ADAPTIVE ROBUST BACKSTEPPING NONLINEAR ALGORITHM APPLIED TO SHIP STEERING

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Abstract: An adaptive robust nonlinear steering controller for ship's course is developed. Incorporating the Nussbaum-type function gain and adaptive backstepping algorithm, the proposed control strategy requires neither known dynamic parameters of a ship's model nor any knowledge about environmental disturbances. It is proved that the designed adaptive robust controller of ship steering can guarantee that the uniform ultimate boundedness of the resulting closed-loop system signals. Dynamic performance can be improved by an appropriate choice of the design parameters. The effectiveness of the presented algorithm has been demonstrated in a simulation involving a ship of 45m in length. *Copyright* © 2005 IFAC

Keywords: ship control, adaptive, robust, nonlinear, tracking.

1. INTRODUCTION

The ship course control directly influences the manoeuvrability, economy and security of ship navigation. An autopilot is a ship's steering controller, which automatically manipulates the rudder to decrease the errors between the desired heading angle and the actual heading angle. Conventional autopilots are based on simple PID control. Modern autopilots using the LQG and H_{∞} control design techniques have been reported in the literature by a large number of authors (Fossen, 2000). In addition, a large number of other advanced control methods, such as model reference adaptive control (Amerongen, 1984) and neural network adaptive control (Unar and Murray-smith, 1999), have been trialled in ship steering applications since the 1980s. So far, the linear Nomoto model has been widely accepted in the design of a ship course controller. However, in some cases such as a coursechanging manoeuvre calling for large rudder movements, ship steering equations of motion are highly nonlinear. For a certain nonlinear ship model, a state feedback linearization control law can be designed (Fossen, 1993), while feedback linearization with saturating and slew rate limiting actuators has been discussed by Tzeng et al. (1999). However they do not possess the robustness to changes of parameter and model. In order to maintain the desired performance of autopilots, it is important for designers to pay attention to the effects on a ship's motions of strong disturbances induced by wind, waves and currents, and to the robustness to changes in a ship's dynamics due to cargo loads, fouling and so on. A nonlinear adaptive autopilot based on backstepping and Nussbaum gain is developed for parametric uncertain ship without a priori knowledge of control gain (Du, 2004). A robust adaptive nonlinear control algorithm for ship steering autopilots, based on the projection approach and the Lyapunov stability theory, was presented by Yang (2000). However, the design procedure requires a priori knowledge of the signs of unknown control coefficients.

In this paper, an adaptive robust nonlinear control

strategy for ship steering is presented based on the Nussbaum-type function (Nussbaum, 1983) and adaptive backstepping algorithm (Krstic, *et al.*,1995). An adaptive robust nonlinear controller of ship steering model with both completely unknown parameters and unknown disturbances. Uniform boundedness of all signals in the resulting closed-loop course error system is guaranteed, and control law is smooth. Suitable transient performance and control action can be achieved by carefully adjusting the design parameters.

The paper is organized as follows: Section 2 discusses nonlinear models for ship steering with uncertainties of parameters and disturbances. In Section 3 the adaptive robust nonlinear controller of ship course-changing is presented. Section 4 contains the simulation study while Section 5 summarizes the conclusion.

2. SHIP STEERING EQUATIONS OF MOTION

For small rudder angles, the transfer function between the rudder angle δ and the yaw rate r of a ship can be described by the linear 1st order model of Nomoto *et al.*.

$$\frac{r(s)}{\delta(s)} = \frac{K}{Ts+1} \tag{1}$$

K is the gain constant and T is time constant. The yaw angle $\psi(t)$ is related to the yaw rate r(t) as: $\dot{\psi}(t) = r(t)$.

To describe large rudder angles and course-unstable ships, an extension of the 1st order model of Nomoto can be made by defining

$$T\ddot{\psi} + \dot{\psi} + \alpha\dot{\psi}^3 = K\delta \tag{2}$$

where α is the Norrbin coefficient which is produced by reverse spiral manoeuvres (Fossen, 1994).

The ship's parameters are basically determined by its size and shape, and may vary depending on operational conditions such as ship speed, draft, trim, and water depth. The main environmental disturbances are wind, waves and currents, which are variable. Accordingly, the following nonlinear ship model is suggested in the controller design discussed in this paper.

$$T\ddot{\psi} + \dot{\psi} + \alpha \dot{\psi}^3 = K(\delta + \delta_{\omega}) \tag{3}$$

where parameters K, T, α are unknown, δ_{ω} represents an unknown bounded equivalent rudder angle induced by environmental disturbances such as waves, wind, ocean currents. δ_{ω} is represented as

$$\left|\delta_{\omega}\right| \le p^* \tag{4}$$

where p^* is an unknown positive constant.

To meet the demands of the later design stages, state variables are selected as $x_1 = \psi$, $x_2 = \dot{\psi}$, and control variable as $u = \delta$. $y = \psi$ is output variable of the ship steering system. Then equation (3) is transformed into the following dynamic equations:

$$\dot{x}_1 = x_2 \tag{5}$$

$$\dot{x}_2 = gu + W^{*T} \varphi(x_2) + g\delta_{\omega}$$
(6)

$$y = x_1 \tag{7}$$

where, g = K/T, $W^* = [-1/T, -\alpha/T]^T$, $\varphi(x_2) = [x_2, x_2^3]^T$. In equation (3), T > 0 when a ship's line-movement is stable; whereas T < 0 when a ship's line-movement is unstable. The sign of control coefficient g is viewed as the unknown in this paper.

3. DESIGN OF NONLINEAR ADAPTIVE ROBUST AUTOPILOT

Ship autopilots can be designed to perform two entirely different functions: course-changing and course-keeping. Particular consideration is given to the course-changing problem in this paper. During course-changing operations it is desirable to specify the dynamics of the desired heading instead of using a constant reference signal. One simple way to do this is by applying model reference techniques. The ideal performance can be given by reference model (8) (Fossen, 1994).

$$\Psi_d = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}\Psi_r \qquad (8)$$

where $\boldsymbol{\xi}$ and $\boldsymbol{\omega}_n$ are the parameters to be determined that describe the closed-loop system response characteristics; the input $\boldsymbol{\psi}_r$ of the reference model is the course-changing heading command, and the output $\boldsymbol{\psi}_d$ of the reference model is the desired smooth course-changing signal. A second-order model is sufficient to generate the desired smooth course signal $\boldsymbol{\psi}_d$ with known and bounded derivatives $\boldsymbol{\psi}_d, \boldsymbol{\psi}_d$.

The adaptive robust nonlinear controller of course tracking of the system (5)-(7) is designed as follows. The design consists of 2 steps.

Step 1: Define the course heading error $z_1 = x_1 - \psi_d$, which satisfies

$$\dot{z}_1 = x_2 - \dot{\psi}_d \tag{9}$$

 x_2 is viewed as the virtual control input. The intermediate control function ϕ_1 for the virtual control x_2 to stabilize (9) is defined as

$$\phi_1(z_1) = -c_1 z_1 + \dot{\psi}_d \tag{10}$$

where c_1 is a positive design constant. However, x_2 is not the actual control, hence a difference between x_2 and the intermediate control function ϕ_1 , which is defined as $z_2 = x_2 - \phi_1$. Accordingly, (9) should be expressed as:

$$\dot{z}_1 = -c_1 z_1 + z_2 \tag{11}$$

Consider Lyapunov function V_1

$$V_1 = \frac{1}{2}z_1^2$$
 (12)

The time derivative of V_1 along the solution of (11) is given by

$$\dot{V}_1 = -c_1 z_1^2 + z_1 z_2 \tag{13}$$

The undesired effects of z_2 on \dot{V}_1 will be controlled at the next step.

Step 2: Using (9), (10) and the definition for z_2 , the derivative of z_2 is expressed as

$$\dot{z}_2 = gu + W^{*T} \varphi(x_2) + g \delta_{\omega} + c_1 x_2 - c_1 \dot{\psi}_d - \ddot{\psi}_d$$
(14)

In the light of (4) and (14), the time derivative of $\frac{1}{2}z_2^2$ is

$$z_{2}\dot{z}_{2} \leq z_{2}[gu + W^{*T}\varphi(x_{2}) + c_{1}x_{2} - c_{1}\dot{\psi}_{d} - \ddot{\psi}_{d}] + |z_{2}|gp^{*}$$
(15)

For notional convenience, let $W_a^* = \begin{bmatrix} 1 & W^{*T} \end{bmatrix}^T$, $\varphi_a = \begin{bmatrix} c_1 x_2 - c_1 \dot{\psi}_d & - \ddot{\psi}_d & \varphi(x_2)^T \end{bmatrix}^T$, then (15) can be further written as

$$z_2 \dot{z}_2 \le z_2 [gu + W_a^{*T} \varphi_a(x_2)] + |z_2| b^*$$
 (16)

where $b^* = gp^*$.

In order to cope with the unknown sign of control coefficient g, the Nussbaum gain technique is

employed in this paper. A differentiable function N(k) is called a Nussbaum-type function if it has the following properties:

$$\lim_{s \to \infty} \sup \frac{1}{s} \int_{0}^{s} N(k) dk = \infty$$
(17)

 $\lim_{s \to \infty} \inf_{s \to \infty} \frac{1}{s} \int_{0}^{0} N(k) dk = -\infty$ (18)

Commonly used Nussbaum functions include: $\exp(k^2)\cos((\pi/2)k)$, $k^2\cos(k)$, $k^2\sin(k)$ (Ge and Wang, 2002).

To complete the later controller design, the following lemma relating to the property of Nussbaum gain and two useful technical lemmas are given.

Lemma1: (Ge and Wang, 2002) Let $V(\cdot)$ and $k(\cdot)$ be smooth functions defined on $[0, t_f)$ with $V(t) \ge 0$, $\forall t \in [0, t_f)$, and $N(\cdot)$ be an even smooth Nussbaum-type function. If the following inequality holds:

$$V(t) \le C_0 + e^{-C_1 t} \int_0^t (\theta N(k) + 1) \dot{k} e^{C_1 \tau} d\tau$$
$$\forall t \in [0, t_f) \quad (19)$$

where constant $C_1 > 0$, θ is a nonzero constant and C_0 represents a suitable constant, then V(t), k(t) and $\int_0^t \theta N(k) \dot{k} d\tau$ must be bounded on $[0, t_f)$.

Consider Lyapunov function V_{20}

$$V_{20} = \frac{1}{2} z_2^2 + \frac{1}{2} (\hat{W}_a - W_a^*)^T \Gamma^{-1} (\hat{W}_a - W_a^*) + \frac{1}{2\lambda} (\hat{b} - b^*)^2$$
(20)

where $\Gamma = \Gamma^T > 0$, $\lambda > 0$, and \hat{W}_a and \hat{b} are the parameter estimates of W_a^* and b^* respectively to be determined later.

Let u be

$$u = N(k)(c_2 z_2 + \hat{W}_a^T \varphi_a + \hat{b} \tanh(\frac{z_2}{\varepsilon})) \quad (21)$$

$$N(k) = k^2 \cos(k) \tag{22}$$

$$\dot{k} = c_2 z_2^2 + z_2 \hat{W}_a^T \varphi_a + z_2 \hat{b} \tanh(\frac{z_2}{\varepsilon}) \quad (23)$$

where c_2 is a positive design constant, \mathcal{E} is a small positive constant and N(k) is an even smooth Nussbaum-type function.

In order to prevent parameter drifts, the following adaptive law incorporating a leakage term based on a variation of σ -modification is presented. Let the parameter adaptation laws be

$$\dot{\hat{W}}_a = \Gamma \varphi_a z_2 - \Gamma \sigma_w (\hat{W}_a - W_a^0) \qquad (24)$$

$$\dot{\hat{b}} = \lambda z_2 \tanh(\frac{z_2}{\varepsilon}) - \lambda \sigma_b (\hat{b} - b^0) \quad (25)$$

where $\sigma_w, \sigma_b, W_a^0, b^0 > 0$ are design constants.

Noting (16), (21), (23), the time derivative of V_{20} is

$$\dot{V}_{20} = z_2 \dot{z}_2 + (\hat{W}_a - W_a^*)^T \Gamma^{-1} \dot{\hat{W}_a} + \frac{1}{\lambda} (\hat{b} - b^*) \dot{\hat{b}}$$

$$\leq gN(k) \dot{k} + z_2 W_a^* \varphi_a + |z_2| b$$

$$+ (\hat{W}_a - W_a^*)^T \Gamma^{-1} \dot{\hat{W}_a} + \frac{1}{\lambda} (\hat{b} - b^*) \dot{\hat{b}} \quad (26)$$

Adding and subtracting (27) on the right hand of (26), and using (23)-(25),

$$c_2 z_2^2 + z_2 \hat{W}_a^T \varphi_a + z_2 \hat{b} \tanh(\frac{z_2}{\varepsilon})$$
(27)

there is

$$\dot{V}_{20} \leq -c_2 z_2^2 + gN\dot{k} + \dot{k} + b^* |z_2| - \hat{b}z_2 \tanh(\frac{z_2}{\varepsilon}) - z_2 (\hat{W}_a - W_a^*)^T \varphi_a + (\hat{W}_a - W_a^*)^T \Gamma^{-1} \dot{W}_a + \frac{1}{\lambda} (\hat{b} - b^*) \dot{\hat{b}} \leq -c_2 z_2^2 + gN\dot{k} + \dot{k} + b^* |z_2| - b^* z_2 \tanh(\frac{z_2}{\varepsilon}) - (\hat{W}_a - W_a^*)^T \sigma_w (\hat{W}_a - W_a^0) - (\hat{b} - b^*) \sigma_b (\hat{b} - b^0)$$
(28)

The following inequalities can be achieved by applying Young's inequality.

$$(\hat{W}_{a} - W_{a}^{*})^{T} (\hat{W}_{a} - W_{a}^{0})$$

$$\geq \frac{1}{2} \left\| \hat{W}_{a} - W_{a}^{*} \right\|^{2} - \frac{1}{2} \left\| W_{a}^{*} - W_{a}^{0} \right\|^{2} \qquad (29)$$

$$(\hat{b} - b^{*}) (\hat{b} - b^{0})$$

$$\geq \frac{1}{2} (\hat{b} - b^{*})^{2} - \frac{1}{2} (b^{*} - b^{0})^{2} \qquad (30)$$

By using (29), (30) and the property of function $\tanh(\cdot): 0 \le |x| - x \tanh(\frac{x}{\varepsilon}) \le 0.2785\varepsilon$ for any $\varepsilon > 0$ and $x \in R$, equation (28) can be further written as:

$$\begin{split} \dot{V}_{20} &\leq -c_{2}z_{2}^{2} + gN\dot{k} + \dot{k} + b^{*}0.2785\varepsilon \\ &- \frac{1}{2}\sigma_{w} \left\| \hat{W}_{a} - W_{a}^{*} \right\|^{2} - \frac{1}{2}\sigma_{w} \left\| \hat{W}_{a} - W_{a}^{*} \right\|^{2} \\ &- \frac{1}{2}\sigma_{b}(\hat{b} - b^{*})^{2} + \frac{1}{2}\sigma_{b}(b^{*} - b^{0})^{2} \\ &\leq -c_{2}z_{2}^{2} + gN\dot{k} + \dot{k} - \frac{1}{2}\sigma_{w} \left\| \hat{W}_{a} - W_{a}^{*} \right\|^{2} \\ &- \frac{1}{2}\sigma_{b}(\hat{b} - b^{*})^{2} + C_{21} \end{split}$$
(31)

where the constant $C_{21} > 0$ is defined as

$$C_{21} = b^* 0.2785\varepsilon + \frac{1}{2}\sigma_W \left\| W_a^* - W_a^0 \right\|^2 + \frac{1}{2}\sigma_b (b^* - b^0)^2$$
(32)

Consider the augmented Lyapunov function V_2

$$V_2 = V_1 + V_{20} \tag{33}$$

Then, the time derivative of V_2 is

$$\begin{split} \dot{V}_{2} &\leq -c_{1}z_{1}^{2} + z_{1}z_{2} - c_{2}z_{2}^{2} + gN\dot{k} + \dot{k} \\ &- \frac{1}{2}\sigma_{w} \left\| \hat{W}_{a} - W_{a}^{*} \right\|^{2} - \frac{1}{2}\sigma_{b}(\hat{b} - b^{*})^{2} + C_{21} \\ &\leq -(c_{1} - \frac{1}{4a^{2}})z_{1}^{2} - (c_{2} - a^{2})z_{2}^{2} + gN\dot{k} + \dot{k} \\ &- \frac{1}{2}\sigma_{w} \left\| \hat{W}_{a} - W_{a}^{*} \right\|^{2} - \frac{1}{2}\sigma_{b}(\hat{b} - b^{*})^{2} + C_{21} \\ &\leq -C_{22}V_{2} + C_{21} + gN\dot{k} + \dot{k} \end{split}$$
(34)

where the constant $C_{22} > 0$ is defined as

$$C_{22} = \min\{2(c_1 - 1/(4a^2)), 2(c_2 - a^2), \\ \frac{\sigma_w}{\lambda_{\max}(\Gamma^{-1})}, \sigma_b \lambda\}, c_1 c_2 > \frac{1}{4}, \forall a \quad (35)$$

Upon multiplication of (34) by $e^{C_{22}t}$, it becomes

$$\frac{d}{dt}(V_2 e^{C_{22}t}) \le (C_{21} + (gN+1)\dot{k}) e^{C_{22}t} \quad (36)$$

Let $\rho = C_{21} / C_{22}$, integrating (39) over [0, t], then

$$0 \leq V_{2}(t) \leq e^{-C_{22}t}V_{2}(0) + \rho(1 - e^{-C_{22}t}) + e^{-C_{22}t} \int_{0}^{t} (gN + 1)\dot{k}e^{C_{22}\tau}d\tau \leq V_{2}(0) + \rho + e^{-C_{22}t} \int_{0}^{t} (gN + 1)\dot{k}e^{C_{22}\tau}d\tau$$
(37)

Applying Lemma 1 to (39), it can be concluded that $\int_{0}^{\tau} gN\dot{k}d\tau$, k(t) and $V_2(t)$ are bounded, hence $\hat{W_a},\hat{b},z_1,z_2,\phi_1$ and the original states x_1,x_2 are bounded $[0, t_f)$. According to (Ryan, 1991, Prop. 2), if the solution of the closed-loop is bounded, then $t_f = \infty$. All signals of the resulting closed-loop system are uniformly ultimately bounded. Furthermore, from the definition of $V_2(t)$ and (37), concluded that for is any given $\mu^* \ge \sqrt{2(V_2(0) + \rho + C)}$, where C is the upper bound of $e^{-C_{22}t} \int_{0}^{t} (gN+1)\dot{k}e^{C_{22}\tau}d\tau$, there exists T such that, for all $t \ge T$, there is $||z|| \le \mu^*$. The compact set $\Omega_z = \{ z \in \mathbb{R}^2 : ||z|| \le \mu^* \}$ can be made as small as required by an appropriate choice of the design constants for achieving suitable transient performance. Hence, the realm of heading error $z_1 = y - \psi_d$ is regulated to a small neighbourhood of $z_1 = 0$, as shown by the following theorem about ship course control systems.

Theorem 1: Applied to ship motion nonlinear systems (5)-(7) with parametric uncertainty and unknown boundedness of environmental disturbances, and without *a priori* knowledge of control direction, the adaptive robust nonlinear control scheme described by (21)-(25) guarantees that all signals of the resulting closed-loop adaptive system are uniformly ultimately bounded, and the tracking error of a ship's course can be regulated to any prescribed accuracy by appropriately choosing the design constants.

4. SIMULATION STUDY

This section validates the proposed nonlinear adaptive robust autopilot on a ship with a length of 45m (Unar and Murray-smith, 1999). The model equation (3) of the ship at a forward speed of 5 m/s has the following set of dynamic parameters: T = 31 s, K = 0.5 1/s, $\alpha = 0.4$ s². δ_{ω} is a $8^{\circ} \times rand(\cdot)$ with $rand(\cdot)$ being zero mean random noise with magnitudes of 1 and frequency of

0.1 rad/s which is a rudder angle that is equivalent to disturbances induced by wind, wave, current. In the simulation, let the input ψ_r of the reference model (8) be the square wave signal whose period is 400s and magnitude is 30°. The design parameters of the reference model are chosen as $\omega_n = 0.05 rad / s$, $\xi = 0.8$. The control design parameters are selected as $c_1 = 0.05$, $c_2 = 5$, $\Gamma = diag([0.5, 0.5, 0.5])$, $\lambda=0.5$, $\sigma_{\scriptscriptstyle b}=0.1$, $\sigma_{\scriptscriptstyle w}=0.1$, $\varepsilon=0.1$, $b^{0} = 0.1$, $W_{c}^{0} = \begin{bmatrix} 0.1 & 0.01 & 0 \end{bmatrix}^{T}$. The initial values are chosen as $x_1(0) = x_2(0) = 0, \hat{b}(0) = 0$, $\hat{W}_{a}(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{T}, k(0) = 0.6 * \pi$ The results of simulation are plotted in Fig. 1 to Fig. 6. Fig. 1 shows the actual heading Ψ (solid line) quickly tracks the desired heading $\boldsymbol{\psi}_d$ (dotted line), and the control rudder angle (dashdot-line) is smooth. The parameter estimates \hat{b} and \hat{W}_a , Nussbaum gain N(k) (solid line) and its argument k (dotted line) are illustrated in Fig. 2 to Fig. 6., respectively. The parameter estimates are convergent. All signals are bounded as proven in Theorem 1. From the above fact, it might be concluded that the presented controller possesses some adaptability and robustness with respect to environment disturbances.

5. CONCLUSION

In order to improve the performance of ship autopilot, this paper firstly discusses a nonlinear mathematical model of a ship steering system with parametric uncertainties unknown environmental and disturbances. Secondly, a nonlinear adaptive robust course-changing autopilot is put forward. The main advantage of the proposed control scheme is that no a priori knowledge is required about the sign of the unknown control coefficient due to the incorporation of Nussbaum gain into the autopilot adaptive backstepping design. In addition, it can deal with the uncertainties of both parameters and boundedness of environmental disturbances. It has been proven that the proposed adaptive robust nonlinear control strategy can guarantee the uniform ultimate boundedness of the resulting closed-loop adaptive system signals and accomplish adaptive tracking control of ship course. The effectiveness of the proposed method was illustrated using a simulation study.

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Fig. 1. Actual heading angle (—), reference heading angle (…), and control rudder angle $(-\cdot)$.



Fig. 2. Adapting parameter: b.



Fig. 3. Adapting parameter: $\hat{W}_{a,1}$.



Fig. 4. Adapting parameter: $\hat{W}_{a,2}$.



Fig. 5. Adapting parameter: $W_{a,3}$.



Fig. 6. Nussbaum gain N(k) (—) and its argument k(t) (…).

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