PRICE-DRIVEN COORDINATION FOR SOLVING PLANT-WIDE MPC PROBLEMS

Ruoyu Cheng [*] **Fraser Forbes** [*,1] **W. San Yip** [**]

[*] *Department of Chemical and Materials Engineering*
University of Alberta, Edmonton, AB T6G 2G6, Canada
[**] *Alberta Research Council*
Edmonton, AB T6N 1E4, Canada

Abstract: Two-level model predictive control (MPC) is the dominant multivariable control technology in the process industries. In large-scale MPC applications, such as plant-wide control, two common approaches are centralized and decentralized MPC schemes, which represent the two extremes in the "trade-off" among the desired characteristics of an implemented MPC system. Alternatively the coordination of decentralized MPC systems may offer the best attributes of each of the extremes. The price-driven coordination scheme requires the existence of "equilibrium prices" and has extensive large-scale applications in economic planning. On-line solutions to large-scale optimization problems require an efficient price-adjustment method. As the coordination problem for decentralized MPC falls into the category of limited resource case, this work develops an efficient price-adjustment algorithm by using Newton's method, in which sensitivity analysis and active set change identification techniques are employed. The proposed price-adjustment strategy is incorporated into a coordinated, decentralized MPC scheme that shows a high degree of accuracy, while retaining the reliability of original decentralized scheme at a reasonable computational load.

Keywords: Model predictive control, Target calculation, Price-driven coordination, Decentralized quadratic programming

1. INTRODUCTION

Model predictive control (MPC) has gained extensive application in industry for controlling multivariable process with constraints. Almost all MPC products have two levels of optimization, a steady-state target calculation and a dynamic control calculation, at each control cycle (Qin and Badgewell, 2003). The target calculation determines the best achievable set-points, both for input and output variables; whereas, the optimum trajectory to move the plant to the steady-state target is determined in the dynamic calculation. In both calculations, a process model is required to perform the optimization. In plant-wide control applications the problem size can be very large.

Two common paradigms for solving plant-wide MPC calculations are centralized and decentralized strategies. The major difference is how they handle interactions among operating units (Cheng *et al.*, 2004). A fully centralized or monolithic MPC for an entire plant is often undesirable and difficult, if not impossible, to implement (Lu, 2003). Such a scheme can exhibit poor fault-tolerance, requires a high performance centralized computational platform, and can be difficult to tune. In the decentralized scheme, the plant-wide MPC problem is decomposed into subprob-

[1] Email : fraser.forbes@ualberta.ca, phone : 780 492-0873, fax : 780 492-2881.

lems by taking advantage of the block-wise structure of the plant model and these subproblems are solved independently. Currently, decentralized MPC strategies are widely used due to their flexibility, reliability and ease of maintenance.

Conventional decentralized schemes may not be able to provide the plant-wide optimum. In general, decentralized schemes approximate or ignore the interactions between operating units, while the downstream units treat the upstream variations as external disturbances. Thus, the decentralized approach solves each subproblem in terms of its own objective function, which may not provide the plant-wide optimum. It is estimated that the potential plant-wide benefit for a typical refinery is 2-10 times more than a decentralized MPC can achieve (Bodington, 1995).

Recently, more effort has been spent on improving the performance of plant-wide decentralized control through coordination. Lu (2003) discussed the need for cross-functional integration for decentralized controllers, in which a coordination "collar" performs centralized steady-state target calculation for decentralized MPC. Venkat *et al.* (2004) proposed several coordination-based MPC algorithms, in which augmented states are used to model interactions, to improve plant-wide performance via the coordination of decentralized MPC dynamic calculation. Kumar and Daoutidis (2002) proposed a controller design framework using a time-scale decomposition approach, in which a supervisory controller deals with the slow-time-scale behavior and coordinates the distributed controllers, which deal with the fast-time-scale behavior. In our previous work (Cheng *et al.*, 2004), Dantzig-Wolfe decomposition was applied to coordinate LP-based MPC target calculation and the performance was improved through coordination.

Our current focus is on improving decentralized MPC performance by coordinating quadratic programming (QP) based target calculation using a price-driven coordination approach (Jose and Ungar, 1998*b*). An efficient coordination method based on a price-adjustment algorithm using Newton's method is developed. Two case studies are then used to illustrate the effectiveness of the proposed approach. The first one emphasizes the computational efficiency, while the latter shows the efficacy on the coordination of decentralized MPC.

2. FORMULATION OF PLANT-WIDE MPC TARGET CALCULATION

Consider the following constrained QP formulation of MPC target calculation for an individual operating unit (Ying and Joseph, 1999)

$$\begin{aligned}\min_{\mathbf{y}_{set},\mathbf{u}_{set}} \quad & z = (\mathbf{y}_{set}-\mathbf{y}^*)^T\mathbf{C}_y(\mathbf{y}_{set}-\mathbf{y}^*) \\ & + (\mathbf{u}_{set}-\mathbf{u}^*)^T\mathbf{C}_u(\mathbf{u}_{set}-\mathbf{u}^*) \\ & + \mathbf{c}_y(\mathbf{y}_{set}-\mathbf{y}^*) + \mathbf{c}_u(\mathbf{u}_{set}-\mathbf{u}^*) + \epsilon^T\mathbf{c}_\epsilon^T\mathbf{c}_\epsilon^T\epsilon \quad (1)\end{aligned}$$

s. t.

$$\begin{aligned}\mathbf{y}_{set} &= \mathbf{K}\mathbf{u}_{set} + \mathbf{d}(k) \\ \mathbf{d}(k) &= \mathbf{d}(k-1) + \delta(k) \\ \mathbf{y}_{min} - \epsilon \le \mathbf{y}_{set} &\le \mathbf{y}_{max} + \epsilon \quad (2) \\ \mathbf{u}_{min} \le \mathbf{u}_{set} &\le \mathbf{u}_{max} \\ \epsilon &\ge 0\end{aligned}$$

where $\mathbf{y}^*$ and $\mathbf{u}^*$ are the optimal nominal "targets" computed by upper level optimizers, $\mathbf{y}_{set}$ and $\mathbf{u}_{set}$ are the achievable targets to be optimized, while $\mathbf{d}(k)$ is the estimated disturbance updated by

$$\delta(k) = \mathbf{y}_m(k) - \mathbf{y}_{set}(k|k-1), \quad (3)$$

where $\mathbf{y}_m(k)$ is the measurement of outputs at time k and $\mathbf{y}_{set}(k|k-1)$ stands for the prediction of outputs in last control execution. Cost parameters can be constructed based on financial or optimization information. ϵ can be defined as a violation tolerance of the output constraints to guarantee a feasible solution to the QP. $\mathbf{K}$ is the steady-state gain matrix calculated via linearization of the nonlinear model used in an upper optimizing layer or abstracted from the linear model used by lower level MPC dynamic control.

Note the above formulation considers only the local unit, which means its solution is optimal with respect to the local objective and constraints. Starting from a conventional decentralized MPC formulation, we introduce the idea of goal-coordination method (D. A. Wismer, 1971) and formulate a block-wise large-scale QP problem for plant-wide target optimization:

$$\begin{aligned}\min_{\mathbf{Y}_{set},\mathbf{U}_{set}} \quad & Z = (\mathbf{Y}_{set}-\mathbf{Y}^*)^T\mathbf{Q}_Y(\mathbf{Y}_{set}-\mathbf{Y}^*) \\ & + (\mathbf{U}_{set}-\mathbf{U}^*)^T\mathbf{Q}_U(\mathbf{U}_{set}-\mathbf{U}^*) + \mathbf{C}_Y(\mathbf{Y}_{set}-\mathbf{Y}^*) \\ & + \mathbf{C}_U(\mathbf{U}_{set}-\mathbf{U}^*) + \eta^T\mathbf{C}_\eta^T\mathbf{C}_\eta^T\eta \quad (4)\end{aligned}$$

s. t.

$$\begin{aligned}\mathbf{Y}_{set} &= \mathbf{A}\mathbf{U}_{set} + \mathbf{D}(k) \\ \mathbf{D}(k) &= \mathbf{D}(k-1) + \Sigma(k) \\ \mathbf{Y}_{min} - \eta \le \mathbf{Y}_{set} &\le \mathbf{Y}_{max} + \eta \quad (5) \\ \mathbf{U}_{min} \le \mathbf{U}_{set} &\le \mathbf{U}_{max} \\ \eta &\ge 0\end{aligned}$$

and an augmented set of equality constraints

$$\mathbf{Y}_i set = \mathbf{U}_i set \quad (6)$$

where equation (6) represents linking constraints and interactions between units in the flowsheet (e.g., the interstream consistency conditions). In the above formulation, decision variables $\mathbf{Y}_{set}$ and

$\mathbf{U}_{set}$ are the aggregations of individual unit input and output set-point variables. The overall steady-state gain matrix $\mathbf{A}$ has a block diagonal structure $diag\{\mathbf{K_1}, \mathbf{K_2}, .., \mathbf{K_n}\}$, as do the weighting matrices $\mathbf{Q}_Y$ and $\mathbf{Q}_U$. Similarly, other vector variables are the corresponding aggregations of operating unit variables. Usually, compared with the dimension of $\mathbf{A}$, the linking constraint (6) is of much lower dimension.

Formulated as above, the plant-wide MPC target calculation falls into one class of problems that can be solved by using the price-driven coordination method.

3. PRICE-DRIVEN COORDINATION

3.1 Price-driven Coordination Strategy

The price-driven coordination method in Jose and Ungar (1998*a*; 1998*b*) is suitable for solving resource distribution or auction problems. In this method, a large-scale optimization problem is decomposed into subproblems by relaxing the resource constraints which connect the subsystems together (the linking constraints in our previous discussions). The general large-scale nonlinear optimization problem considered here is:

$$\begin{aligned} &\max_{\mathbf{x}_1,\ldots,\mathbf{x}_n} \sum_{i}^{n} \mathbf{f}_i(\mathbf{x}_i) \\ &\text{s.t.} \quad \sum_{i}^{n} R_i(\mathbf{x}_i) \le \overline{R} \\ &\qquad \mathbf{x}_i \in X_i \end{aligned} \tag{7}$$

where $\mathbf{x_i}$ is the vector of decision variables, X is the feasible set, $\mathbf{f}_i$ is the objective function for subproblem i, R_i is the vector of resource demands, and $\overline{R}$ represents the availability of common resources. The subproblems are formulated using the local constraints and the objective function obtained by augmenting $\mathbf{f}_i$ and the subsystem demands $R_i(\mathbf{x}_i)$ as shown in equation (8),

$$\max_{\mathbf{x}_i \in X_i} \quad \mathbf{f}_i(\mathbf{x}_i) - (\mathbf{p} + qR_i(\mathbf{x}_i))^T R_i(\mathbf{x}_i) \tag{8}$$

where $\mathbf{p}$ is a given price vector and q is a small positive scalar. It was shown in Jose and Ungar (1998*b*) that if the subproblems have concave, continuous objective functions and compact, convex feasible sets, there exist equilibrium augmented prices in the form of $\mathbf{p}+qR$ that optimally coordinate the subproblems for given resource availability $\overline{R}$. For a given q, the equilibrium prices satisfy the nonlinear complementarity (NLC) problem:

$$\begin{aligned} &\text{For } \mathbf{p} \in R_+ \text{ such that:} \\ &\qquad \Delta(\mathbf{p}; q) \le 0 \\ &\qquad \mathbf{p}^T(-\Delta(\mathbf{p}; q)) = 0 \end{aligned} \tag{9}$$

where $\Delta(\mathbf{p}; q) = \sum_i R_i(\mathbf{p}; q) - \overline{R}$, which is the corresponding *excess resource demand* (i.e., the difference between the total demand of all subproblems and the plant-wide resource availability). Therefore, if we can find the equilibrium prices from the above NLC problem, the optimum of (7) can be obtained by solving the subproblems independently in equation (8).

The mechanism for adjusting $\mathbf{p}$ until it satisfies equation (9) can be considered as the coordination in price-driven approach. For the plant-wide MPC problem in equation (4) to (6), the linking constraints are formulated as the equality constraints, and (9) is simplified to

$$\Delta(\mathbf{p}; q) = 0 \tag{10}$$

Therefore, in coordinating MPC subproblems, the prices can be adjusted by numerically solving the system of equations in (10).

3.2 An Efficient Price-adjustment Algorithm

An efficient price-adjustment algorithm is developed based on Newton's method for solving equation system (10). During iterations, the coordinator adjusts the prices as follows

$$\mathbf{p}(k+1) = \mathbf{p}(k) - \alpha(\frac{d\Delta(k)}{d\mathbf{p}(k)})^{-1}\Delta(k) \tag{11}$$

where $\Delta(\mathbf{p}; q) = \sum_i R_i(\mathbf{p}; q) - \overline{R}$, α is the step size, and

$$\mathbf{J} = \frac{d\Delta(k)}{d\mathbf{p}(k)} = \sum_i \frac{dR_i(k)}{d\mathbf{p}(k)} \tag{12}$$

Figure 1 explains the information exchange between the coordinator and subsystems. The coordination process is similar to setting up the prices for selling common resources to different consumers. The coordinator sends a price vector of the resources to every subsystem. After solving

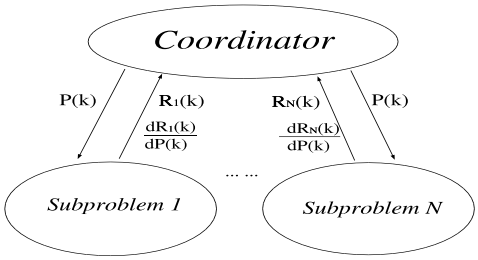

Fig. 1. Price Adjustment Schematic Diagram

local optimization problems, the subsystems inform the coordinator of the resource demands (R_i) at current prices and their responses to the price change ($dR_i/d\mathbf{p}$). The coordinator then collects these two pieces of information to evaluate Δ and $d\Delta/d\mathbf{p}$, and the prices are updated using equation(11). This process of information exchange continues until the total demand is equal to overall supply, i.e., $\Delta = 0$.

The derivative information $dR_i/d\mathbf{p}$ of local subproblems is obtained by sensitivity analysis. Procedures for sensitivity analysis of nonlinear programming problems can be found in McCormick (1983) and Wolbert *et al.* (1994). In this work, the QP subproblems have the form

$$\min_{\mathbf{x}_i} \quad (\mathbf{c}_i^T - \mathbf{p}^T\mathbf{B}_i)\mathbf{x}_i - \frac{1}{2}\mathbf{x}_i^T(\mathbf{Q}_i + q\mathbf{B}_i^T\mathbf{B}_i)\mathbf{x}_i \quad (13)$$

subject to

$$\begin{aligned} \mathbf{A}_i^{eq}\mathbf{x}_i &= \mathbf{b}_i^{eq} \\ \mathbf{A}_i^{ineq}\mathbf{x}_i &\le \mathbf{b}_i^{ineq} \end{aligned} \quad (14)$$

where $R_i(\mathbf{x}_i) = \mathbf{B}_i\mathbf{x}_i$, and

$$\sum_i \frac{dR_i(k)}{d\mathbf{p}(k)} = \sum_i \mathbf{B}_i \frac{d\mathbf{x}_i(k)}{d\mathbf{p}(k)} \quad (15)$$

where $\mathbf{B}_i$ is the coefficient matrix in the linking constraints corresponding to the variables of the i^{th} subproblem. Then the sensitivity matrix, $d\mathbf{x}_i/d\mathbf{p}$, can be obtained by solving the following system of linear equations assuming that the matrix Γ is full column-rank

$$\Gamma \begin{bmatrix} \nabla_{\mathbf{p}}\mathbf{x}_i \\ \nabla_{\mathbf{p}}\lambda_i \\ \nabla_{\mathbf{p}\,A}\mu_i \\ \nabla_{\mathbf{p}\,I}\sigma_i \end{bmatrix} = - \begin{bmatrix} \mathbf{B}_i^T \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (16)$$

where

$$\Gamma = \begin{bmatrix} \mathbf{Q} + q\mathbf{B}_i^T\mathbf{B}_i & \mathbf{A}_i^{eqT} & {}_A\mathbf{A}_i^{ineqT} & \mathbf{0} \\ \mathbf{A}_i^{eq} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ {}_A\mathbf{A}_i^{ineq} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ {}_I\mathbf{A}_i^{ineq} & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \quad (17)$$

The vectors λ and ${}_A\mu$ are the Lagrange multipliers for the equality constraints and active inequality constraints in (14), respectively, and the vector ${}_I\sigma$ is the slack variable corresponding to inactive constraints in (14). The subscripts ${}_A$ and ${}_I$ indicate the active and inactive status of the inequality constraints.

The Jacobian matrix $\mathbf{J}$ used in equation (11), which is evaluated by linear sensitivity analysis, is valid only when there is no active set change; however, during price-searching, there is no guarantee that the active set for any subproblem does not change. Therefore, a full Newton step is taken only when no active set change in each subproblem is identified. Otherwise, a step size α, less than one, should be taken.

The largest step size that could be taken before a change in active set occurs can be determined from the sensitivity information. When there is an active set change, one of the slack variables and Lagrange multipliers in the subproblems will become zero. The slack variables or Lagrange multipliers, denoted by θ, as a function of $\mathbf{p}$ is given as follows

$$\theta = \theta^*(\mathbf{p}(k)) + \nabla_{\mathbf{p}}\theta(\mathbf{p} - \mathbf{p}(k)) \quad (18)$$

Then equation (11) can be substituted into equation (18) to express θ in terms of α. We can equate θ to 0 for every slack variable and Lagrange multiplier for each subproblem to determine the value of α which makes individual constraint change its activity. The smallest positive α will be taken as the step size candidate for the current iteration. If it is less than 1, it will be chosen as the step size, otherwise, a full Newton's step is taken. Although this procedure is not shown in Figure 1 for clarity, the above calculation can be implemented in the subproblems as α can be determined independently for each subproblem.

4. ILLUSTRATIVE CASE STUDIES

4.1 Coordination Strategies Numerical Efficiency

A simple case study is used to compare the proposed algorithm and the P-control price-update scheme (Jose and Ungar, 1998*b*), where the prices are updated by

$$\mathbf{p}(k+1) = \mathbf{p}(k) + k_c\Delta(k) \quad (19)$$

Consider the following QP problem

$$\min_{\mathbf{x}} \quad \frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x} - \mathbf{c}^T\mathbf{x}$$

s. t.

$$\begin{aligned} \mathbf{A}\mathbf{x} &\le \mathbf{b} \\ \mathbf{x} &\ge \mathbf{0} \end{aligned} \quad (20)$$

where

$$\mathbf{A} = \begin{bmatrix} 2 & 5 & 7 & 3 \\ 3 & 5 & 3 & 4 \\ 1 & 3 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1.5 & 4 \\ 0 & 0 & 2 & 1 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 14 \\ 10 \\ 6 \\ 5 \\ 12 \\ 6 \end{bmatrix} \quad (21)$$

$$\mathbf{Q} = diag\{2, 4, 3, 8\} \qquad \mathbf{c}^T = [2 \;\; 5 \;\; 6 \;\; 8]$$

The problem can be decomposed into two subproblems with two linking constraints. In Table 1, performance of these two algorithms is compared based on the number of iterations required to reach the equilibrium prices. The initial guess $\mathbf{P} = 0$ is used for both cases. The proposed price-adjustment algorithm shows a substantial convergent rate improvement.

Table 1. Performance of Price-Update

Methods	Tuning	Iterations	Convergence
Newton's	NA	2	Yes
P-control	$k_c = 0.02$	400	Yes
P-control	$k_c = 0.04$	139	Yes
P-control	$k_c = 0.1$	NA	No

Furthermore, the proposed algorithm provides a guideline for the tuning of proportional gain in P-control scheme. Comparing equation (11) and (19)

shows that the proposed price-adjustment scheme adaptively updates the P-control gain within coordination, letting $k_c = -\alpha \mathbf{J}^{-1}$. Moreover, for multi-variable systems (i.e., multiple resources), k_c is a diagonal matrix in the P-control scheme, while it is a full matrix in the proposed scheme.

4.2 Coordination of QP-based Target Calculation

In this subsection, the performance of three MPC strategies, the centralized, decentralized, and coordinated decentralized strategies are compared. The system model discussed in Cheng *et al.* (2004) is used in this case study. We compare their performance in terms of objective function values and computational effort.

Shown in Figure 2, the disturbances enter the plant where dashed lines indicate, and they cause changes in the optimal set-points values. In this case study, we consider disturbances directly imposed on the outputs every 10 control executions.

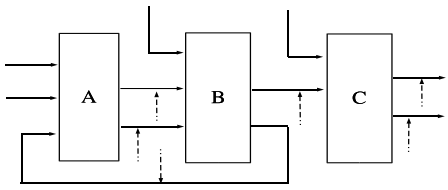

Fig. 2. Interacting MIMO Operating Units

Each operating unit has its own objective, which is a subset of information used by plant-wide optimizers. Following the QP formulation (4) to (6), and using a velocity form of decision variables, the objective is to drive the inputs and outputs as close to zero as possible. To simplify the discussion, only quadratic terms related to $\mathbf{Y}_{set}$ and $\mathbf{U}_{set}$ are included in the objective function

$$\mathbf{Y}_{max} = \mathbf{1}, \mathbf{Y}_{min} = -\mathbf{1}, \mathbf{U}_{max} = \mathbf{1}, \mathbf{U}_{min} = -\mathbf{1}$$
$$\mathbf{Q}_Y = \mathbf{I}, \quad \mathbf{Q}_U = \mathbf{I} \qquad (22)$$

Three MPC strategies (centralized, decentralized, and coordinated, decentralized MPC) are evaluated for their abilities to keep the plant at desired setpoints in steady-state target calculations. Note that, in the target calculation of decentralized scheme, the upstream decisions are considered as disturbances in the downstream units.

In this case study, disturbances are generated in a similar way as was described in Cheng *et al.* (2004), and the magnitude in ± 0.1. We use the method in equation (3) to update the disturbance information at every control execution.

The following function is used to determine cumulative performance for comparison purpose:

$$P = \sum Z(k) * T_s + \sum V(k) * T_s \qquad (23)$$

where $Z(k)$ is the actual plant objective at the k^{th} target calculation; $V(k)$ represents the penalty for output constraint violations when we implement the calculated targets; and T_s is the sampling period between two target calculations. Thus the smaller the cumulative performance function value, the closer the set-points are to the desired targets from upper level optimization, i.e., better performance is achieved.

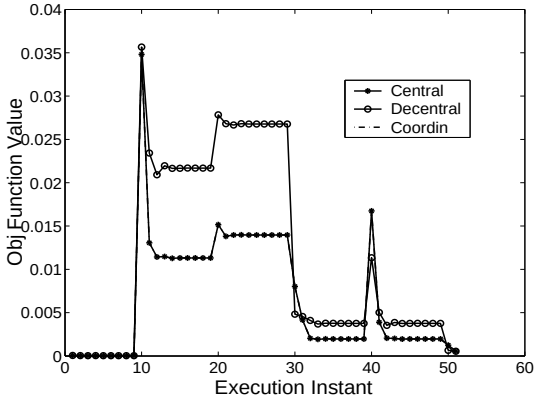

Fig. 3. Tracking Performance of MPC Strategies

Figure 3 shows the ability of different MPC steady-state target calculation strategies to track the best achievable targets. Note that the coordinated scheme gives the same performance as the centralized MPC scheme and the two curves overlap. However, the decentralized scheme shows much lower performance.

Table 2 compares the performance of different strategies for a simulation of 50 target calculation executions. From the reported objective function values in the table, we can see that the centralized and the coordinated, decentralized target calculation keep the plant running closer to desired setpoints.

Table 2. Performance Comparison

Control Strategy	Objective Function	Computation Effort* (s)	Problem Dimension
Central	0.3457	0.7073	38×14
Decentral	0.5885	0.6477	$10 \times 4 \times 3$
Coordin	0.3457	0.4096 + 1.7524	$12 \times 5 \times 3$

* Monte-Carlo simulations performed in Matlab 6.5, Pentium III 1.0G Hz, 512M RAM machine.

The computational effort is also reported in table 2 as the average simulation time for 50 MPC executions. Since all the simulations were performed in a single-processor machine, we approximate the computational load in the decentralized scheme by summing up the maximum time spent on solving each individual subproblem at one interval. We report the computational load of the coordinated scheme in two components. The first is the time taken for coordination and the second part is the time used for solving the most computationally intensive subproblem at each iteration. Two

points should be noted from this report. Firstly, the coordination time takes up a rather small part of the total time in the coordinated scheme, because price update (solving a system of linear equations) is computationally less expensive than solving local optimization problems. Secondly, the overall time for the coordinated case is only a factor of 3 more than the other two cases. This was accomplished without using any of the available coding techniques for decentralized computing. Thus the above computational efficiency suggests some promise of the proposed coordination strategy for industrial on-line application.

We discuss the problem size in order to show the ease of implementation of different MPC strategies. The problem sizes are reported as the dimension of equality constraints and inequality constraints in a QP problem formulated for an MPC controller. Thus, the centralized MPC scheme has an overall QP problem, while the other two schemes have several smaller scale problems. Note the problem size in the coordinated scheme is larger than that in the decentralized scheme due to the inclusion of tear variables. In general, the larger a QP problem, the harder to develop, tune and maintain an MPC controller.

5. CONCLUSIONS AND FUTURE WORK

In this paper, an efficient price-adjustment algorithm is developed for price-driven coordination methods. By introducing the idea of price-driven coordination, a new approach to solving plant-wide MPC target calculation has been proposed.

Many industrial problems have a block-wise structure and as a result, belong to the set of decomposable problems, which may be solved with price-driven coordination methods. Most importantly, the determination of equilibrium prices is a key factor for on-line application of the price-driven coordination techniques. This work presents a novel price-update method, which shows promise in increasing the convergence rate of the price-driven approach. This increase in computational efficiency provides an acceptable online calculation speed for solving industrial plant-wide MPC control and optimization problems. At the same time, the proposed coordinated MPC scheme retains the good features of decentralized schemes such as easy tuning of decentralized MPC controllers and a high degree of reliability.

In future work, industrial-scale MPC applications will be used to evaluate the performance of the proposed coordination method. The algorithm may be extended to cases where not all the resources are limited, so as to broaden the applicability of the technique beyond MPC.

6. ACKNOWLEDGMENTS

This study is part of a project on decomposition and coordination approaches to large-scale operations optimization. The authors would like to express their gratitude to NSERC and the Chemical and Materials Engineering Department at University of Alberta for financial support.

REFERENCES

Bodington, E. (1995). *Planning, Scheduling, and Control Integration in the Process Industries.* 1 ed.. McGraw-Hill.

Cheng, R., J. F. Forbes and W. S. Yip (2004). Dantzig-Wolfe Decomposition and Large-scale Constrained MPC Problems. In: *DYCOPS 7, paper 117.* Boston, USA.

D. A. Wismer, Editor (1971). *Optimization Methods for Large-Scale Systems – with Applications.* McGraw-Hill.

Jose, R. A. and L. H. Ungar (1998*a*). Auction-driven Coordination for Plantwide Optimization. *Foundations of Computer-aided Process Operation FOCAPO.*

Jose, R. A. and L. H. Ungar (1998*b*). Pricing Interprocess Streams Using Slack Auctions. *AIChE Journal* **46**, 575–587.

Kumar, A. and P. Daoutidis (2002). Nonlinear Dynamics and Control of Process Systems with Recycle. *Journal of Process Control* **12**, 475–484.

Lu, J. Z. (2003). Challenging Control Problems and Emerging Technologies in Enterprise Optimization. *Control Engineering Practice* **11**, 847–858.

McCormick, G. P. (1983). *Nonlinear Programming – Theory, Algorithms, and Applications.* 1 ed.. John Wiley & Sons, Inc.

Qin, S. J. and T. A. Badgewell (2003). A Survey of Industrial Model Predictive Control Technology. *Control Engineering Practice* **11**, 733–764.

Venkat, A. N., J. B. Rawlings and S. J. Wright (2004). Plant-wide Optimal Control with Decentralized MPC. In: *DYCOPS 7, paper 190.* Boston, USA.

Wolbert, D., X. Joulia, B. Koehret and L. T. Biegler (1994). Flowsheet Optimization and Optimal Sensitivity Analysis Using Analytical Derivatives. *Computer and Chemical Engineering* **18**, 1083–1095.

Ying, C. and B. Joseph (1999). Performance and Stability Analysis of LP-MPC and QP-MPC Cascade Control Systems. *AIChE Journal* **45**, 1521–1534.