DESIGN OF $\mathcal{H}_\infty$ FEEDBACK CONTROL SYSTEMS WITH QUANTIZED SIGNALS

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Abstract: In this paper, we consider design of $\mathcal{H}_\infty$ feedback control systems with quantized signals. We first assume that a state feedback has been designed for a continuous-time LTI system so that the closed-loop system is (Hurwitz) stable and a desired $\mathcal{H}_\infty$ disturbance attenuation level is achieved, and that the states are quantized before they are passed to the controller. We propose a state-dependent strategy for updating the quantizer's parameter, so that the system is asymptotically stable and achieves the same $\mathcal{H}_\infty$ disturbance attenuation level. We then extend the result to the case of observer-based dynamic output feedback where the measurement outputs are quantized, and propose an output-dependent strategy for updating the quantizer's parameter.

Keywords: continuous-time LTI system, $\mathcal{H}_\infty$ control, quantizer, quantization, Riccati equation, state feedback, observer-based output feedback.

1. INTRODUCTION

In classical feedback control theory, various signals or data in the control loop have been assumed to be passed directly without data loss, except in saturated systems. However, this is not true in many real applications. For example, in networked control systems (Bushnell, 2001; Ishii and Francis, 2002) where all signals are transferred through network, package dropouts or data transfer rate limitations always happen. Another important aspect, which is well known in signal processing area, is signal quantization. Since quantization always exists in computer based control systems, many researchers have begun to study the analysis and design problems for control systems involving various quantization methods. (Delchamps, 1990) addressed the problem of stabilizing an unstable linear system by means of quantized state feedback, i.e., state feedback where the measurements of the system state are quantized. The quantizer in (Delchamps, 1990) takes value in a countable set. (Brockett and Liberzon, 2000) defined a quantizer taking value in a finite set and considered quantized feedback stabilization for linear systems. It has been shown there that if it is possible to change the sensitivity of the quantizer on the basis of available quantized measurements, then a hybrid control strategy, for both continuous- and discrete-time systems, can be designed to guarantee global asymptotic stability. Noting that the approach in (Brockett and Liberzon, 2000) relies on the possibility of making discrete online adjustments of quantizer parameters, (Liberzon, 2003) extended the approach for more general nonlinear systems with general types of quantizers involving the states of the system, the measured outputs, and the control inputs. The idea and results in (Liberzon, 2003) are applied for stabilization of discrete-time LTI systems with quantized measurement outputs in (Matsumoto, Zhai and Mi, 2003). Recently, (Zhai, Matsumoto, Chen and Mi, 2004) considered the stabilization problem for a discrete-time LTI system via state feedback involving both quantized states and control inputs. As assumed in (Liberzon, 2003), the system considered in (Zhai *et al.*, 2004) is supposed to be stabilizable and a stabilizing state feedback has been designed without taking quantization into account. However, the system's states are quantized before they are passed to the controller, and the control inputs are quantized before they are passed to the system. This is a natural setting in networked control systems, where all informations (reference input, plant output, control input, etc.) are exchanged through a network among control system components (sensors, controller, actuators, etc.). Due to the quantization effects, the desired system stability can not be guaranteed. For this reason, (Zhai *et al.*, 2004) defined the two quantizers with general forms as in (Liberzon, 2003) and then proposed a hybrid quantized state feedback strategy where the values of the quantizer parameters are updated at discrete instants of time.

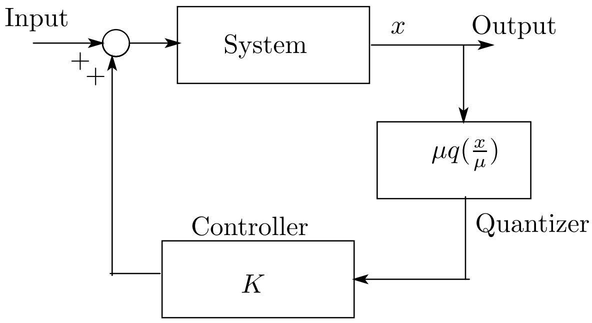

Fig.1 Feedback System with Quantized State or Measurement Output

Noticing that the above works deal with only stability/stabilization problems, we aim to extend the results to $\mathcal{H}_\infty$ feedback control systems in this paper. We consider both state feedback and dynamic output feedback. First, we assume that a state feedback has been designed for a continuous-time LTI system so that the closed-loop system is (Hurwitz) stable and a desired $\mathcal{H}_\infty$ disturbance attenuation level γ is achieved. However, the states are quantized before they are passed to controller (see Fig.1), and due to the quantization error, the system does not have the same performance as the case where no quantization is involved. For this reason, we propose a state-dependent strategy for updating the quantizer's parameter, so that the system is asymptotically stable and achieves the same $\mathcal{H}_\infty$ disturbance attenuation level γ. Then, we extend the consideration to the case of observer-based dynamic output feedback and assume that the measurement outputs are quantized in the controller. To deal with the quantization error, we propose an output-dependent strategy for updating the quantizer's parameter, so that the system has the same performance as before. We note that the control strategies of updating the quantizer's parameter are dependent on time in the existing works (Brockett and Liberzon, 2000; Liberzon, 2003; Matsumoto *et al.*, 2003; Zhai *et al.*, 2004), and such control strategies can not be applied for the case of $\mathcal{H}_\infty$ control systems since we do not know the value of the disturbance inputs and thus can not drive the state into an invariant region, as done in (Liberzon, 2003). As a great contrast, the control strategy in this paper is state or output dependent, which is usually regarded to have more robustness.

The rest of this paper is organized as follows. Section 2 gives the definition and the property of generalized quantizer. Section 3 considers state quantization in state feedback, and proposes state-dependent strategy for updating the quantizer's parameter, so that the system is asymptotically stable and achieves the same $\mathcal{H}_\infty$ disturbance attenuation level. Section 4 extends the consideration to the case of observer-based dynamic output feedback, and obtain parallel result. Finally, Section 5 gives some concluding remarks.

2. QUANTIZER DESCRIPTION

First, we give the definition of a quantizer with general form as introduced in (Liberzon, 2003). Let $z \in \Re^l$ be the variable being quantized. A *quantizer* is defined as a piecewise constant function $q : \Re^l \to \mathcal{D}$, where $\mathcal{D}$ is a finite subset of $\Re^l$. This leads to a partition of $\Re^l$ into a finite number of quantization regions of the form $\{z \in \Re^l : q(z) = i\}$, $i \in \mathcal{D}$. These quantization regions are not assumed to have any particular shapes. We assume that there exist positive real numbers M and Δ such that the following conditions hold:

(1) If

$$|z| \le M \tag{1}$$

then

$$|q(z) - z| \le \Delta\,. \tag{2}$$

(2) If

$$|z| > M$$

then

$$|q(z)| > M - \Delta\,.$$

Throughout this paper, we denote by $|\cdot|$ the standard Euclidean norm in the n-dimensional vector space $\Re^n$, and denote by $\|\cdot\|$ the corresponding induced matrix norm in $\Re^{n\times n}$. Condition 1 gives a bound on the quantization error when the quantizer does not saturate. Condition 2 provides a way to detect the possibility of saturation. We will refer to M and Δ as *the range of* q and the *quantization error*, respectively. We also assume that $q(x) = 0$ for x in some neighborhood of the origin. The example of satisfying the above requirements is given by the quantizer with rectangular quantization regions in (Brockett and Liberzon, 2000; Liberzon, 2000).

In the control strategy to be developed below, we will use quantized measurements of the form

$$q_\mu(z) \triangleq \mu q(\frac{z}{\mu}), \tag{3}$$

where $\mu > 0$ is the parameter. The extreme case of $\mu = 0$ is regarded as setting the output of the quantizer as zero. The range of this quantizer is $M\mu$ and the quantization error is $\Delta\mu$. We can view μ as a "zoom" variable: increasing μ corresponds to zooming out and essentially obtaining a new quantizer with larger range and larger quantization error, while decreasing μ corresponds to zooming in and obtaining a quantizer with smaller range but also smaller quantization error. We will update μ later depending on the system state (or the measurement output). In this sense, it can be considered as another state of the resultant closed-loop system.

3. STATE QUANTIZATION IN STATE FEEDBACK

In this section, we consider the continuous-time LTI system described by

$$\begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z = C_1 x, \end{cases} \tag{4}$$

where $x \in \Re^n$ is the state, $w \in \Re^h$ is the disturbance input, $z \in \Re^p$ is the controlled output, and $u \in \Re^m$ is the control input. The matrices A, B_1, B_2, C_1 are constant and of appropriate dimension. We assume that the pair (A, B_2) is stabilizable.

Suppose that for the system (4), we have designed a state feedback

$$u = Kx \tag{5}$$

so that the closed-loop system, composed of (4) and (5), is stable and the $\mathcal{H}_\infty$ norm of the transfer function from w to z is less than a specified level γ. More precisely, the closed-loop system is writen as

$$\begin{cases} \dot{x} = \bar{A}x + B_1 w \\ z = C_1 x \end{cases} \tag{6}$$

where $\bar{A} = A + B_2 K$. Then, the hypothesis is that, without taking quantization into consideration, the gain K in (5) is designed so that $\bar{A}$ is (Hurwitz) stable and $\|C_1(sI - \bar{A})^{-1}B_1\|_\infty < \gamma$. Therefore, according to the well known Bounded Real Lemma (Iwasaki, Skelton and Grigoriadis, 1998), there exist two positive definite matrices P and Q satisfying the Riccati equation

$$\bar{A}^T P + P\bar{A} + \gamma^{-2} P B_1 B_1^T P + C_1^T C_1 + Q = 0. \tag{7}$$

We will let $\lambda_m(\cdot)$ and $\lambda_M(\cdot)$ denote the smallest and the largest eigenvalue of a symmetric matrix, respectively. Since P and Q are positive definite, the inequalities

$$\begin{aligned} \lambda_m(P)\,|x|^2 \le x^T P x \le \lambda_M(P)\,|x|^2 \\ \lambda_m(Q)\,|x|^2 \le x^T Q x \le \lambda_M(Q)\,|x|^2 \end{aligned} \tag{8}$$

holds for any x.

Here, we deal with the case where only quantized state information is available. For this reason, we modify the state feedback (5) using quantized information of x as

$$u = K\mu q(\frac{x}{\mu}). \tag{9}$$

For any fixed positive scalar μ, the closed-loop system composed of the system (4) and the new state feedback (9) is given by

$$\begin{cases} \dot{x} = \bar{A}x + B_1 w + D(\mu, x) \\ z = C_1 x, \end{cases} \tag{10}$$

where

$$D(\mu, x) = \mu B_2 K \left(q(\frac{x}{\mu}) - \frac{x}{\mu} \right). \tag{11}$$

Now, the control problem is very natural. Due to the existence of quantization error, the stability and the desired $\mathcal{H}_\infty$ disturbance attenuation level γ is not guaranteed. For this reason, we propose a control strategy which adjusts μ appropriately online, depending on the state, so that the same $\mathcal{H}_\infty$ disturbance attenuation level is achieved.

Theorem 1. *Assume that M is chosen large enough compared to Δ so that we have*

$$M > 2\Delta \frac{\|PB_2K\|}{\lambda_m(Q)}. \tag{12}$$

Then, there exists a control strategy for updating μ, which is dependent on the state, that makes the closed-loop system (10) *asymptotically stable and achieves $\mathcal{H}_\infty$ disturbance attenuation level γ.*

Proof. Since $\frac{x}{\mu}$ is quantized before going to the state feedback, we obtain by using the properties of general quantizers in (1) and (2) that whenever $|x| \le M\mu$, the following holds.

$$\left| q(\frac{x}{\mu}) - \frac{x}{\mu} \right| \le \Delta \tag{13}$$

We consider the Lyapunov function candidate

$$V(x) = x^T P x \tag{14}$$

for the closed-loop system (10). By using the Riccati equation (7), we obtain that when $|x| \le M\mu$, the derivative of $V(x)$ along solutions of (10) satisfies

$$
\begin{aligned}
\dot V &= \left(\bar A x + B_1 w + D(\mu,x)\right)^T P x \\
&\quad + x^T P\left(\bar A x + B_1 w + D(\mu,x)\right) \\
&= -x^T\left(Q + \gamma^{-2} P B_1 B_1^T P + C_1^T C_1\right) x \\
&\quad + w^T B_1^T P x + x^T P B_1 w \\
&\quad + D(\mu,x)^T P x + x^T P D(\mu,x) \\
&\le -z^T z + \gamma^2 w^T w - \lambda_m(Q)|x|^2 \\
&\quad + 2|x|\|PB_2K\|\Delta\mu \\
&= -z^T z + \gamma^2 w^T w \\
&\quad -\lambda_m(Q)|x|\left(|x| - 2\Delta\frac{\|PB_2K\|}{\lambda_m(Q)}\mu\right). \quad (15)
\end{aligned}
$$

According to (12), we can always find a scalar $\epsilon \in (0,1)$ such that

$$M > 2\Delta\frac{\|PB_2K\|}{\lambda_m(Q)} \times \frac{1}{1-\epsilon}, \quad (16)$$

which is equivalent to

$$\frac{1}{1-\epsilon} \times 2\Delta\frac{\|PB_2K\|}{\lambda_m(Q)}\mu < M\mu. \quad (17)$$

Therefore, for any nonzero x, we can find a positive scalar μ such that

$$\frac{1}{1-\epsilon} \times 2\Delta\frac{\|PB_2K\|}{\lambda_m(Q)}\mu \le |x| \le M\mu. \quad (18)$$

This is also true in the case of $x = 0$, where we set $\mu = 0$ as an extreme case and consider the output of the quantizer as zero.

In other words, if we always choose μ so that (18) is satisfied, then (15) holds and thus

$$
\begin{aligned}
\dot V &\le -z^T z + \gamma^2 w^T w - \epsilon\lambda_m(Q)|x|^2 \\
&\le -z^T z + \gamma^2 w^T w - \epsilon\frac{\lambda_m(Q)}{\lambda_M(P)}V \\
&= -\epsilon\frac{\lambda_m(Q)}{\lambda_M(P)}V - \Gamma(t), \quad (19)
\end{aligned}
$$

where $\Gamma(t) \stackrel{\triangle}{=} z^T(t)z(t) - \gamma^2 w^T(t)w(t)$.

First, by setting $w = 0$ in (19), we see clearly that the system is asymptotically stable.

Next, since $V(t) \ge 0$, we obtain from (19) that $\dot V \le -\Gamma(t)$, and thus for any $t > t_0$,

$$V(t) - V(t_0) \le -\int_{t_0}^{t}\Gamma(\tau)d\tau. \quad (20)$$

Using $V(t) \ge 0$ again, we obtain

$$\int_{t_0}^{t} z^T(\tau)z(\tau)d\tau \le V(t_0) + \gamma^2\int_{t_0}^{t} w^T(\tau)w(\tau)d\tau, \quad (21)$$

which implies that $\mathcal{H}_\infty$ disturbance attenuation level γ is achieved. This completes the proof. ∎

Remark 1. In the existing references (for example, (Liberzon, 2003), (Zhai *et al.*, 2004)), the value of μ is updated in a time-controlled manner, i.e., when to change the value of μ is dependent only on time. This is not possible for the present situation because we do not know the value of $w(t)$ and thus we can not drive $x(t)$ into a specified invariant region, as done in (Liberzon, 2003; Zhai *et al.*, 2004). To overcome this difficulty, we have proposed a state-dependent strategy (18) for adjusting the value of μ. As also pointed out in many other references, such a state-dependent strategy is usually more robust to modelling imperfection than time-dependent one.

Remark 2. There is an important observation concerning the implementation of the quantizer proposed in this section, and it is also valid for the quantizer in the next section. We assume that the function $q(\cdot)$, which may be very complicated, has been designed and we implement $\mu q(\frac{x}{\mu})$ (NOT $q(\frac{x}{\mu})$ only) as a parameter-dependent quantizer. Since the variable of the function $q(\cdot)$ is $\frac{x}{\mu}$, the quantizer can flexibly deal with large or small state x by adjusting the value of μ, so that the condition (18) is satisfied. This is very important in $\mathcal{H}_\infty$ control problems since the state x may be very large temporarily due to unexpected disturbance input. In the case where only $q(\frac{x}{\mu})$ is viewed as a quantizer, the output of the quantizer has to be scalared by μ before it is passed to the controller. The function $q(\cdot)$ in this paper is a general concept for quantization, and thus careful consideration is required in real implementation.

Remark 3. Although the $\mathcal{H}_\infty$ disturbance attenuation level γ is fixed in this paper, the same discussion is applicable for any positive $\gamma > \gamma_{opt}$, where γ_{opt} is the optimal $\mathcal{H}_\infty$ norm that the system (4) can reach via state feedback.

4. OUTPUT QUANTIZATION IN OBSERVER-BASED OUTPUT FEEDBACK

In the case where the state information is not available in the feedback loop and also in the quantizer, we need to pull out certain output information from the system and then consider output feedback. For this reason, we consider in this section the continuous-time LTI system described by

$$\begin{cases} \dot x = Ax + B_1 w + B_2 u \\ z = C_1 x \\ y = C_2 x, \end{cases} \quad (22)$$

where $y \in \Re^q$ is the measurement output, and all the other vectors are the same as before. We assume that the triple (A, B_2, C_2) is stabilizable and detectable.

Suppose that for the system (22), we have designed a full order Luenberger observer described by

$$\begin{cases} \dot{\hat{x}} = (A + LC_2)\hat{x} + B_2 u - Ly \\ u = K\hat{x} \end{cases} \tag{23}$$

so that the closed-loop system, composed of (22) and (23), is stable and the $\mathcal{H}_\infty$ norm of the transfer function from w to z is less than a specified level γ. Since the closed-loop system is writen as

$$\begin{cases} \dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}_1 w \\ z = \tilde{C}_1 \tilde{x} \end{cases} \tag{24}$$

where $\tilde{x} = \left[x^T \ (x - \hat{x})^T\right]^T$ and

$$\begin{aligned} \tilde{A} &= \begin{bmatrix} A + B_2 K & -B_2 K \\ 0 & A + LC_2 \end{bmatrix} \\ \tilde{B}_1 &= \begin{bmatrix} B_1 \\ B_1 \end{bmatrix}, \quad \tilde{C}_1 = \begin{bmatrix} C_1 & 0 \end{bmatrix}, \end{aligned} \tag{25}$$

the hypothesis is that, without taking quantization into consideration, the gains K and L in (23) are designed so that $\tilde{A}$ is (Hurwitz) stable and $\|\tilde{C}_1(sI - \tilde{A})^{-1}\tilde{B}_1\|_\infty < \gamma$. Therefore, according to the well known Bounded Real Lemma (Iwasaki *et al.*, 1998), there exist two positive definite matrices $\tilde{P}$ and $\tilde{Q}$ satisfying the Riccati equation

$$\tilde{A}^T\tilde{P} + \tilde{P}\tilde{A} + \gamma^{-2}\tilde{P}\tilde{B}_1\tilde{B}_1^T\tilde{P} + \tilde{C}_1^T\tilde{C}_1 + \tilde{Q} = 0\,. \tag{26}$$

Here, we deal with the case where only quantized measurements of the output y are available. For this reason, we modify the observer (23) using quantized information of y as

$$\begin{cases} \dot{\hat{x}} = (A + LC_2)\hat{x} + B_2 u - L\mu q(\frac{y}{\mu}) \\ u = K\hat{x}\,. \end{cases} \tag{27}$$

Then, the closed-loop system composed of the system (22) and the new observer (27) is given by

$$\begin{cases} \dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}_1 w + D(\mu, y) \\ z = \tilde{C}_1\tilde{x}\,, \end{cases} \tag{28}$$

where

$$D(\mu, y) = -\mu\tilde{L}\begin{bmatrix} 0 \\ \frac{y}{\mu} - q(\frac{y}{\mu}) \end{bmatrix}, \quad \tilde{L} = \begin{bmatrix} 0 & 0 \\ 0 & L \end{bmatrix}. \tag{29}$$

Using the state of the closed-loop system, we write the measurement output y as

$$y = \tilde{C}_2\tilde{x}\,, \quad \tilde{C} = \begin{bmatrix} C_2 & 0 \end{bmatrix}. \tag{30}$$

Also, due to the existence of quantization error, the stability and the desired $\mathcal{H}_\infty$ disturbance attenuation level γ is not guaranteed. Next, we propose a control strategy which adjusts the quantizer's parameter μ appropriately, depending on the measurement output, so that the stability and the desired $\mathcal{H}_\infty$ disturbance attenuation level γ is achieved.

We are in the position to state and prove the main result in this section.

Theorem 2. *Assume that M is chosen large enough compared to Δ so that we have*

$$M > 2\Delta\frac{\|\tilde{P}\tilde{L}\|\|C_2\|}{\lambda_m(\tilde{Q})}\,. \tag{31}$$

Then, there exists a control strategy for updating μ, which is dependent on the measurement output, that makes the closed-loop system (28) *asymptotically stable and achieves $\mathcal{H}_\infty$ disturbance attenuation level γ.*

Proof. Since $\frac{y}{\mu} = \frac{C_2 x}{\mu}$ is quantized before being passed to the observer, we obtain by using the properties of general quantizers in (1) and (2) that whenever $|y| \le M\mu$, the inequality

$$\left|\frac{y}{\mu} - q(\frac{y}{\mu})\right| \le \Delta \tag{32}$$

is true. We consider the Lyapunov function candidate

$$V(\tilde{x}) = \tilde{x}^T\tilde{P}\tilde{x} \tag{33}$$

for the closed-loop system (28). By using the Riccati equation (26), we obtain that when $|y| \le M\mu$, the derivative of $V(\tilde{x})$ along solutions of (28) satisfies

$$\begin{aligned} \dot{V} &= \left(\tilde{A}\tilde{x} + \tilde{B}_1 w + D(\mu, y)\right)^T\tilde{P}\tilde{x} \\ &\quad + \tilde{x}^T\tilde{P}\left(\tilde{A}\tilde{x} + \tilde{B}_1 w + D(\mu, y)\right) \\ &= -\tilde{x}^T\left(\tilde{Q} + \gamma^{-2}\tilde{P}\tilde{B}_1\tilde{B}_1^T\tilde{P} + \tilde{C}_1^T\tilde{C}_1\right)\tilde{x} \\ &\quad + w^T\tilde{B}_1^T\tilde{P}\tilde{x} + \tilde{x}^T\tilde{P}\tilde{B}_1 w \\ &\quad + D(\mu, y)^T\tilde{P}\tilde{x} + \tilde{x}^T\tilde{P}D(\mu, y) \\ &\le -z^T z + \gamma^2 w^T w - \lambda_m(\tilde{Q})|\tilde{x}|^2 + 2|\tilde{x}|\|\tilde{P}\tilde{L}\|\Delta\mu \\ &= -\Gamma(t) - \lambda_m(\tilde{Q})|\tilde{x}|\left(|\tilde{x}| - 2\Delta\frac{\|\tilde{P}\tilde{L}\|}{\lambda_m(\tilde{Q})}\mu\right) \\ &\le -\Gamma(t) - \lambda_m(\tilde{Q})|\tilde{x}|\left(\frac{|y|}{\|\tilde{C}_2\|} - 2\Delta\frac{\|\tilde{P}\tilde{L}\|}{\lambda_m(\tilde{Q})}\mu\right) \\ &\le -\Gamma(t) - \frac{\lambda_m(\tilde{Q})|\tilde{x}|}{\|C_2\|}\left(|y| - 2\Delta\frac{\|\tilde{P}\tilde{L}\|\|C_2\|}{\lambda_m(\tilde{Q})}\mu\right). \end{aligned} \tag{34}$$

According to (31), we can always find a scalar $\tilde{\epsilon} \in (0,1)$ such that

$$M > 2\Delta \frac{\|\tilde{P}\tilde{L}\| \|C_2\|}{\lambda_m(\tilde{Q})} \times \frac{1}{1-\tilde{\epsilon}}\,, \qquad (35)$$

which is equivalent to

$$\frac{1}{1-\tilde{\epsilon}} \times 2\Delta \frac{\|\tilde{P}\tilde{L}\| \|C_2\|}{\lambda_m(\tilde{Q})} \mu < M\mu\,. \qquad (36)$$

Similarly as in Theorem 1, if we choose the quantizer's parameter μ for any y such that

$$\frac{1}{1-\tilde{\epsilon}} \times 2\Delta \frac{\|\tilde{P}\tilde{L}\| \|C_2\|}{\lambda_m(\tilde{Q})} \mu \le |y| \le M\mu\,, \qquad (37)$$

then (34) is true and thus

$$\dot{V} \le -\Gamma(t) - \tilde{\epsilon} \frac{\lambda_m(\tilde{Q})|\tilde{x}|}{\|C_2\|} |y|\,. \qquad (38)$$

The remaining proof, concerning the asymptotic stability and $\mathcal{H}_\infty$ disturbance attenuation level, is the same as in Theorem 1, and is thus omitted. ∎

Remark 4. The difference between the control strategies (18) and (37) is that (18) is dependent on the state while (37) is dependent on the measurement output. This is natural since in the present situation we can not obtain the state information directly.

Remark 5. Although we focused our attention on Luenberger observer here, the result in this section can be easily extended to the case of the general dynamic output feedback

$$\begin{cases} \dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}y \\ u = \hat{C}\hat{x} + \hat{D}y \end{cases} \qquad (39)$$

where $\hat{x} \in \Re^{\hat{n}}$ ($\hat{n}$ is a fixed order on which there is no limitation), y is assumed to be quantized in the closed-loop system.

5. CONCLUDING REMARKS

In this paper, we have studied stabilization and $\mathcal{H}_\infty$ disturbance attenuation problem for feedback control systems where the states or the measurement outputs are quantized before they go to the controller. We have proposed a state-dependent (or output-dependent) control strategy for updating the quantizer's parameter on line so that the system is asymptotically stable and achieves the same $\mathcal{H}_\infty$ disturbance attenuation level as in the case where no quantization is involved.

Our future interest is $\mathcal{H}_\infty$ disturbance attenuation problem for feedback control systems with two quantizers (quantization of both states/outputs and control inputs), as shown in Fig.2. Furthermore, the application of these results for design of networked control systems is an interesting and challenging problem.

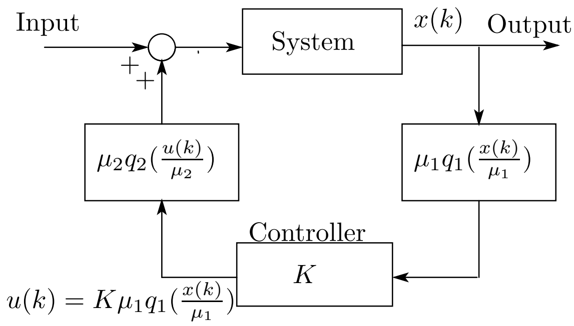

Fig.2 Feedback Control Systems with Two Quantizers

6. ACKNOWLEDGEMENT

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