix} \quad (12)$$

$$B(z^{-1}) = \begin{bmatrix} b_1 z^{-1} + b_2 z^{-2} & b_3 z^{-1} + b_4 z^{-2} \\ b_5 z^{-1} + b_6 z^{-2} & b_7 z^{-1} + b_8 z^{-2} \end{bmatrix}$$

This model proved to be effective. Expression (12) can be transcribed to the difference equations of the model

$$y_1(k) = -a_1 y_1(k-1) - a_2 y_1(k-2) - a_3 y_2(k-1) - a_4 y_2(k-2) + b_1 u_1(k-1) + b_2 u_1(k-2) + b_3 u_2(k-1) + b_4 u_2(k-2)$$

$$y_2(k) = -a_5 y_1(k-1) - a_6 y_1(k-2) - a_7 y_2(k-1) - a_8 y_2(k-2) + b_5 u_1(k-1) + b_6 u_1(k-2) + b_7 u_2(k-1) + b_8 u_2(k-2) \quad (13)$$

4.2. Design of 2DOF Controller

The structure of the closed loop, shown in Fig. 6, was presented in (Ortega and Kelly, 1984).

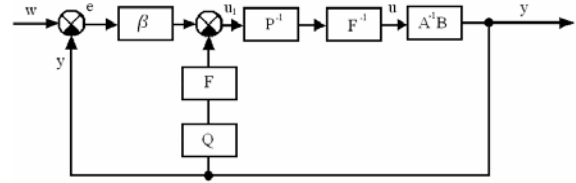


Fig. 6. Block diagram of 2DOF configuration

Generally, the vector $W(z)$ of input reference signals is specified as

$$W(z) = F_w^{-1}(z^{-1})h(z^{-1}) \quad (14)$$

Here, the reference signals are considered as a class of step functions. In this case $h(z^{-1})$ is a vector of constants and $F_w(z^{-1})$ is expressed as

$$F_w(z^{-1}) = \begin{bmatrix} 1 - z^{-1} & 0 \\ 0 & 1 - z^{-1} \end{bmatrix} \quad (15)$$

The compensator $F(z^{-1})$ is a component formally separated from the controller. It has to be included in the controller to fulfil the requirement on the asymptotic tracking. If the reference signals are of the same class as the step functions are, then $F(z^{-1})$ is an integrator.

It is possible to derive the following equation for the system output (operator z^{-1} will be omitted from some operations for the purpose of simplification)

$$Y = A^{-1}BU = A^{-1}BF^{-1}P^{-1}U_1 \quad (16)$$

Where

$$U_1 = \beta(W - Y) - QFY \quad (17)$$

The corresponding equation for the controller's output, as shown in the block diagram in Fig. 6, follows as

$$U = F^{-1}P^{-1}U_1 \quad (18)$$

The substitution of U_1 and Y results in

$$U = F^{-1}P^{-1}[\beta(W - A^{-1}BU) - QFA^{-1}BU] \quad (19)$$

The equation (19) can be modified using the right matrix fraction of the controlled system into the form

$$U = A_1 [PFA_1 + (\beta + FQ)B_1] BW \quad (20)$$

The closed loop system is stable when the following diophantine equation is satisfied

$$PFA_1 + (\beta + FQ)B_1 = M \quad (21)$$

Where $M \in R_{22}[z^{-1}]$ is a stable diagonal polynomial matrix

$$M(z^{-1}) = \begin{bmatrix} 1 + m_1 z^{-1} + m_2 z^{-2} + & 0 \\ + m_3 z^{-3} + m_4 z^{-4} & \\ 0 & 1 + m_1 z^{-1} + m_2 z^{-2} + \\ & + m_3 z^{-3} + m_4 z^{-4} \end{bmatrix} \quad (22)$$

and the structure of the matrices P , Q and β was chosen as follows

$$P(z^{-1}) = \begin{bmatrix} 1 + p_1 z^{-1} & p_2 z^{-1} \\ p_3 z^{-1} & 1 + p_4 z^{-1} \end{bmatrix} \quad (23)$$

$$Q(z^{-1}) = \begin{bmatrix} q_1 + q_2 z^{-1} & q_3 + q_4 z^{-1} \\ q_5 + q_6 z^{-1} & q_7 + q_8 z^{-1} \end{bmatrix}$$

$$\beta(z^{-1}) = \begin{bmatrix} \beta_1 & \beta_2 \\ \beta_3 & \beta_4 \end{bmatrix}$$

The solution of the diophantine equation results in a set of sixteen algebraic equations with unknown controller parameters. Using matrix notation the algebraic equations are expressed in the following form

$$\begin{bmatrix} 1 & 0 & b_9 & 0 & b_{13} & 0 & b_9 & b_{13} \\ a_9 - 1 & a_{13} & b_{10} - b_9 & b_9 & b_{14} - b_{13} & b_{13} & b_{10} & b_{14} \\ a_{10} - a_9 & a_{14} - a_{13} & -b_{10} & b_{10} - b_9 & -b_{14} & b_{14} - b_{13} & 0 & 0 \\ -a_{10} & -a_{14} & 0 & -b_{10} & 0 & -b_{14} & 0 & 0 \\ 0 & 1 & b_{11} & 0 & b_{15} & 0 & b_{11} & b_{15} \\ a_{11} & a_{15} - 1 & b_{12} - b_{11} & b_{11} & b_{16} - b_{15} & b_{15} & b_{12} & b_{16} \\ a_{12} - a_{11} & a_{16} - a_{15} & -b_{12} & b_{12} - b_{11} & -b_{16} & b_{16} - b_{15} & 0 & 0 \\ -a_{12} & -a_{16} & 0 & -b_{12} & 0 & -b_{16} & 0 & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ q_1 \\ q_2 \\ q_3 \\ q_4 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} m_1 - a_9 + 1 \\ m_2 + a_9 - a_{10} \\ m_3 + a_{10} \\ m_4 \\ -a_{11} \\ a_{11} - a_{12} \\ a_{12} \\ 0 \end{bmatrix} \quad (24)$$

$$\begin{bmatrix} 1 & 0 & b_9 & 0 & b_{13} & 0 & b_9 & b_{13} \\ a_9 - 1 & a_{13} & b_{10} - b_9 & b_9 & b_{14} - b_{13} & b_{13} & b_{10} & b_{14} \\ a_{10} - a_9 & a_{14} - a_{13} & -b_{10} & b_{10} - b_9 & -b_{14} & b_{14} - b_{13} & 0 & 0 \\ -a_{10} & -a_{14} & 0 & -b_{10} & 0 & -b_{14} & 0 & 0 \\ 0 & 1 & b_{11} & 0 & b_{15} & 0 & b_{11} & b_{15} \\ a_{11} & a_{15} - 1 & b_{12} - b_{11} & b_{11} & b_{16} - b_{15} & b_{15} & b_{12} & b_{16} \\ a_{12} - a_{11} & a_{16} - a_{15} & -b_{12} & b_{12} - b_{11} & -b_{16} & b_{16} - b_{15} & 0 & 0 \\ -a_{12} & -a_{16} & 0 & -b_{12} & 0 & -b_{16} & 0 & 0 \end{bmatrix} \begin{bmatrix} p_3 \\ p_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \\ \beta_3 \\ \beta_4 \end{bmatrix} = \begin{bmatrix} -a_{13} \\ a_{13} - a_{14} \\ a_{14} \\ 0 \\ m_5 - a_{15} + 1 \\ m_6 + a_{15} - a_{16} \\ m_7 + a_{16} \\ m_8 \end{bmatrix}$$

The controller parameters are derived by solving these equations. The control law apparent from the block diagram is defined as

$$FPU = \beta E - FQY \quad (25)$$

The control law in the form of difference equations is defined by the following expression

$$u_1(k) = \beta_1 e_1(k) + \beta_2 e_2(k) - q_1 y_1(k) - (q_2 - q_1) y_1(k-1) + q_2 y_1(k-2) - q_3 y_2(k) - (q_4 - q_3) y_2(k-1) + q_4 y_2(k-2) - (p_1 - 1) u_1(k-1) + p_1 u_1(k-2) - p_2 u_2(k-1) + p_2 u_2(k-2) \quad (26)$$

$$u_2(k) = \beta_3 e_1(k) + \beta_4 e_2(k) - q_5 y_1(k) - (q_6 - q_5) y_1(k-1) + q_6 y_1(k-2) - q_7 y_2(k) - (q_8 - q_7) y_2(k-1) + q_8 y_2(k-2) - (p_4 - 1) u_2(k-1) + p_4 u_2(k-2) - p_3 u_1(k-1) + p_3 u_1(k-2)$$

5. EXPERIMENTAL RESULTS

The Twin Rotor MIMO System is a nonlinear system with variable parameters that is practically too complex to be controlled using controllers with fixed parameters. The nonlinear dynamics was in the neighbourhood of a steady state described by the linear models given in previous sections. Adaptive control was performed using the designed controllers.

The sampling period was chosen in both cases $T_0 = 0.5s$. The other parameters for the decentralized control - damping factor $\xi = 10$ and natural frequency $\omega_n = 1$ were chosen in virtue of several realized experiments. The matrix M (the pole placement of the multivariable controller) was also obtained from a number of experiments in the form

$$M(z^{-1}) = \begin{bmatrix} 1 - 0,9z^{-1} + 0,19z^{-2} - & 0 \\ -0,009z^{-3} - 0,002z^{-4} & \\ 0 & 1 - 0,9z^{-1} + 0,19z^{-2} - \\ & -0,009z^{-3} - 0,002z^{-4} \end{bmatrix} \quad (27)$$

The initial parameter estimates were chosen without any previous information in both cases.

The control courses of twin rotor MIMO system using both controllers are presented in Fig. 7, Fig. 8, Fig. 9 and Fig. 10. These figures demonstrate that the strong nonlinear, unstable and high order system can be stabilized and also quite good asymptotic tracking can be achieved by using adaptive control without apriori information about model of the process.

Table 1 contains the values of the control quality criterions. A criterion is a sum of powers of tracking errors and a sum of increments of manipulated variables. The table contains the values after 50 s when the identified parameters became steady.

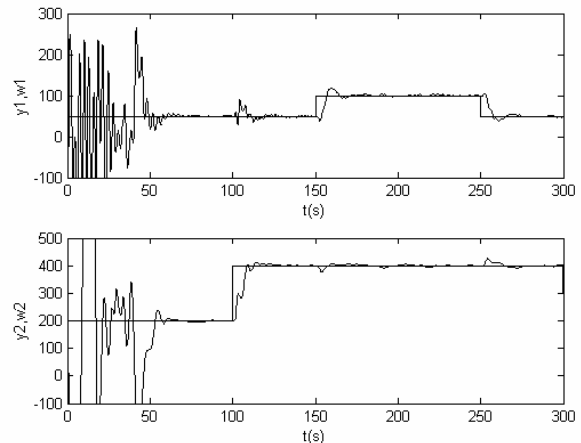


Fig. 7. Control courses using the matrix controller

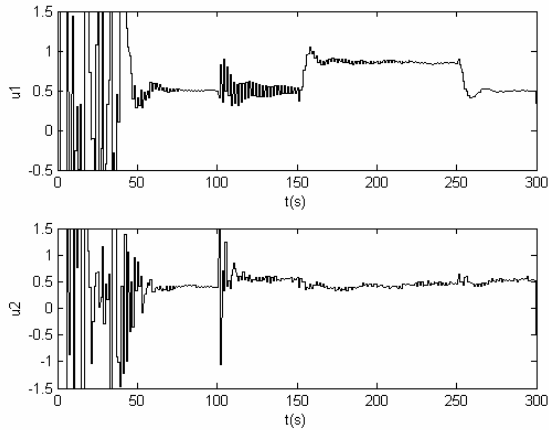


Fig. 8. Control courses using the matrix controller – controllers output

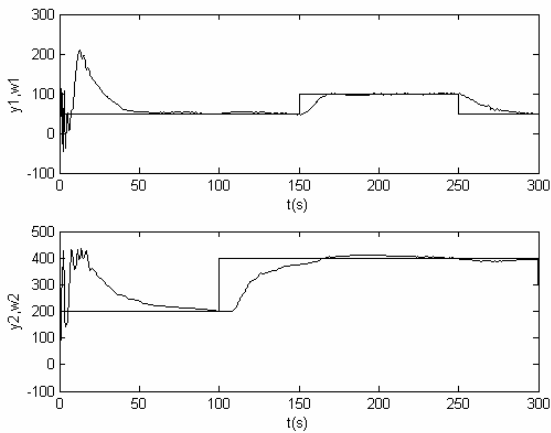


Fig. 9. Control courses using the decentralized controller

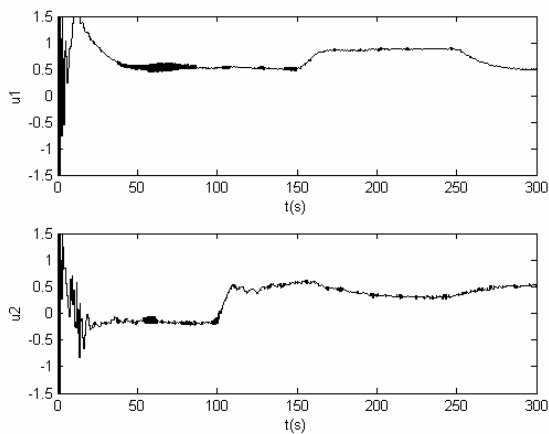


Fig. 10. Control courses using the decentralized controller – controllers output

Table 1 Control quality criterions

Controller	$\sum e_1^2$	$\sum e_2^2$	$\sum \Delta u_1^2$	$\sum \Delta u_2^2$
Multivariable	136,75	694,45	0,0012	0,0074
Decentralized	182,66	2784,2	0,0004	0,0003

6. CONCLUSIONS

In case of the decentralized control it was necessary to choose a large value of the damping factor ξ . It means approximation of the dynamic behaviour of the process by an over – damped second order model. It caused a slow approach of the controlled variables to the reference signals. On the other hand, oscillations of both controlled variables and the manipulated variables were significantly damped. This is quite important for this kind of process. The multivariable controller ensured faster asymptotic tracking of the reference signals. The courses of the controlled variables and the manipulated variables appeared to have more oscillatory characters. These facts are also evident from the values in Table 1. Sums of powers of tracking errors are greater in case of the decentralized control (slow courses), whilst sums of increments of manipulated variables are greater for multivariable control (oscillations of manipulated variables).

Despite the fact, that relatively simple control algorithms without any sophisticated numerical computation were used, quite good control results were achieved by both approaches. An advantage of the proposed strategies lies in their simplicity and applicability in industrial practice.

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