

## WINDUP PREVENTION WHEN USING DAVISON'S APPROACH TO DISTURBANCE REJECTION

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Abstract. Presented is a two step windup prevention scheme with proven stability for observer based compensators possibly containing signal models for disturbance rejection. If the undesired effects of input saturation can solely be attributed to badly damped or unstable compensator dynamics (controller windup), they are prevented by simple structural measures not increasing the compensator order. Only if the destabilizing influence of input saturation is attributable to plant windup, additional dynamic elements are added to the controller. *Copyright © 2005 IFAC*

Keywords: Nonlinear systems, Windup prevention, Signal models for disturbance rejection.

### 1. INTRODUCTION

In a well damped linear loop, where an input constraint is the only nonlinearity, saturation can cause badly damped or unstable transients. This effect was originally observed in the presence of integral controller action. During saturation, the (stabilizing) feedback is interrupted, and consequently, the integrator output can attain enormous amplitudes: it is winding up. This namesake effect appears not only in integrating controllers but in all compensators containing badly damped or unstable modes. Since it can be attributed to the controller dynamics, it is a *controller windup*. There is a considerable amount of literature devoted to the prevention of this windup and basically the approaches boil down to “stabilize the compensator in case of input saturation”. This stabilization can be achieved without augmenting the controller order and nearly all the existing methods are contained in the generalized treatment by Kothare et. al (1994). But also in the absence of controller windup, or if the controller has no dynamic elements (as in proportional or constant state feedback control), input saturation can cause badly damped transients or closed loop instability. This effect is due to

system states that cannot be transferred to their stationary values fast enough because of the input signal limitation. Obviously in such cases the plant states are winding up, so that this effect is a *plant windup*.

Whereas controller windup is removable without order augmentation, plant windup prevention calls for additional dynamic elements (Hippe and Wurmthaler, 1999). So in a first step, controller windup prevention can be achieved by structural measures. Only if there is the possible danger of plant windup, additional dynamics for its removal have to be introduced in a second step. In the framework of the Conditioning Technique (Hanus et al., 1987), plant windup was called “short sightedness of the conditioning technique”, and the measure for its prevention is the so-called “filtered setpoint” (Rönnbäck et al., 1991). A one step windup prevention scheme was presented in Teel and Kapoor (1997). It prevents controller and plant windup at the same time by augmenting the controller by a plant model.

When considering observer based controllers, the so-called *Observer Technique* seems to be the most systematic approach to controller windup prevention,

since after an application of this observer technique (Anderson and Moore, 1979, Hippe and Wurmthaler, 1999), the possibly remaining (plant) windup effects are the same as if static state feedback control without observer had been applied. If constantly acting disturbances have to be attenuated, either a disturbance observer (Johnson, 1971) or a disturbance model (Davison, 1976) can be incorporated in the observer based controller. Using Johnson's approach, the observer technique gives controller windup prevention. Johnson's disturbance observer approach, however, is not robust. Davison's approach to compensate constantly acting disturbances is both robust to plant parameter variations and to changing input locations of the external disturbances. The standard observer technique, however, does not give controller windup prevention here.

In this contribution, the modifications necessary to remove controller windup also in the presence of internal signal models for disturbance rejection are presented, and they assure, that the possibly remaining windup effects are the same as if static state feedback without signal models had been applied. Thus, the measures for removing plant windup can be designed independent of the fact, whether the controller contains an observer and signal models for the robust disturbance rejection or not. This is also the case in the one step approach of Teel and Kapoor (1997).

In Section 2 the prevention of controller windup for observer based controllers is investigated and in Section 3, the same problem is solved for controllers with signal models for robust disturbance rejection. Plant windup prevention is discussed in Section 4 and Section 5 contains some concluding remarks.

## 2. CONTROLLER WINDUP PREVENTION FOR OBSERVER BASED COMPENSATORS

Given a linear, time invariant, stable MIMO system having state  $x \in \mathfrak{R}^n$ , control input  $u_s \in \mathfrak{R}^m$ , controlled variables  $y_c \in \mathfrak{R}^m$ , measurements  $y \in \mathfrak{R}^p$ , with  $p \geq m$ , and a completely controllable and observable state space representation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu_s(t) + B_d d(t) \\ y_c(t) &= C_c x(t) + D_{cd} d(t) \\ y(t) &= Cx(t) + D_d d(t) \end{aligned} \quad (1)$$

where  $d \in \mathfrak{R}^p$  is a disturbance input. The measurements  $y$  are supposed to be subdivided according to

$$y(t) = \begin{bmatrix} y^1(t) \\ y^2(t) \end{bmatrix} = \begin{bmatrix} C^1 \\ C^2 \end{bmatrix} x(t) + \begin{bmatrix} D_d^1 \\ D_d^2 \end{bmatrix} d(t) \quad (2)$$

with  $y^1 \in \mathfrak{R}^{p-\kappa}$  and  $y^2 \in \mathfrak{R}^\kappa$ ,  $0 \leq \kappa \leq p$  and the controlled variable  $y_c$  is supposed to be measurable, i. e. it is contained in the output vector  $y$ .

In view of tracking constant reference signals the (square) transfer matrix

$$G(s) = C_c(sI - A)^{-1}B = N(s)D^{-1}(s) \quad (3)$$

is such that  $\det(N(s))$  does not have zeros at  $s = 0$ . Located at the plant input there is a nonlinearity  $u_s = \text{sat}_{u_0}(u)$  described by

$$\text{sat}_{u_0}(u) = \begin{cases} u_{0i} & \text{if } u_i > u_{0i} \\ u_i & \text{if } -u_{0i} \leq u_i \leq u_{0i}; \quad u_{0i} > 0 \\ -u_{0i} & \text{if } u_i < -u_{0i} \quad i = 1, 2, \dots, m \end{cases} \quad (4)$$

Let the nominal state feedback be denoted by

$$u(t) = -\tilde{u}(t) + Mr(t) \quad (5)$$

with

$$\tilde{u}(t) = Kx(t) \quad (6)$$

and

$$M = \{C_c(-A + BK)^{-1}B\}^{-1} \quad (7)$$

The control (5), characterized by the feedback interconnection  $u_s = u$ , is supposed to give a desired reference and disturbance behavior, where (7) assures vanishing tracking errors for step-like reference inputs  $r_i(t) = r_{si}1(t)$ ,  $i = 1, 2, \dots, m$ .

If not all states are measurable ( $p < n$ ), one needs a (stable) state observer

$$\dot{z}(t) = Fz(t) + Dy(t) + TBu(t) \quad (8)$$

having order  $n_0 = n - \kappa$  with  $0 \leq \kappa \leq p$  (i. e. it is assumed that only the  $\kappa$  outputs  $y^2$  are directly used to reconstruct the state  $x$ ) yielding

$$z(t) = Tx(t) \quad (9)$$

in steady state (and for  $d \equiv 0$ ), if the equation

$$TA - FT = DC \quad (10)$$

holds, if the pair  $(A, C)$  is completely observable, if the pair  $(F, D)$  is completely controllable, and if no eigenvalues of  $A$  and  $F$  coincide (Luenberger, 1971).

When the rows of  $C^2$  and  $T$  are linearly independent, the estimate  $\hat{x}$  for the state  $x$  is

$$\hat{x}(t) = \begin{bmatrix} C^2 \\ T \end{bmatrix}^{-1} \begin{bmatrix} y^2(t) \\ z(t) \end{bmatrix} = [\Psi_2 \quad \Theta] \begin{bmatrix} y^2(t) \\ z(t) \end{bmatrix} \quad (11)$$

For simplicity introduce  $\Psi = [0 \quad \Psi_2]$ , so that (11) can also be written as

$$\hat{x}(t) = \Psi y(t) + \Theta z(t) \quad (12)$$

Substituting in (6) the state  $x$  by  $\hat{x}$  according to (12), and using this in (5) and (8) one obtains the observer based compensator

$$\begin{aligned} \dot{z}(t) &= (F - TBK\Theta)z(t) + (D - TBK\Psi)y(t) + TBMr(t) \\ u(t) &= -K\Theta z(t) - K\Psi y(t) + Mr(t) \end{aligned} \quad (13)$$

The following example is chosen to demonstrate a controller windup, which is not related to an integral part in the controller or to other signal models for disturbance rejection.

**Example 1.** Considered is an observer based state control for a 3<sup>rd</sup> order SISO system with

$$A = \begin{bmatrix} 2 & -6 & -7 \\ 3 & -7 & -8 \\ -1 & 2 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, C = [4.7 \quad -3.7 \quad 4.8]$$

where  $C_c = C \equiv C^2$ . The nominal state feedback  $K = [2 \quad 4 \quad 30]$  assigns all system eigenvalues to  $s = -3$ , and  $M = 270/85$  assures vanishing steady state errors for reference step inputs. The observer of order  $n_0 = n - p = 2$  is supposed to have eigenvalues at  $s = -4$ , and it is easy to check that the matrices

$$F = \begin{bmatrix} -5 & -1 \\ 1 & -3 \end{bmatrix}, D = \begin{bmatrix} 9 \\ 0 \end{bmatrix} \text{ and } T = \begin{bmatrix} 10 & -8.6 & 5.9 \\ 4 & -2.3 & 3.1 \end{bmatrix}$$

solve eqn. (10). With these parameters the observer based compensator (13) is completely parametrized. The broken line in Figure 1 shows the reference step response for an unconstrained input signal.

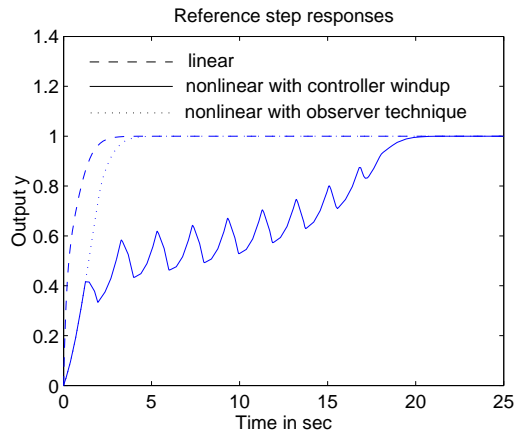


Fig. 1. Reference step responses of the linear and of the nonlinear loop

When there is an input saturation with  $u_0 = 0.2$  at the plant input, the transient response shown in full lines in Fig. 1 results. This is a consequence of a controller windup, as the eigenvalues of  $F - TBK\Theta$  (poles of the compensator transfer function) are located near the imaginary axis at  $-0.05 \pm 2.8j$ . Usually (controller) windup is said to cause big and long lasting overshoots. This one is different, but also due to unfavourable compensator dynamics.  $\therefore$

Controller windup can e. g. be removed by the so-called *Observer Technique* (Hippe and Wurmthaler, 1999). It consists of inserting a model  $u_s = \text{sat}_{u_0}(u)$  of the input saturation at the compensator output, and of feeding the limited signal  $u_s(t)$  instead of  $u(t)$

into the observer (8). Due to this, there are no observation errors triggered by the input saturation and consequently, the reference behavior of the observer based loop is the same as if a constant state feedback (6), (5) of measurable states had been applied (see also Fig. 2). In a constant state feedback loop there are no compensator states, so that controller windup is systematically removed by the observer technique. The possibly remaining undesired effects of input saturation are consequently a plant windup.

**Example 1 cont.:** Applying the observer technique for controller windup prevention in Example 1 gives the dotted reference transient in Fig. 1. The windup effects are completely removed because the undesired effects of input saturation can solely be attributed to a controller windup in this Example.  $\therefore$

### 3. CONTROLLER WINDUP PREVENTION IN THE PRESENCE OF SIGNAL MODELS FOR DISTURBANCE REJECTION

The rejection of constantly acting disturbances modeled by a known  $q^{\text{th}}$  order signal process

$$\begin{aligned} \dot{\bar{v}}(t) &= \bar{S}_q \bar{v}(t), \quad \bar{v}(0) = v_0 \\ d(t) &= H\bar{v}(t) \end{aligned} \quad (14)$$

with unknown initial conditions  $v_0$ , can be achieved in two different ways. One is the disturbance observer approach by Johnson (Johnson, 1971). When augmenting the plant by a model of the signal process, its states can be observed and used to counteract the effects of the disturbances in the controlled variables  $y_c(t)$ . Controller windup is easily prevented in this approach by feeding the limited input signal into the state plus disturbance observer (i. e. by the observer technique). Johnson's disturbance accommodation, however, is neither robust to changing disturbance inputs nor to plant parameter variations.

Using Davison's approach instead (Davison, 1976), one obtains a robust disturbance rejection for all modeled disturbances no matter where they actually attack, and also for modified system parameters (provided they do not cause closed loop instability). But applying the observer technique (to the usually necessary state observer as described above) does not remove controller windup, since the signal model is not driven by the plant input signal in Davison's approach and thus it cannot be stabilized in case of input saturation. However, when modifying Davison's approach as presented in this paper, this becomes possible.

Davison (1976) suggested to drive a model of the assumed signal process by the tracking error  $y_c(t) - r(t)$  and to stabilize the plant augmented by this process model. This gives robust disturbance rejection for all modeled signals but also a robust tracking of all such reference signals. However, the joint disturbance attenuation and tracking for all modeled disturbance and reference signals may have undesired consequences. If, e. g., constant reference signals and sinusoidal disturbances had to be accommodated, the

standard Davison approach would call for a signal model with  $\det(sI - \bar{S}_d) = s(s^2 + \omega_0^2)$ . This assures vanishing tracking errors for step-like reference inputs and the rejection of sinusoidal disturbances of frequency  $\omega_0$ . It would also assure, however, a suppression of step-like disturbances and a vanishing tracking error for sinusoidal reference signals of frequency  $\omega_0$ . At one hand, the latter is not required, and on the other hand, this may have disastrous consequences on the transients for step-like inputs (for a demonstrating example see Hippe and Wurmthaler, 1985). Therefore, a modified approach is suggested, yielding the following properties:

- (i) Disturbance rejection of all signals modeled by (14)
- (ii) Tracking of constant reference signals such that only the controlled plant dynamics, characterized by  $\det(sI - A + BK)$ , are influential
- (iii) Controller windup prevention such, that the remaining windup effects of the closed loop are again the same as if constant state feedback control without observer had been applied (see also the block diagram in Fig. 2).

Assume the acting disturbances can be modeled by (14) with a characteristic polynomial  $\det(sI - \bar{S}_d) = s^q + \psi_{q-1}s^{q-1} + \dots + \psi_0$  and the system (1) is augmented by the signal model

$$\dot{v}(t) = Sv(t) + B_\varepsilon y_C(t) - B_\sigma [\tilde{u}(t) + u_s(t)] \quad (15)$$

with the  $(mq, mq)$  matrix  $S = \text{diag}(S_d)$  and the  $(mq, m)$  matrix  $B_\varepsilon = \text{diag}(b_\varepsilon)$  where, for simplicity

$$S_d = \begin{bmatrix} 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 \\ -\psi_0 & -\psi_1 & \dots & -\psi_{q-1} \end{bmatrix}; b_\varepsilon = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (16)$$

and  $B_\sigma$  as defined in Lemma 1. Assume that the augmented plant with state  $\begin{bmatrix} x^T & v^T \end{bmatrix}^T$  is completely controllable and observable and that no zero of  $\det N(s)$  in (3) coincides with a zero of  $\det(sI - S_d)$ . Since the state  $v$  is directly obtainable from the model (15), only an observer (8) for the state  $x$  is required and it is assumed for the following, that the observer technique has been applied, i. e. that  $u(t)$  in (8) is replaced by  $u_s(t)$ .

**Lemma 1:** Given a “nominal” state feedback (6), (5) for the non augmented plant (1). With  $X$  a solution to the Ljapunov equation

$$X(A - BK) - SX = B_\varepsilon C_C \quad (17)$$

define

$$B_\sigma = -XB \quad (18)$$

and compute the state feedback  $K_v v(t)$  such that  $S - B_\sigma K_v$  has stable eigenvalues. Then with

$$K_x = K - K_v X \quad (19)$$

the feedback (5) with

$$\tilde{u}(t) = \begin{bmatrix} K_x & K_v \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} \quad (20)$$

and  $M$  according to (7) gives the above stated properties (i) through (iii), and the characteristic closed loop polynomial is

$$\det(sI - A + BK) \det(sI - S + B_\sigma K_v). \quad ./.$$

**Proof:** First consider the linear case, i. e.  $u_s = u$ . Inserting the control (5), (20) in (1) and in (15), one obtains

$$\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} A - BK_x & -BK_v \\ B_\varepsilon C_C & S \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} BM \\ -B_\sigma M \end{bmatrix} r(t) + \begin{bmatrix} B_d \\ B_\varepsilon D_{Cd} \end{bmatrix} d(t) \\ y_C(t) = \begin{bmatrix} C_C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + D_{Cd} d(t) \quad (21)$$

Applying the similarity transformation

$$\begin{bmatrix} x(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} I & 0 \\ X & I \end{bmatrix} \zeta(t) \quad (22)$$

to (21) it becomes obvious, that the closed loop characteristic polynomial is indeed  $\det(sI - A + BK) \det(sI - S + B_\sigma K_v)$ , and that with  $M$  according to (7), there is a tracking of constant reference signals as if static state feedback (6) and (5) had been applied (which shows (ii)). Using Rosenbrock's system matrix for the disturbance inputs  $d_j$  in (21) it can be shown, that all transfer functions from the disturbance inputs  $d_j$ ,  $j = 1, 2, \dots, p$  to the controlled variables  $y_{Ci}(t)$   $i = 1, 2, \dots, m$  contain the polynomial  $\det(sI - S_d)$  in the numerator, so that asymptotic disturbance rejection is assured for all modeled disturbance signals (i. e. (i) holds).

In the nonlinear case, i. e. when input saturation is active, the behavior between the output  $u_s$  of the saturation element and its input  $\tilde{u}$  (see (20)) is characterized by

$$\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ B_\varepsilon C_C - B_\sigma K_x & S - B_\sigma K_v \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} B \\ -B_\sigma \end{bmatrix} u_s(t) \\ \tilde{u}(t) = \begin{bmatrix} K_x & K_v \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} \quad (23)$$

When applying the similarity transformation (22) to (23) it obtains a form which directly shows that (iii) holds.  $./.$

Thus, when applying Lemma 1, the controller windup is also systematically prevented for Davison's approach to disturbance rejection, and the possibly re-

maintaining plant windup effects are the same as if the nominal control (5), (6) without state observer and without disturbance model had been applied.

#### 4. PLANT WINDUP PREVENTION

Using the above controller windup prevention, the reference behavior of the closed loop is the same as if constant state feedback without observer had been applied, i. e. the possibly remaining effects of input saturation can be investigated by inspection of the closed loop shown in Fig. 2.

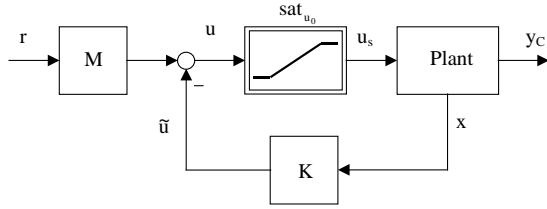


Fig. 2. State feedback with input saturation

Due to the absence of controller states, all possibly remaining undesired effects of input saturation are now attributable to the dynamics of the controlled plant, i.e. they depend on the feedback matrix  $K$ .

The loop in Fig. 2 is globally asymptotically stable if the transfer matrix

$$G_L(s) = K(sI - A)^{-1}B \quad (24)$$

meets the circle criterion (Vidyasagar, 1993). Here with the input nonlinearity limited by the sectors 0 and 1, this is the case, if  $G_L(j\omega)$  stays right of a vertical line passing through  $-1$  for SISO systems. If, however,  $G_L(s)$  violates the circle criterion, stability of the nonlinear loop is no longer guaranteed, i. e. there is the danger of *plant windup*.

**Remark 1.** The circle criterion has been chosen as it gives less conservative results than the requirement that  $G_L(s)$  is positive real. Of course also the Popov criterion could be used to discuss the stability of the loop in Fig. 2.  $\therefore$

**Theorem 1.** Assume a nominal state feedback (6) s. th.  $G_L(s)$  violates the circle criterion. Then linear performance recovery and closed loop stability are guaranteed when substituting  $u(t) = -\tilde{u}(t) + Mr(t)$  in Fig. 2 by

$$u(t) = -\tilde{u}(t) + Mr(t) - \eta(t) \quad (25)$$

where

$$\begin{aligned} \dot{\xi}(t) &= (A - BK_S)\xi(t) + B[u(t) - u_s(t)] \\ \eta(t) &= (K - K_S)\xi(t) \end{aligned} \quad (26)$$

and  $K_S$  is a “safe” state feedback such that  $G_{LS}(s) = K_S(sI - A)^{-1}B$  meets the circle criterion.  $\therefore$

**Proof:** Linear performance recovery results, as the additional dynamics (26) are only excited when saturation becomes active. If so, the transfer behavior from  $u_s$  to  $-u$  is given by

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{\xi}(t) \end{bmatrix} &= \begin{bmatrix} A & 0 \\ -BK & A - BK \end{bmatrix} \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix} + \begin{bmatrix} B \\ -B \end{bmatrix} u_s(t) \\ -u(t) &= \begin{bmatrix} K & (K - K_S) \end{bmatrix} \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix} \end{aligned} \quad (27)$$

Applying the similarity transformation

$$\begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix} = \begin{bmatrix} I & 0 \\ -I & I \end{bmatrix} \zeta(t) \quad (28)$$

to (27) gives

$$\begin{aligned} \dot{\zeta}(t) &= \begin{bmatrix} A & 0 \\ 0 & A - BK \end{bmatrix} \zeta(t) + \begin{bmatrix} B \\ 0 \end{bmatrix} u_s(t) \\ -u(t) &= \begin{bmatrix} K_S & (K - K_S) \end{bmatrix} \zeta(t) \end{aligned} \quad (29)$$

which shows that (27) contains a stable subsystem not controllable from  $u_s$  and that the transfer behavior  $-u(s) = G_L(s)u_s(s)$  is characterized by the “safe” state feedback, i. e. one has

$$G_L(s) \triangleq G_{LS}(s) = K_S(sI - A)^{-1}B. \quad \therefore$$

Thus, windup prevention can be achieved in a two step procedure.

- 1.) Remove a possibly existing controller windup (by the observer technique, and if there is a signal model for disturbance rejection, by using Lemma 1) and then
- 2.) If there is the danger of plant windup (i. e. (24) violates the circle criterion), it can be prevented by additional dynamics according to Theorem 1.

**Remark 2.** The suggested two step procedure and the Teel/Kapoor (1997) approach give identical results, provided the (linear) feedback  $K_\xi$  for the model states in the latter approach coincides with the safe state feedback  $K_S$  above (In the Teel/Kapoor approach, the transfer behavior of the linear part in case of input saturation is  $G_L(s) = K_\xi(sI - A)^{-1}B$ ). In the light of this, the two step procedure has the advantage, that additional dynamics are only required if the danger of plant windup exists, whereas in the Teel/Kapoor approach, a plant model is always used with the compensator. If there is no danger of plant windup,  $K$  would be a “safe” state feedback and (26) shows that for  $K_S = K$  the additional dynamics can be removed from the windup prevention scheme  $\therefore$ .

**Example 2.** Considered is again the system of Example 1, but with an output vector  $C = C_c = [1 \ -1 \ 0]$ . There is an input saturation with  $u_0 = 3$  and the observer based compensator is required to suppress

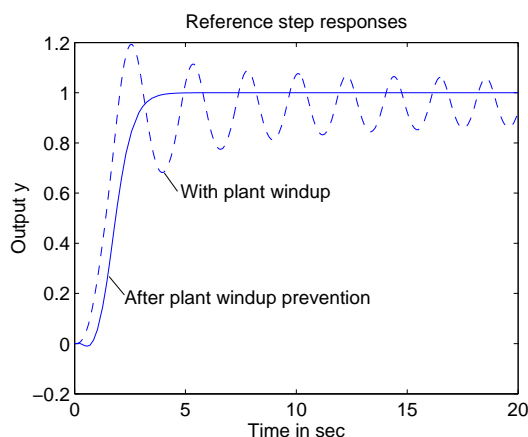
sinusoidal disturbances with  $\det(sI - S_d) = s^2 + 9$ . The nominal state feedback  $K = [2702 \quad -2660 \quad 714]$  with  $M = 3375$  gives the reference transfer function  $y_c(s) \equiv y(s) = \frac{3375}{(s+15)^3} r(s)$ . The state observer is parametrized by

$$F = \begin{bmatrix} -10 & 1 \\ 0 & -10 \end{bmatrix}, D = \begin{bmatrix} 243 \\ -243 \end{bmatrix}, \text{ and} \\ T = \frac{1}{9} \begin{bmatrix} 214 & -212 & -15 \\ -240 & 237 & 18 \end{bmatrix}$$

With the solution  $X$  of the equation (17) one obtains

$$B_\sigma = \frac{1}{1423656} \begin{bmatrix} -74 \\ 330 \end{bmatrix} \text{ where } K_v = [150390 \quad 120006]$$

assures  $\det(sI - S + B_\sigma K_v) = (s+10)^2$ . With this (19) gives  $K_x = [19126 \quad -19064 \quad 1725]$ . But also after applying the above measures for controller windup prevention, the closed loop reference step response for  $r(t) = 1(t)$  is oscillating (dashed lines in Fig. 3). This is due to a plant windup, indicated by a severe violation of the circle criterion, since the frequency response  $G_L(j\omega)$  intersects the negative real axis left of  $-1$ . However, with additional dynamics according to (25), (26), and  $K_s = [2 \quad 4 \quad 30]$  the step res-



ponse in full lines in Fig. 3 results.

Fig. 3. Example 2 before and after plant windup prevention

## 5. CONCLUSIONS

Presented is a two step approach to windup prevention in observer based controllers possibly incorporating signal models for robust disturbance rejection. This two step technique has the advantage of using additional compensator dynamics only when there is the danger of plant windup, indicated by the open loop frequency response of the nominal state feedback loop. Controller windup prevention uses the observer technique and a modification of Davison's approach to robust disturbance rejection (Lemma 1). This facilitates plant windup prevention, as it is now independent of whether one has a constant state feedback control with measured states, with observed states or with signal models for disturbance rejection.

Zaccarian and Teel (2004) demonstrate, that the Teel/Kapoor approach to windup prevention also constitutes a *systematic* solution to the bumpless transfer problem. The arguments are based on the *target response*, characterizing the ideal behavior of the system after the switch. The bumpless transfer design goal is formally stated as "the goal of recovering that response (in an  $L_2$  sense) with a bound dependent on the size of the mismatch between the actual plant state and the ideal target plant state at the switching time". In other words, the closed loop transients after the switching are the same as those resulting with the nominal controller, given the initial states at the switching time. Since the presented scheme gives the same results as the Teel/Kapoor approach (see Remark 2), it also constitutes a systematic solution to the bumpless transfer problem when switching at the input of the saturating element.

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