A ROBUST DECONVOLUTION LMI PROCEDURE FOR FAULT DETECTION AND ISOLATION OF UNCERTAIN LINEAR SYSTEMS: AN LMI APPROACH

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Abstract: A robust Fault Detection and Isolation (FDI) scheme for uncertain polytopic linear systems based on optimal H_∞ deconvolution filters is discussed. The filter must be capable to satisfy two sets of H_∞ constraints: the first is a disturbance-to-fault decoupling requirement, whereas the second expresses the capability of the filter to track the fault signals in a prescribed frequency range. By means of the Projection Lemma, a quasi-convex formulation of the problem is obtained via LMIs. Finally, a FDI logic consisting of an adaptive thresholds scheme based on the on-line rms evaluation of relevant system variables is proposed. The effectiveness of the design technique is illustrated via a numerical example.

Keywords: Fault detection, Fault isolation, Uncertainty, Linear systems, Convex programming

1. INTRODUCTION

Fault Detection and Isolation (FDI) techniques are important topics in systems engineering from the viewpoint of improving the system reliability. A fault represents any kind of malfunction in a plant that leads to unacceptable anomalies in the overall system behavior. Such a malfunction may occur due to component failures inside the main frame of the process, sensors and/or actuators.

The issue of fault detection and isolation has been addressed by many authors in several books and survey articles where many different design methodologies have been exploited (model based approach, parameter estimation, generalized likelihood ratio etc.). See (Frank, 1990; Patton *et al.*, 1989; Qiu *et al.*, 1993) and references therein for comprehensive and up-to-date tutorials.

In this paper a novel robust H_∞ FDI design procedure is proposed for polytopic uncertain LTI systems where the residual generator is a deconvolution filter whose dynamic does not depend on any nominal plant realization. Such a filter will be designed so as to robustly decouple the residuals (Fault Detection) from the disturbances and conversely to enhance the sensitivity to each fault signal by properly separating classes of different faults (Fault Isolation). The satisfaction of requirements on disturbances decoupling and fault sensitivity enhancement leads to the minimization of standard H_∞-norm optimization problems. In particular, the first consists in minimizing the H_∞-norm of the disturbance-to-residual map whereas the second corresponds to solve an optimal H_∞ tracking problem where the objective is that the residual optimally tracks the fault signal over a prescribed frequency range.

The design methodology presented in this paper is a two steps procedure. In the first step, the synthesis of a robust FDI filter via LMI optimization techniques is described. Via the Projection Lemma and Congruence transformations (see (Tuan *et al.*, 2003) for details), the H_∞ norm constraints can be converted into quasi-LMIs feasibility conditions and efficiently solved by standard semidefinite programming solvers. The second step consists in equipping the residual generator with the capacity of discriminating between real and false alarms. This is done by resorting to decision logics based on adaptive thresholds, computed on-line on the basis of time-windowed rms-norms of the residual responses. It worth pointing out that the use of standard Luenberger observers based on nominal models (Casavola *et al.*, 2003), instead of deconvolution filters as here proposed, would have led to nonlinear matrix conditions and nonconvex optimization problems, much more difficult to be solved.

2. PROBLEM FORMULATION

Consider the following uncertain continuous-time linear system described by the following state-space model

$$P:\begin{cases}\dot{x}(t) = Ax(t)+B_u u(t)+[B_f\,B_d]\begin{bmatrix}f(t)\\ d(t)\end{bmatrix}\\ y(t) = Cx(t)+D_u u(t)+[D_f\,D_d]\begin{bmatrix}f(t)\\ d(t)\end{bmatrix}\end{cases}\quad (1)$$

where $x(t)\in\mathbb{R}^n$ represents the state, $y(t)\in\mathbb{R}^p$ the measured output, $f(t)\in\mathbb{R}^m$ a set of detectable fault signals, $d(t)\in\mathbb{R}^d$ bounded energy sensors/actuator disturbances and $u(t)\in\mathbb{R}^r$ an external manipulable command. The plant matrices belong to the polytopic family

$$\begin{bmatrix}A & B_u & B\\ C & D_u & D\end{bmatrix}\in\left\{\begin{bmatrix}A(\alpha) & B(\alpha)\\ C(\alpha) & D(\alpha)\end{bmatrix} = \sum_{i=1}^{s}\alpha_i\begin{bmatrix}A_i & B_{u,i} & B_i\\ C_i & D_{u,i} & D_i\end{bmatrix},\ \alpha\in\Gamma\right\}\quad (2)$$

where $B:=[B_f\,B_d]$, $D:=[D_f\,D_d]$ and Γ is the unitary simplex

$$\Gamma:=\left\{(\alpha_1,\ldots,\alpha_s):\sum_{i=1}^{s}\alpha_i=1,\alpha_i\geq 0\right\}$$

W.l.o.g. we can assume that the polytope system is quadratically stable (see (Boyd *et al.*, 1994)). This is the case e.g. when the system is pre-compensated. Notice that such a condition is necessary in order to satisfy the Bounded Real Lemma (see inequalities (19)-(20) in Section III). In this case, $u(t)$ plays the role of a tracking reference signal.

Fault detection and isolation (FDI) relies on the generation of a signal, referred to as residual, which must be sensitive to failures, viz. capable to distinguish failures from disturbances and discriminate failures each other. Specifically, the design must ensure that residuals are "close" to zero in fault-free situations while suitably deviating from zero in the presence of faults. With these premises, the idea is to consider a residual generator based on a deconvolution filter having the following general structure.

$$F:\begin{cases}\dot{x}_F(t) = A_F x_F(t)+B_F s(t)\\ z(t) = L_F x_F(t)+H_F s(t)\end{cases}\quad (3)$$

where $x_F(t)\in\mathbb{R}^{n_F}$, $z(t)\in\mathbb{R}^p$ and

$$s(t):=[y^T(t)\,u^T(t)]^T\in\mathbb{R}^{p+r}\quad (4)$$

Note that in the above filter structure the information coming from the reference input $u(t)$ is directly used for estimation purposes.

Let

$$r(t):=z(t)-y(t)\quad (5)$$

be the residual vector. Accordingly, the augmented system of becomes

$$G:\begin{cases}\dot{x}_{cl}(t) = A_{cl}x_{cl}(t)+B_{cl,u}u(t)+B_{cl}\begin{bmatrix}f(t)\\ d(t)\end{bmatrix}\\ r(t) = C_{cl}x_{cl}(t)+D_{cl,u}u(t)+D_{cl}\begin{bmatrix}f(t)\\ d(t)\end{bmatrix}\end{cases}\quad (6)$$

$$x_{cl}(t):=\begin{bmatrix}x(t)\\ x_F(t)\end{bmatrix},$$

$$A_{cl}:=\begin{bmatrix}A & 0\\ B_F\begin{bmatrix}C\\0\end{bmatrix} & A_F\end{bmatrix}, B_{cl}=\begin{bmatrix}B\\ B_F\begin{bmatrix}D_f & D_d\\ 0 & 0\end{bmatrix}\end{bmatrix},$$

$$B_{cl,u}:=\begin{bmatrix}B_u\\ B_F\begin{bmatrix}D_u\\ I_r\end{bmatrix}\end{bmatrix}, C_{cl}:=\begin{bmatrix}H_F\begin{bmatrix}C\\0\end{bmatrix}-C & L_F\end{bmatrix},$$

$$D_{cl}:=H_F\begin{bmatrix}D_f & D_d\\ 0 & 0\end{bmatrix}-\begin{bmatrix}D_f & D_d\end{bmatrix}, D_{cl,u}:=H_F\begin{bmatrix}D_u\\ I_r\end{bmatrix}-D_u$$

and the residual vector, depending on disturbances, fault signals and command inputs, can be rewritten as

$$r(s)=G_{rf}(s)\,f(s)+G_{rd}(s)\,d(s)+G_{ru}(s)\,u(s)\quad (7)$$

in terms of uncertain transfer functions $G_{rd}(s)$, $G_{rf}(s)$ and $G_{ru}(s)$. We shall assume hereafter that the number of faults to be isolated is less than or equal to the number of outputs. Such an assumption is necessary because we want to consider simultaneous fault occurrences.

The objectives of robust residual generation are partially conflicting each other. In fact, there exists a trade-off between the minimization of the effects of the disturbance and reference input on the residual and the maximization of the residual sensitivity to faults. The first leads to the minimization of the H_∞-norms of G_{rd} and G_{ru}. Notice that $G_{ru}\neq 0$ here because of the model uncertainty, whereas it would be zero in the uncertainty-free case thanks to the separation principle. The fault sensitivity enhancement would correspond to the maximization of the minimum singular values of G_{rf}, which is a nonconvex function of the convolution filter matrices. This complication can be gone around via the Smallest Gain Lemma (Rank, 1998), which allows the replacement of a maximization problem regarding the minimum singular value with the minimization of a standard H_∞-norm model-matching problem, specifically the minimization of the H_∞-norm of the difference between the residuals $r(s)$ and the faults $f(s)$. The latter transformation involves a certain degree of conservativeness up an extent that depends on γ_f in (10), the lower the best. For solvability reasons, the above problem only makes sense over a prescribed frequency range, $\Omega:=[\omega_i,\omega_s]$. Frequency weighting is also important from practical points of view if the disturbances and faults have known spectra. Then, for prescribed suitable levels γ_d, γ_u and γ_f, the above problems can be recast in the simultaneous satisfaction of the following conditions:

$$\max_{\alpha\in\Gamma} \overline{\sigma}_{\omega\in\Omega_1} \left(G_{rd}(j\omega)\right) \leq \gamma_d,\, \gamma_d > 0. \quad (8)$$

$$\max_{\alpha\in\Gamma} \overline{\sigma}_{\omega\in\Omega_1} \left(G_{ru}(j\omega)\right) \leq \gamma_u,\, \gamma_u > 0. \quad (9)$$

$$\max_{\alpha\in\Gamma} \overline{\sigma}_{\omega\in\Omega_2} \left(W_f(j\omega) - G_{rf}(j\omega)\right) \leq \gamma_f,\, \gamma_f > 0. \quad (10)$$

Note that conditions (8) and (9) translate into the robust decoupling of the residual w.r.t. disturbances and reference inputs in the frequency interval Ω_1, whereas condition (10) means that, in the frequency interval Ω_2, $r(s)$ robustly tracks a filtered version of the fault signal, $W_f(s)\,f(s)$, with $W_f(s)$ stable appropriately chosen. The detection and isolation problems can be recast into the following multi-objective H_∞ optimization problem:

Optimal FDI design problem (OFDP) - Given positive reals a, b and c, find a filter realization $F(s)$ such that

$$\min_{F(s)} a\gamma_d + b\gamma_f + c\gamma_u$$

subject to

$$\max_{\alpha\in\Gamma} \left\| F(s)P_{sd}(s) - P_{yd}(s) \right\|_\infty \leq \gamma_d, \quad (11)$$

$$\max_{\alpha\in\Gamma} \left\| F(s)P_{su}(s) - P_{yu}(s) \right\|_\infty \leq \gamma_u, \quad (12)$$

$$\max_{\alpha\in\Gamma} \left\| W_f(s) - \left(F(s)P_{sf}(s) - P_{yf}(s)\right) \right\|_\infty \leq \gamma_f \quad (13)$$

where $P_{sd}(s) := \begin{bmatrix} P_{yd}(s) \\ 0 \end{bmatrix}$, $P_{su}(s) := \begin{bmatrix} P_{yu}(s) \\ I_r \end{bmatrix}$, $P_{sf}(s) := \begin{bmatrix} P_{yf}(s) \\ 0 \end{bmatrix}$.

Constants a, b and c are used to trade-off between the conflicting requirements (11), (12) and (13).

The second important task for FDI consists in the evaluation of the generated residuals. One widely adopted approach is to choose a threshold $J_{th} > 0$ and use the logic

$$\begin{array}{l} J_r(t) > J_{\text{th}} \;\Rightarrow\; \text{faults} \\ J_r(t) \leq J_{\text{th}} \;\Rightarrow\; \text{no faults} \end{array} \quad (14)$$

for fault detection and

$$\begin{array}{l} J_{r,i}(t) > J_{\text{th},i} \;\Rightarrow\; \text{i-th fault} \\ J_{r,i}(t) \leq J_{\text{th},i} \;\Rightarrow\; \text{without i-th fault} \end{array} \quad (15)$$

for fault isolation where

$$J_r(t) = \sqrt{\frac{1}{t}\int_0^t r^T(\tau)\,r(\tau)\,d\tau} \quad (16)$$

See (Frank *et al.*, 1997) for a detailed discussion about this index and similarly for $J_{r_i}(t)$. Details and properties of the detection and isolation logic used in this paper will be given in next Section 4.

3. LMI FORMULATION

The design of the residual observer (3) is accomplished by recurring to the above **OFDP** optimization problem. Here it will be shown, following similar lines as in (Tuan *et al.*, 2003), that **OFDP** can be reformulated as an LMI optimization problem when the frequency constraints (11) and (13) are replaced by an equivalent set of μ-parameterized LMI (quasi-LMI) feasibility conditions

To this end, let a minimal state-space realization of the tracking filter $W_f(s)$ be given by

$$W_f(s) := \left[\begin{array}{c|c} A_r & B_{r,f} \\ \hline C_r & D_{r,f} \end{array}\right]$$

while

$$P_{sd}(s) := \left[\begin{array}{c|c} A & B_d \\ \hline \begin{bmatrix} C \\ 0 \end{bmatrix} & \begin{bmatrix} D_d \\ 0 \end{bmatrix} \end{array}\right], P_{su}(s) := \left[\begin{array}{c|c} A & B_u \\ \hline \begin{bmatrix} C \\ 0 \end{bmatrix} & \begin{bmatrix} D_u \\ I_r \end{bmatrix} \end{array}\right], P_{sf}(s) := \left[\begin{array}{c|c} A & B_f \\ \hline \begin{bmatrix} C \\ 0 \end{bmatrix} & \begin{bmatrix} D_f \\ 0 \end{bmatrix} \end{array}\right]$$

are the state-space realizations of $P_{sd}(s)$, $P_{su}(s)$ and $P_{sf}(s)$ in (11), (12) and (13). It follows that $W_f(s) - (F(s)P_{sf}(s) \;\; -P_{sf}(s))$ can be realized as

$$\left[\begin{array}{ccc|c} A_r & 0 & 0 & B_{r,f} \\ 0 & A & 0 & B_f \\ 0 & B_F\begin{bmatrix} C \\ 0 \end{bmatrix} & A_F & B_F\begin{bmatrix} D_f \\ 0 \end{bmatrix} \\ \hline C_r & -H_F\begin{bmatrix} C \\ 0 \end{bmatrix} + C & -L_F & -(H_F\begin{bmatrix} D_f \\ 0 \end{bmatrix} - D_f) \end{array}\right] =: \left[\begin{array}{cc|c} \tilde{A} & 0 & \tilde{B}_f \\ B_F\tilde{C} & A_F & B_F\begin{bmatrix} D_f \\ 0 \end{bmatrix} \\ \hline -\tilde{L}_f & -L_F & -\tilde{H}_f \end{array}\right] \quad (17)$$

for some matrices A_F, B_F, L_F and H_F to be determined with A_F asymptotically stable. Coherently, by adding the unobservable/uncontrollable modes of A_r, non-minimal state-space realizations of $F(s)P_{sd}(s) - P_{sd}(s)$ and $F(s)P_{su}(s) - P_{su}(s)$ assume the following form

$$\left[\begin{array}{ccc|c} A_r & 0 & 0 & 0 \\ 0 & A & 0 & B_d \\ 0 & B_F\begin{bmatrix} C \\ 0 \end{bmatrix} & A_F & B_F\begin{bmatrix} D_d \\ 0 \end{bmatrix} \\ \hline 0 & H_F\begin{bmatrix} C \\ 0 \end{bmatrix} - C & L_F & H_F\begin{bmatrix} D_d \\ 0 \end{bmatrix} - D_d \end{array}\right] =: \left[\begin{array}{cc|c} \tilde{A} & 0 & \tilde{B}_d \\ B_F\tilde{C} & A_F & B_F\begin{bmatrix} D_d \\ 0 \end{bmatrix} \\ \hline \tilde{L}_d & L_F & \tilde{H}_d \end{array}\right] \quad (18)$$

$$\left[\begin{array}{ccc|c} A_r & 0 & 0 & 0 \\ 0 & A & 0 & B_u \\ 0 & B_F\begin{bmatrix} C \\ 0 \end{bmatrix} & A_F & B_F\begin{bmatrix} D_u \\ I_r \end{bmatrix} \\ \hline 0 & H_F\begin{bmatrix} C \\ 0 \end{bmatrix} - C & L_F & H_F\begin{bmatrix} D_u \\ I_r \end{bmatrix} - D_u \end{array}\right] =: \left[\begin{array}{cc|c} \tilde{A} & 0 & \tilde{B}_u \\ B_F\tilde{C} & A_F & B_F\begin{bmatrix} D_u \\ I_r \end{bmatrix} \\ \hline \tilde{L}_u & L_F & \tilde{H}_u \end{array}\right] \quad (19)$$

The addition of the unobservable/uncontrollable modes of A_r in (18) and (19) makes it possible to use the same matrix vertices $\tilde{A}_i$ for the tracking and the decoupling objectives. This choice is also necessary in order to apply the Projection Lemma and obtain a convex design. For the same reason, note that the filter dimension (the dimension of A_F) must satisfy

$$n_F = n_r \text{ (tracking)} + n \text{ (plant)} \quad \text{(Full Order Filter)}$$

Then, by exploiting the Bounded Real Lemma, conditions (8), (9) and (10) are jointly satisfied iff there exist filter matrices A_F, B_F, L_F and H_F, with A_F

asymptotically stable, and an auxiliary matrix $X = X^T \in \mathbb{R}^{2n_F \times 2n_F}$, $X > 0$, such that the following matrix inequalities

$$\begin{bmatrix} \begin{pmatrix} \tilde{A} & 0 \\ B_F \tilde{C}_f & A_F \end{pmatrix}^T X + X \begin{pmatrix} \tilde{A} & 0 \\ B_F \tilde{C}_f & A_F \end{pmatrix} & X \begin{pmatrix} \tilde{B}_f \\ B_F \begin{bmatrix} D_f \\ 0 \end{bmatrix} \end{pmatrix} & \begin{pmatrix} -\hat{L}_f^T \\ -L_F^T \end{pmatrix} \\ \begin{pmatrix} \tilde{B}_f^T & \begin{bmatrix} D_f \\ 0 \end{bmatrix}^T B_F^T \end{pmatrix} X & -\gamma_f I & -\tilde{H}_f^T \\ \begin{pmatrix} -\tilde{L}_f & -L_F \end{pmatrix} & -\tilde{H}_f & -\gamma_f I \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} \begin{pmatrix} \tilde{A} & 0 \\ B_F \tilde{C}_d & A_F \end{pmatrix}^T X + X \begin{pmatrix} \tilde{A} & 0 \\ B_F \tilde{C}_d & A_F \end{pmatrix} & X \begin{pmatrix} \tilde{B}_d \\ B_F \begin{bmatrix} D_d \\ 0 \end{bmatrix} \end{pmatrix} & \begin{pmatrix} \hat{L}_d^T \\ L_F^T \end{pmatrix} \\ \begin{pmatrix} \tilde{B}_d^T & \begin{bmatrix} D_d \\ 0 \end{bmatrix}^T B_F^T \end{pmatrix} X & -\gamma_d I & \tilde{H}_d^T \\ \begin{pmatrix} \tilde{L}_d & L_F \end{pmatrix} & \tilde{H}_d & -\gamma_d I \end{bmatrix} \tag{21}$$

$$\begin{bmatrix} \begin{pmatrix} \tilde{A} & 0 \\ B_F \tilde{C}_u & A_F \end{pmatrix}^T X + X \begin{pmatrix} \tilde{A} & 0 \\ B_F \tilde{C}_u & A_F \end{pmatrix} & X \begin{pmatrix} \tilde{B}_u \\ B_F \begin{bmatrix} D_u \\ I_r \end{bmatrix} \end{pmatrix} & \begin{pmatrix} \hat{L}_u^T \\ L_F^T \end{pmatrix} \\ \begin{pmatrix} \tilde{B}_u^T & \begin{bmatrix} D_u \\ I_r \end{bmatrix}^T B_F^T \end{pmatrix} X & -\gamma_u I & \tilde{H}_u^T \\ \begin{pmatrix} \tilde{L}_u & L_F \end{pmatrix} & \tilde{H}_u & -\gamma_f I \end{bmatrix} \tag{22}$$

will be negative definite. Notice that, for any given quadruple (A_F, B_F, L_F, H_F) with A_F stable, the above inequalities are jointly solvable for some symmetrical matrix $X > 0$ and for sufficiently large γ_f, γ_d and γ_u. As a usual in the multiobjective optimization, a single matrix X is used in both the LMI conditions (20), (21) and (22).

By using the Projection Lemma (PL) and following similar lines of (Tuan *et al.*, 2003), inequalities (20), (21) and (22) are satisfied iff (see (Tuan *et al.*, 2003) for details) the following matrix inequalities

$$\begin{bmatrix} -\mu(V+V^T) & V^T A_{cl,f}+X & V^T B_{cl,f} & 0 & \mu V^T \\ A_{cl,f}^T V+X & -X & 0 & \begin{pmatrix} -\tilde{L}_f^T \\ -L_F^T \end{pmatrix} & 0 \\ B_{cl,f}^T V & 0 & -\gamma_f I & -\tilde{H}_f^T & 0 \\ 0 & \begin{pmatrix} -\tilde{L}_f & -L_F \end{pmatrix} & -\tilde{H}_f & -\gamma_f I & 0 \\ \mu V & 0 & 0 & 0 & -X \end{bmatrix} \tag{23}$$

$$\begin{bmatrix} -\mu(V+V^T) & V^T A_{cl,d}+X & V^T B_{cl,d} & 0 & \mu V^T \\ A_{cl,d}^T V+X & -X & 0 & \begin{pmatrix} \tilde{L}_d^T \\ L_F^T \end{pmatrix} & 0 \\ B_{cl,d}^T V & 0 & -\gamma_d I & \tilde{H}_d^T & 0 \\ 0 & \begin{pmatrix} \tilde{L}_d & L_F \end{pmatrix} & \tilde{H}_d & -\gamma_d I & 0 \\ \mu V & 0 & 0 & 0 & -X \end{bmatrix} \tag{24}$$

$$\begin{bmatrix} -\mu(V+V^T) & V^T A_{cl,u}+X & V^T B_{cl,U} & 0 & \mu V^T \\ A_{cl,u}^T V+X & -X & 0 & \begin{pmatrix} \tilde{L}_u^T \\ L_F^T \end{pmatrix} & 0 \\ B_{cl,u}^T V & 0 & -\gamma_u I & \tilde{H}_u^T & 0 \\ 0 & \begin{pmatrix} \tilde{L}_u & -L_F \end{pmatrix} & \tilde{H}_u & -\gamma_u I & 0 \\ \mu V & 0 & 0 & 0 & -X \end{bmatrix} \tag{25}$$

are negative definite. Here, V is a slack variable of proper dimensions and $\mu \geq 0$ is a scalar that can be selected to be sufficiently large to render (23), (24) and (25) feasible

$$A_{cl,f} := \begin{pmatrix} \tilde{A} & 0 \\ B_F \tilde{C}_f & A_F \end{pmatrix}, B_{cl,f} := \begin{pmatrix} \tilde{B}_f \\ B_F \begin{bmatrix} D_f \\ 0 \end{bmatrix} \end{pmatrix},$$

$$A_{cl,d} := \begin{pmatrix} \tilde{A} & 0 \\ B_F \tilde{C}_d & A_F \end{pmatrix}, B_{cl,d} := \begin{pmatrix} \tilde{B}_d \\ B_F \begin{bmatrix} D_d \\ 0 \end{bmatrix} \end{pmatrix}.$$

$$A_{cl,u} := \begin{pmatrix} \tilde{A} & 0 \\ B_F \tilde{C}_u & A_F \end{pmatrix}, B_{cl,u} := \begin{pmatrix} \tilde{B}_u \\ B_F \begin{bmatrix} D_u \\ I_r \end{bmatrix} \end{pmatrix}.$$

By partitioning V and X as 2×2 block-matrices

$$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}, X = \begin{bmatrix} X_1 & X_3^T \\ X_3 & X_2 \end{bmatrix}$$

and by taking into account the structure of $A_{cl,f}$, $B_{cl,f}$, $A_{cl,d}$ and $B_{cl,d}$, (23)-(25) become

$$\left[\begin{array}{cccc} -\mu(V_{11}+V_{11}^T) & -\mu(S_2+S_2^T) & V_{11}^T\tilde{A}+\hat{B}_F\tilde{C}_f+\hat{X}_1 & \hat{A}_F+\hat{X}_3^T \\ -\mu(S_2+S_2^T) & -\mu(S_1+S_1^T) & V_{12}^T\tilde{A}+\hat{B}_F\tilde{C}_f+X_3 & \hat{A}_F+X_2 \\ (*) & (*) & -\hat{X}_1 & -\hat{X}_3^T \\ (*) & (*) & -\hat{X}_3 & -\hat{X}_2 \\ (*) & (*) & (*) & (*) \\ (*) & (*) & (*) & (*) \\ (*) & (*) & (*) & (*) \\ (*) & (*) & (*) & (*) \end{array}\right.$$

$$\left.\begin{array}{cccc} V_{11}^T\tilde{B}_f+\hat{B}_F D_f & 0 & \mu V_{11}^T & \mu S_2 \\ V_{12}^T B+\hat{B}_F D_f & 0 & \mu S_2^T & \mu S_1 \\ 0 & \begin{pmatrix} C_f^T \\ -C_f^T H_F^T + C_f^T \end{pmatrix} & 0 & 0 \\ 0 & -\hat{L}_F^T & 0 & 0 \\ -\gamma_f I & -(H_F \begin{bmatrix} D_f \\ 0 \end{bmatrix} - D_f)^T & 0 & 0 \\ (*) & -\gamma_f I & 0 & 0 \\ (*) & (*) & -\hat{X}_1 & -\hat{X}_3^T \\ (*) & (*) & -\hat{X}_3 & -\hat{X}_2 \end{array}\right] < 0$$

$$\left[\begin{array}{cccc} -\mu(V_{11}+V_{11}^T) & -\mu(S_2+S_2^T) & V_{11}^T\tilde{A}+\hat{B}_F\tilde{C}_d+\hat{X}_1 & \hat{A}_F+\hat{X}_3^T \\ -\mu(S_2+S_2^T) & -\mu(S_1+S_1^T) & V_{12}^T\tilde{A}+\hat{B}_F\tilde{C}_d+X_3 & \hat{A}_F+X_2 \\ (*) & (*) & -\hat{X}_1 & -\hat{X}_3^T \\ (*) & (*) & -\hat{X}_3 & -\hat{X}_2 \\ (*) & (*) & (*) & (*) \\ (*) & (*) & (*) & (*) \\ (*) & (*) & (*) & (*) \\ (*) & (*) & (*) & (*) \end{array}\right.$$

$$\left.\begin{array}{cccc} V_{11}^T\tilde{B}_d+\hat{B}_F D_d & 0 & \mu V_{11}^T & \mu S_2 \\ V_{12}^T B+\hat{B}_F D_d & 0 & \mu S_2^T & \mu S_1 \\ 0 & \begin{pmatrix} 0 \\ C_d^T H_F^T - C_d^T \end{pmatrix} & 0 & 0 \\ 0 & \hat{L}_F^T & 0 & 0 \\ -\gamma_d I & (H_F \begin{bmatrix} D_d \\ 0 \end{bmatrix} - D_d)^T & 0 & 0 \\ (*) & -\gamma_d I & 0 & 0 \\ (*) & (*) & -\hat{X}_1 & -\hat{X}_3^T \\ (*) & (*) & -\hat{X}_3 & -\hat{X}_2 \end{array}\right] < 0$$

$$\left[\begin{array}{cccc} -\mu(V_{11}+V_{11}^T) & -\mu(S_2+S_2^T) & V_{11}^T\tilde{A}+\hat{B}_F\tilde{C}_d+\hat{X}_1 & \hat{A}_F+\hat{X}_3^T \\ -\mu(S_2+S_2^T) & -\mu(S_1+S_1^T) & V_{12}^T\tilde{A}+\hat{B}_F\tilde{C}_d+X_3 & \hat{A}_F+X_2 \\ (*) & (*) & -\hat{X}_1 & -\hat{X}_3^T \\ (*) & (*) & -\hat{X}_3 & -\hat{X}_2 \\ (*) & (*) & (*) & (*) \\ (*) & (*) & (*) & (*) \\ (*) & (*) & (*) & (*) \\ (*) & (*) & (*) & (*) \end{array}\right.$$

$$\begin{array}{cccc} V_{11}^T \tilde{B}_u + \hat{B}_F D_u & 0 & \mu V_{11}^T & \mu S_2 \\ V_{12}^T B + \hat{B}_F D_u & 0 & \mu S_2^T & \mu S_1 \\ 0 & \begin{pmatrix} 0 \\ C_u^T H_F^T - C_u^T \end{pmatrix} & 0 & 0 \\ 0 & \hat{L}_F^T & 0 & 0 \\ -\gamma_u I & (H_F \begin{bmatrix} D_u \\ I_r \end{bmatrix} - D_u)^T & 0 & 0 \\ (*) & -\gamma_u I & 0 & 0 \\ (*) & (*) & -\hat{X}_1 & -\hat{X}_3^T \\ (*) & (*) & -\hat{X}_3 & -\hat{X}_2 \end{array} \Bigg] < 0$$

where $\hat{A}_F = V_{21}^T A_F V_{22}^{-1} V_{21}$, $\hat{B}_F = V_{21}^T B_F$, $\hat{L}_F = L_F V_{22}^{-1} V_{21}$, $S_1 = V_{21}^T V_{22}^{-T} V_{21}$, $S_2 = V_{21}^T V_{22}^{-T} V_{12}$

$$\hat{X} = \begin{bmatrix} \hat{X}_1 & \hat{X}_3^T \\ \hat{X}_3 & \hat{X}_2 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & V_{21}^T V_{22}^{-T} \end{bmatrix} X \begin{bmatrix} I & 0 \\ 0 & V_{22}^{-1} V_{21} \end{bmatrix}$$

The previous three inequalities are indeed quasi-linear (μ-parameterized LMIs) in $\hat{X}, S_1, S_2, \hat{A}_F, \hat{B}_F, V_{11}, \hat{L}_F, H_F$. The next main result summarizes the above discussion and provides a procedure for solving **OFDP**:

Theorem 1. - A feasible solution to the OFDP problem is obtained by solving a sequence of μ-parameterized optimization problems

$$\min_{\hat{X}, S_1, S_2, \hat{A}_F, \hat{B}_F, V_{11}, \hat{L}_F, H_F} a\gamma_d + b\gamma_f + c\gamma_u$$

subject to

$$\begin{array}{c} \hat{X} > 0 \\ (23)-(25), \\ \text{evaluated over the polytope vertices (2).} \end{array} \tag{26}$$

For any choice of $\mu > 0$, if solvable, the above problem is convex and admits a unique solution.

Remark 1. The matrices A_F, B_F, L_F, H_F, defining the residual generator can be derived by means of the following procedure. Let us denote $\hat{X}, S_1, S_2, \hat{A}_F, \hat{B}_F, V_{11}, \hat{L}_F$ a solution of (26):

(1) compute V_{22}, V_{21} ($n_F \times n_F$) by solving the following factorization problem

$$S_1 = V_{21}^T V_{22}^{-1} V_{21},$$

(2) compute A_F, B_F, L_F

$$A_F = V_{21}^{-T} \hat{A}_F V_{21}^{-1} V_{22}, B_F = V_{21}^{-T} \hat{B}_F, L_F = \hat{L}_F V_{21}^{-1} V_{22}.$$

4. THRESHOLDS COMPUTATION

The detection and isolation decision logic is based on the adaptive residual thresholds evaluation proposed by (Frank *et al.*, 1997). Consider first the detection problem and let the time-windowed rms-norm

$$\begin{aligned} J_r(t) = \ & \|r\|_{\text{rms},t} = \sqrt{\frac{1}{t}\int_0^t r^T(\tau) r(\tau)\, d\tau}, \\ J_{r_i}(t) = \ & \|r_i\|_{\text{rms},t} = \sqrt{\frac{1}{t}\int_0^t r_i^T(\tau) r_i(\tau)\, d\tau} \end{aligned}$$

be a convenient residual measure. Under fault-free conditions, (7) becomes $r(s) = G_{rd}\, d(s) + G_{ru}\, u(s)$ and via the Perseval's Theorem one has that

$$\begin{aligned} \|r\|_{\text{rms},t,f=0} &= \|r_d + r_u\|_{\text{rms},t} \\ &\leq \|G_{rd}\|_\infty \|d\|_{\text{rms}} + \|G_{ru}\|_\infty \|u\|_{\text{rms},t} \\ &= \gamma_d \nu + \gamma_u \|u\|_{\text{rms},t} \end{aligned}$$

where γ_f is the solution of (10) and ν is a convenient upper-bound to the rms-norm of the worst disturbance acting on the plant. As a consequence, the following threshold results

$$J_{th}(t) := \gamma_d \nu + \gamma_u \|u\|_{\text{rms},t}\,. \tag{27}$$

Isolation thresholds can be derived in a similar way. Let us consider the i-th component of residual vector $r(t)$. When the i-th fault signal is equal to zero in $[0,t]$, one has that

$$\begin{aligned} \|r_i\|_{\text{rms},t,f_i=0} &= \Big\| r_{i,d} + \sum_{j=1, j\neq i}^m r_{i,f_j} + r_{i,u} \Big\|_{\text{rms},t,f_i=0} \\ &\leq \|G_{rd}\|_\infty \|d\|_{\text{rms}} + \sum_{j=1, j\neq i}^m \|e_i^T G_{rf} e_j\|_\infty \|f_j\|_{\text{rms},t} + \gamma_u \|u\|_{\text{rms},t} \end{aligned}$$

where e_i is the canonical basis of $\mathbb{R}^m$ and $r_{i,d}$ and $r_{i,u}$ denote the distinct amounts of r_i depending on disturbances and, respectively, on the external input $u(t)$. By denoting with $\xi_{ij} := \|e_i^T G_{rf} e_j\|_\infty$ the H_∞-norm of the map between the j-th fault to the i-th residual, a convenient isolation threshold is given by

$$J_{th,i}(t) := \gamma_d \nu + \sum_{j=1, j\neq i}^m \xi_{ij} \beta_j + \gamma_u \|u\|_{\text{rms},t} \tag{28}$$

where β_j denotes an upper bound to the rms-norm of the $j-th$ fault class.

5. AN ISOLATION EXAMPLE

This example aims at illustrating the isolation capability of filters designed by the proposed method. To this end, consider the following uncertain LTI system

$$P_0 \begin{cases} y_1(s) = \dfrac{k_1}{s^2 + \theta_1 s + \theta_2}(u(s) + f_1(s)) \\ \qquad + \dfrac{k_1}{(sT_1+1)(s^2+\theta_1 s+\theta_2)} d_1(s) \\ y_2(s) = \dfrac{k_3 k_1}{s^2 + \theta_1 s + \theta_2}(u(s) + f_1(s)) + \dfrac{k4}{sT_4+1} f_2(s) \\ \qquad + \dfrac{k_1}{(sT_1+1)(s^2+\theta_1 s+\theta_2)} d_1(s) + \dfrac{k2(sT_2+1)}{sT_3+1} d_2(s) \end{cases}$$

where: $T_1 = 0.1$, $T_2 = 10$, $T_3 = 0.2$, $T_4 = 1$, $k_1 = 1$, $k_2 = 0.2$, $k_3 = k_4 = 10$ and the parameters θ_1, θ_2 belonging to the intervals $0.5 \leq \theta_1 \leq 1.2$, $1 \leq \theta_2 \leq 1.5$ The signals $d_1(t)$ and $d_2(t)$ are assumed to be unitary variance white noises. We are interested here to isolate faults over the frequency interval $\Omega = [0, 1]\frac{\text{rad}}{\text{s}}$. Here, we consider the following output filter ($P(s) = H(s) P_0(s)$) and tracking filter $H(s) = \text{diag}([\frac{1}{s+1}, \frac{1}{s+1}])$, $W_f(s) = \text{diag}([\frac{1}{s^2+s+1}, \frac{1}{s+1}])$. Note that a polytopic state-space realization of $P(s)$ consisting of four vertices results. By solving the quasi-convex problem (26) for $a = 1, b = 1, c = 1$, it results that the lowest value of μ ensuring the feasibility of (26) equals $\mu = 1.2$, and the corresponding optimal values of the objective function terms are $\gamma_d = 0.0230$, $\gamma_f = 0.7749$ and $\gamma_u = 1.3351$. In this example, the simulations have been carried out by taking the uncertain plant parameters constant at one of its vertices ($\theta_1 = 0.5$, $\theta_2 = 1$). The detection and isolation capability of the filter can be observed in Fig. 1, where the time responses $r_i(t)$, $i = 1, 2$ for $u(t) = 0$ are reported under superimposed unitary variance white noises d_1 and d_2 and for the following faults occurrence

$$f_1(t) = \begin{cases} 0 & t < 5\text{s}. \\ 1 & 5\text{s}. \leq t \leq 50\text{s}. \\ 0 & 50\text{s}. < t \leq 100\text{s}. \end{cases}, \quad f_2(t) = \begin{cases} 0 & t \leq 50\text{s}. \\ 1 & 50\text{s}. < t \leq 100\text{s}. \end{cases}$$

The residual $r(t)$, fault indexes $J_{r_i}(t)$ and thresholds $J_{th,i}(t), i = 1,2$ for an external signal $u(t) = 1 - e^{-0.01t}$, $t \geq 0$, are finally shown in Figs. 2-3.

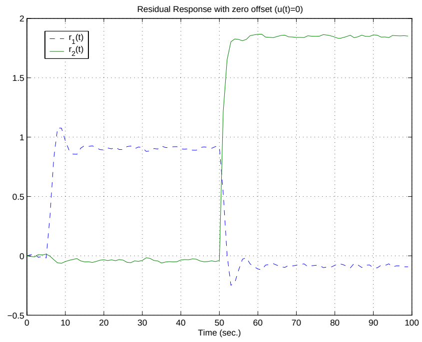

Fig. 1. Residual response, dashed line $r_1(t)$, continuous line $r_2(t)$ for $u(t) \equiv 0$

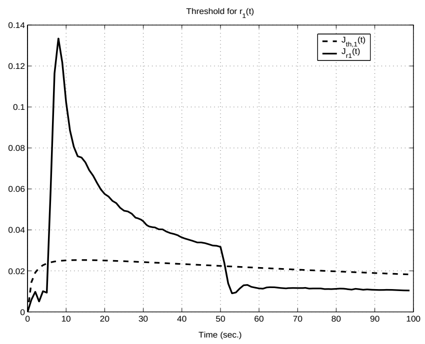

Fig. 2. Threshold on the first residual $J_{th,1}(t)$ for $u(t) = 1 - e^{-0.01t}$

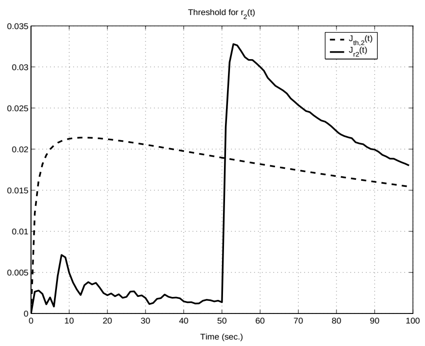

Fig. 3. Threshold on the second residual $J_{th,2}(t)$ for $u(t) = 1 - e^{-0.01t}$

6. CONCLUSIONS

A novel solution for Robust Fault Detection and Isolation for Linear Polytopic Uncertain plants via deconvolution filters has been proposed. By taking advantage of the Projection Lemma and using Congruence transformations, the FDI problem has been converted into a quasi-LMI optimization problem. An adaptive threshold logic has been proposed in order to to discriminate between real and false alarms. A numerical example showing the effectiveness of the proposed approach has been described in details and the results have shown good detection and isolation capabilities.

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