DECENTRALISED DIAGNOSIS OF AUTOMATA NETWORKS

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Abstract: Automata-theoretic models have been used successfully in model-based process supervision and diagnosis. From a practical viewpoint, their main drawback is their complexity, which increases fast with the size of the original discrete-event system. This complexity can be reduced by compositional modelling resulting in an automata network. The reduced complexity of the network leads to a complexity reduction of the diagnostic algorithm, as the fault diagnosis can be performed in a decentralised way. The paper develops such a diagnostic method for nondeterministic and stochastic automata networks.

Keywords: Automata theory, diagnosis, decentralised systems, discrete-time systems

1. INTRODUCTION

The automaton is a well-known model type for discrete-event and discrete-time systems. It is easy to use and straightforward to implement. In recent years automata have also been used in process monitoring and fault diagnosis (Förstner and Lunze, 2001; Lamperti and Zanella, 2003). The drawback of the automaton is its size (number of possible state transitions), which increases fast with the size of the original system. This phenomenon is known as state space explosion. One way to counteract is to describe the behaviour of the system in terms of the behaviour of its components (compositional modelling). The result is an automata network which will be used in this paper for model-based fault diagnosis.

Automata networks, in principle, allow for distributed diagnosis which requires no global diagnoser, but uses instead multiple local diagnosers to determine the diagnostic result componentwise. Decentralised diagnosis of networks of nondeterministic discrete-event automata has been investigated by a number of groups e.g. (Lamperti and Zanella, 2003; Pencolé, 2000; Debouk *et al.*, 2000; Su and Wonham, 2004). This paper presents methods for decentralised diagnosis of networks of stochastic and nondeterministic discrete-time automata which are synchronised by a clock. The presented approach differs from the cited, because no explicit unfolding is done and the usage of stochastic information changes the diagnostic method and result fundamentally.

The structure of the paper is as follows. In Section 2 the automaton is defined formally and the solution to the diagnostic problem is presented. In Section 3 the automata network will be introduced. Decentralised diagnosis of networks is then presented in Section 4.

2. DIAGNOSIS OF DISCRETE-TIME AUTOMATA

In this section the stochastic and nondeterministic automaton are presented and it is shown how they can be used for diagnosis. Parts of this section have been described in more detail in (Blanke *et al.*, 2003; Schröder, 2003).

2.1 Stochastic Automata

The stochastic automaton (SA) is described by the tuple

$$\mathcal{S} = (\mathcal{N}_z, \mathcal{N}_v, \mathcal{N}_w, \mathcal{N}_f, L, \boldsymbol{p}(z_0), \boldsymbol{p}(f_0)) \qquad (1)$$

with the sets $\mathcal{N}_z$=$\{1, \ldots, N\}$ of automaton states, $\mathcal{N}_v$=$\{1, \ldots, M\}$ of input symbols, $\mathcal{N}_w$=$\{1, \ldots, R\}$ of output symbols, and $\mathcal{N}_f$=$\{1, \ldots, S\}$ of faults. The state of the automaton is denoted by $z \in \mathcal{N}_z$, the input by $v \in \mathcal{N}_v$, the output by $w \in \mathcal{N}_w$, and the fault by $f \in \mathcal{N}_f$. The initial state and initial fault are given as probability distributions $\boldsymbol{p}(z_0)$, $\boldsymbol{p}(f_0)$. At every time-step k for every signal a symbol from the respective domain is selected as its value.

The dynamics of the SA is given by the behavioural relation

$$L : \mathcal{N}_z \times \mathcal{N}_w \times \mathcal{N}_z \times \mathcal{N}_v \times \mathcal{N}_f \to [0, 1]. \qquad (2)$$

It describes the probability P for a state transition from $z(k)$ to $z(k+1)$ for a given input $v(k)$ while producing output $w(k)$ affected by fault f:

$$\begin{aligned} L(z(k{+}1), w(k)|z(k), v(k), f) &= L(z', w|z, v, f) \\ &= P(z(k{+}1), w(k)|z(k), v(k), f(k)). \end{aligned} \qquad (3)$$

Although all approaches presented in this paper allow non-constant faults, for notational convenience it will be assumed throughout the paper that all faults are constant. The relation will be abbreviated by

$$L(k) := L(z(k{+}1), w(k)|z(k), v(k), f).$$

The presented model is a discrete-time model, meaning a clock determines the progress of time (Cassandras and Lafortune, 1999).

State sequences are denoted by

$$Z(0 \ldots k_h) = (z(0), z(1), \ldots, z(k_h)).$$

The input and output sequences $V(0 \ldots k_h)$, $W(0 \ldots k_h)$ are defined analogously.

2.2 Nondeterministic Automata

The nondeterministic automaton (NA) can be derived from the SA by omitting the probabilistic information and merely stating if a transition is *possible* or not. The NA is described by the tuple

$$\mathcal{N} = (\mathcal{N}_z, \mathcal{N}_v, \mathcal{N}_w, \mathcal{N}_f, \bar{L}, z_0, f_0) \qquad (4)$$

with the sets defined as above. With the assumption of a constant fault (without restriction of generality) the dynamics of the NA is given by

$$\bar{L} : \mathcal{N}_z \times \mathcal{N}_w \times \mathcal{N}_z \times \mathcal{N}_v \times \mathcal{N}_f \to \{0, 1\}, \qquad (5)$$

which describes which state transition from $z(k)$ to $z(k+1)$ *may* occur given input $v(k)$ while producing the output $w(k)$ under the influence of fault f. Since the possibility of an event is described by the function

$$\text{Poss}(\bullet) : \text{domain}(\bullet) \to \{0, 1\}$$

which assumes the value 1 for a possible and 0 for an impossible event, the transition is possible if the relation

$$\bar{L}(z(k+1), w(k), z(k), v(k), f) = 1 \qquad (6)$$

holds, otherwise $\bar{L} = 0$.

2.3 Solution to the Diagnostic Problem

The aim of diagnosis is to determine the fault f which causes a system to operate abnormally. Since the initial state z_0 is in general unknown and the model is nondeterministic, the solution to the diagnostic problem is not a single fault, but a probability distribution $\boldsymbol{p}(f)$ is found for a SA or a set of possible faults $\mathcal{F}$ for a NA. The diagnostic problem for a single SA is stated as follows:

Fault diagnostic problem (SA).
Given:- Stochastic automaton $\mathcal{S}$
- Measurements $V(0 \ldots k_h)$, $W(0 \ldots k_h)$
- Probability distribution of initial state $\boldsymbol{p}(z_0)$ and fault $\boldsymbol{p}(f_0)$

Find: - Probability distribution $\boldsymbol{p}_{k_h}(f|V, W)$

The probability of a sequence of states $Z(0 \ldots k)$ given the measured input and output sequences $V(0 \ldots k)$, $W(0 \ldots k)$ and a fault f can be calculated by

$$P(Z(0 \ldots k), f|V(0 \ldots k), W(0 \ldots k)) = \frac{\sum\limits_{z(k+1)} L(k) \cdots L(1)L(0)P(z_0)P(f_0)}{\sum\limits_{Z(0 \ldots k+1)} L(k) \cdots L(1)L(0)P(z_0)P(f_0)}. \qquad (7)$$

The probability that the current state is z and the actual fault is f, given the measurements, is calculated using the acquired state sequences:

$$P(z(k), f|V(0 \ldots k), W(0 \ldots k)) = \sum_{Z(0 \ldots k-1)} P(Z(0 \ldots k), f|V(0 \ldots k), W(0 \ldots k)).$$

This calculation summarises the probabilities of all state sequences ending in state z. This is done, because it is of no interest *how* the system moved to the actual state, but only how *likely* it is for the system to reach the state z under the influence of the fault f. The probability of the fault is obtained by projecting the result onto the fault space:

$$P(f|V(0 \ldots k), W(0 \ldots k)) = \sum_{z(k)} P(z(k), f|V(0 \ldots k), W(0 \ldots k)). \qquad (8)$$

The distribution $\boldsymbol{p}_k(f)$ is a vectorial notion of the result (8) for all faults $f \in \mathcal{N}_f$.

The solution for the NA can be derived from (8) by omitting the probabilistic information:

$$\text{Poss}(f|V(0\dots k),W(0\dots k)) = \bigvee_{z(k)} \text{Poss}(z(k),f|V(0\dots k),W(0\dots k)). \quad (9)$$

The diagnostic result is the set of all possible faults:

$$\mathcal{F}(k_h)=\{f|\text{Poss}(f|V(0\dots k_h),W(0\dots k_h))=1\}.$$

Note that a recursive solution of the diagnostic problem has been found in (Schröder, 2003) which allows to calculate the current probability distribution $\boldsymbol{p}_k(f)$ by using the calculation result of the last step $\boldsymbol{p}_{k-1}(f)$ and the additional measurements $v(k)$, $w(k)$. This implies that no explicit unfolding is computed. It has also been proven that the diagnostic result is both complete and sound, meaning no fault is excluded wrongly and all fault candidates explain the observation.

3. AUTOMATA NETWORKS

An automata network consists of several interconnected automata (e.g. Figure 1). In this section the network is formally introduced and it is investigated how a network can be composed into a single automaton.

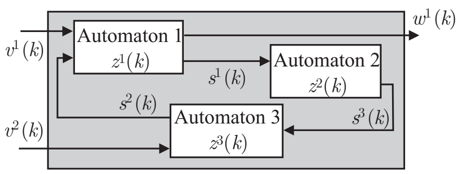


Figure 1. Example of an automaton network

3.1 Automata with Multiple Inputs and Outputs

To be able to connect several automata to a network, the definition of the automaton has to be extended to include multiple input and output signals. Therefore the behavioural relation of the SA is modified to

$$L(z',\boldsymbol{w}|z,\boldsymbol{v},f) = P(z',\boldsymbol{w}|z,\boldsymbol{v},f), \qquad (10)$$

where $\boldsymbol{v}$ is a set containing all input signals und $\boldsymbol{w}$ a set containing all output signals. All signals have a unique name. The sets $\mathcal{N}_v$ and $\mathcal{N}_w$ no longer contain single symbols, but the domains of the different signals. Every one of these domains is assigned uniquely to the specific signal. [1] Using this formalism, multiple faults can also be introduced easily. The same extension also applies to the NA.

3.2 Modelling the Network

A stochastic automata network (SAN) consists of γ SA. To distinguish between the network signals and the signals of an automaton the network's signals are marked by superscripts as opposed to subscripts for the automata's signals. Network input signals are denoted by $v^1,\dots,v^\mu$, network output signals by $w^1,\dots,w^\rho$ and coupling signals by $s^1,\dots,s^\kappa$. The network state $\boldsymbol{z}$ consists of the states of all γ automata of the network.

The i-th SA ($i\in\{1,\dots,\gamma\}$) of the network is defined by

$$\mathcal{S}_i = (\mathcal{N}_{zi},\mathcal{N}_{vi},\mathcal{N}_{wi},\mathcal{N}_{fi},L_i,\boldsymbol{p}(z_{i0}),\boldsymbol{p}(f_{i0})) \quad (11)$$

with

$$L_i(z_i',\boldsymbol{w}_i|z_i,\boldsymbol{v}_i,f_i) = P(z_i',\boldsymbol{w}_i|z_i,\boldsymbol{v}_i,f_i).$$

The stochastic automata network is then given by

$$\mathcal{SAN} = (\mathcal{S}_1,\dots,\mathcal{S}_\gamma). \qquad (12)$$

The nondeterministic automaton network (NAN) is defined analogously. As the single automaton is a clocked system the same holds true for the network. [2] An extended introduction to automata networks can be found in (Lunze and Neidig, 2003; Lunze and Schröder, 2003).
The couplings are defined by using the same symbols as input and output for different automata. E.g. a coupling from Automaton 1 to Automaton 2 in Fig. 1 occurs through signal s_1, which is an output signal of Automaton 1 and an input signal of Automaton 2. In other words, the network topology is completely defined by the sets of input and output signals $\boldsymbol{v},\boldsymbol{w}$ of all automata of the network.

Example: The network $(\mathcal{S}_1,\mathcal{S}_2,\mathcal{S}_3)$ in Fig. 1 consists of three automata and has two input signals v^1 and v^2 and one output signal w^1. All other signals are coupling signals. Automaton 1 is given by

$$\mathcal{S}_1 = (\mathcal{N}_{z1},\mathcal{N}_{v1},\mathcal{N}_{w1},L_1,z_{10})$$

with the behavioural relation $L_1(z_1',\boldsymbol{w}_1|z_1,\boldsymbol{v}_1)$ and $\boldsymbol{v}_1 = \{v^1,s^2\}$, $\boldsymbol{w}_1 = \{w^1,s^1\}$.

3.3 Composition Rules

Composing an automaton network means to create a single automaton with the same input/output behaviour as the original network. The principal approach to composition can be

[1] Formally the maps $\text{dom}_v : \boldsymbol{v} \to \mathcal{N}_v$ and $\text{dom}_w : \boldsymbol{w} \to \mathcal{N}_w$ which assign to every signal the respective domain have to be defined. However, to keep the formalism as simple as possible it is assumed that every signal v_i has its own domain $\mathcal{N}_{vi}$.

[2] Synchronised signals in combination with feedback connections may result in conflicts. The problem is known and has been dealt with, however, this is beyond the scope of this paper, cf. (Schröder, 2003; Pache, 2004).

found e.g. in (Lee and Varaiya, 2003). The composition of the network $\mathcal{SAN} = (\mathcal{S}_1, \ldots, \mathcal{S}_\gamma)$ results in the automaton

$$\tilde{\mathcal{S}} = (\tilde{\mathcal{N}}_z, \mathcal{N}_v, \mathcal{N}_w, \mathcal{N}_f, \tilde{L}, \boldsymbol{p}(\tilde{z}_0), \boldsymbol{p}(\tilde{f}_0))$$

with $\tilde{\mathcal{N}}_z = \mathcal{N}_{z1} \times \cdots \times \mathcal{N}_{z\gamma}$ and the sets of network inputs and outputs $\mathcal{N}_v$, $\mathcal{N}_w$. The behavioural relation $\tilde{L}$ of the composed SA is calculated by multiplying all behavioural relations of the SAN (element by element), evaluating this expression for all possible values of all coupling signals, and summing up the results. For a serial connection as shown in Fig. 2 this amounts to

$$\tilde{L}(\tilde{z}', w_2 | \tilde{z}, v_1) = \sum_{s_1} L_1(z_1', s_1 | z_1, v_1) \cdot L_2(z_2', w_2 | z_2, s_1) \quad (13)$$

for the stochastic automaton network and to

$$\tilde{\bar{L}}(\tilde{z}', w_2, \tilde{z}, v_1) = \bigvee_{s_1} \bar{L}_1(z_1', s_1, z_1, v_1) \wedge \bar{L}_2(z_2', w_2, z_2, s_1) \quad (14)$$

for the nondeterministic network (cf. (Lunze and Schröder, 2003; Plateau and Atif, 1991)).

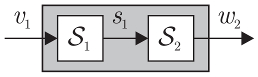
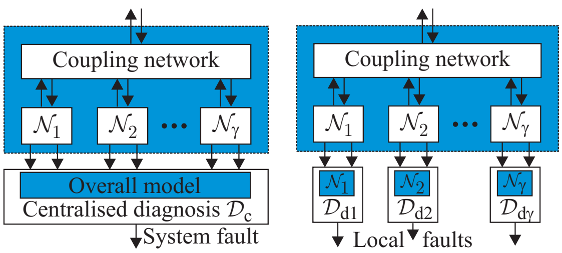


Figure 2. Serial connection

4. DIAGNOSING AUTOMATA NETWORKS

In this section several approaches to diagnose automata networks are presented. In Section 4.1 it is shown how automata networks can be diagnosed in a centralised way, whereas in Section 4.2 decentralised diagnosis is presented. It will be proven that decentralised diagnosis can be applied to all NAN, but only to a small class of stochastic automata networks.



a) Centralised diagnosis b) Decentralised diagnosis

Figure 3. Different approaches to diagnosis

4.1 *Centralised Diagnosis*

In centralised diagnosis one diagnoser $\mathcal{D}_c$ has access to all network inputs and outputs, which are lumped to vector-valued sequences $\boldsymbol{V}$ and $\boldsymbol{W}$, and the complete network model (cf. Fig. 3a)). The result of the centralised diagnosis is a global probability distribution $\boldsymbol{p}(f)$ for the SAN or a set of global fault candidates $\mathcal{F}$ for the NAN. The centralised diagnostic problem can be stated as follows:

Centralised diagnosis (SAN).

Given:- Stochastic automata network $\mathcal{SAN}$

- Measurements $\boldsymbol{V}(0 \ldots k_h)$, $\boldsymbol{W}(0 \ldots k_h)$
- Probability distribution of initial state $\boldsymbol{p}(\boldsymbol{z_0})$ and fault $\boldsymbol{p}(f_0)$

Find: - Global probability distribution $\boldsymbol{p}^c_{k_h}(f)$ [3]

The diagnostic problem can be solved using solution (8) developed for the single stochastic automaton if the composition rule is applied to the network. However, the resulting SA might be too large to handle even with modern computers and therefore must not be built explicitly. Instead the composition rule has to be integrated into the diagnostic algorithm to calculate the diagnostic result directly. The probability of a sequence of network states $\boldsymbol{Z}(0 \ldots k)$ for given network input and output sequences $\boldsymbol{V}(0 \ldots k)$, $\boldsymbol{W}(0 \ldots k)$ and a fault f is then calculated in case of the serial connection as shown in Fig. 2 by

$$P(\boldsymbol{Z}(0 \ldots k), f | \boldsymbol{V}(0 \ldots k), \boldsymbol{W}(0 \ldots k)) = \frac{\sum\limits_{z(k+1)} \sum\limits_{s_1} L_1(k) L_2(k) \cdots L_1(0) L_2(0) P(z_{10}, f_{10}, z_{20}, f_{20})}{\sum\limits_{\boldsymbol{Z}(0 \ldots k+1)} \sum\limits_{s_1} L_1(k) L_2(k) \cdots L_1(0) L_2(0) P(z_{10}, f_{10}, z_{20}, f_{20})}.$$

The probability distribution $\boldsymbol{p}^c_{k_h}(f)$ is computed as described in Section 2.3. The approach can be applied to NAN analogously. Since it has been proven that solution (8) is complete and sound and that a SAN can be composed to a single SA with the identical behaviour using the composition rule, the solution for centralised diagnosis is also complete and sound. This solution uses far less memory capacities compared to the usage of a single automaton since all calculations are done element by element. However, it is rather time consuming.

4.2 *Decentralised Diagnosis*

Although the presented approach to centralised diagnosis does not calculate the global model explicitly, it treats the model as a single unit and not as a collection of components. In decentralised diagnosis every component i is diagnosed independently by a local diagnoser $\mathcal{D}_{di}$ as shown in Fig. 3b). Every local diagnoser has only a model of the respective component and access to the component signals $\boldsymbol{V}_i \subset \boldsymbol{V}$ and $\boldsymbol{W}_i \subset \boldsymbol{W}$. The diagnostic result is a local probability distribution

[3] The superscript c marks the result of a centralised diagnosis

$\boldsymbol{p}(f_i)$ for every component. The main advantages of decentralised diagnosis are lower memory usage, good scalability, robustness, reusability of component models, and possibility to distribute calculations locally. The decentralised diagnostic problem can be given as a collection of local diagnostic problems:

Local diagnosis (SAN).
Given:- Stochastic automaton $\mathcal{S}_i$ of a network
- Local measurements $\boldsymbol{V}_i(0\dots k_h) \subset \boldsymbol{V}$, $\boldsymbol{W}_i(0\dots k_h) \subset \boldsymbol{W}$
- Probability distribution of local initial state $\boldsymbol{p}(z_{i0})$ and faults $\boldsymbol{p}(f_{i0})$

Find: - Local probability distribution $\boldsymbol{p}_{k_h}(f_i)$

Decentralised diagnosis (SAN).
Given:- Local probability distribution $\boldsymbol{p}_{k_h}(f_i), \forall i$
Find: - Global probability distribution $\boldsymbol{p}^d_{k_h}(f)$ [4]

The local and the decentralised diagnostic problem as stated above can easily be adapted to NAN. Here the result of a local diagnosis is a local set of fault candidates $\mathcal{F}_i$ and, hence, the result of decentralised diagnosis a global set $\mathcal{F}^d$.

The problem statement is straightforward, however, before a local diagnosis can be performed it has to be investigated if the global diagnostic problem can be separated into several local problems at all. Additionally, it has to be examined how the subdivision and neglecting of the coupling signals alters the diagnostic result, meaning how $\boldsymbol{p}^c_{k_h}(f)$ and $\boldsymbol{p}^d_{k_h}(f)$ and accordingly $\mathcal{F}^c$ and $\mathcal{F}^d$ are related. This is done in the remainder of the paper by proving the following two theorems.

Theorem 1. *Two automata in a stochastic automata network are* stochastically independent*, iff in every time-step k the value of the coupling signal s is unambiguously defined through the measured input v and output w.*

Proof 1. This proof will be shown for two SA in a series connection without loss of generality, since every network can be composed to a single automaton using the composition rule. Because SA possess the Markov property the proof can be restricted to $k = 0$. The symbols are denoted as in Fig. 2. Let

$$\mathcal{Z}_1'(v_1) = \{z_1' \mid L_1(z_1', s_1|z_1, v_1, f_1) > 0\}$$

be the set of all states the automaton $\mathcal{S}_1$ can be in at time $k+1$ with a given input v_1. This automaton has an output function $H_1 : \mathcal{N}_{v1} \rightarrow \mathcal{N}_{s1}$. The set

$$\mathcal{Z}_2'(w_2) = \{z_2' \mid L_2(z_2', w_2|z_2, s_1, f_2) > 0\}$$

and the output function $H_2 : \mathcal{N}_{s1} \rightarrow \mathcal{N}_{w2}$ are defined analogously for automaton $\mathcal{S}_2$. Let $\mathcal{Q}_v$ be the set of all possible values of s_1 given input v_1 and $\mathcal{Q}_w$ of s_1 given output w_2:

$$\begin{aligned} L_1 > 0 \quad \forall\, z_1' \in \mathcal{Z}_1'(v_1) \quad &\Leftrightarrow \quad \mathcal{Q}_v = H_1(v_1) \\ L_2 > 0 \quad \forall\, z_2' \in \mathcal{Z}_2'(w_2) \quad &\Leftrightarrow \quad \mathcal{Q}_w = H_2^{-1}(w_2). \end{aligned}$$

When calculating $P(z(0), f(0)|v(0), w(0))$ one gets the following equation for the nominator in (7)

$$\begin{aligned} &\sum_{z_1',z_2'} \tilde{L}\,(z_1', z_2', w_2|z_1, z_2, v_1, f_1, f_2) P(z_{10}, z_{20}) P(f_{10}, f_{20}) \\ &= \sum_{z_1',z_2',s_1 \in (\mathcal{Q}_v \cap \mathcal{Q}_w)} L_1 L_2 P(z_{10}) P(f_{10}) P(z_{20}) P(f_{20}) \\ &= \sum_{s_1 \in (\mathcal{Q}_v \cap \mathcal{Q}_w)} (\sum_{z_1'} L_1 P(z_{10}) P(f_{10}) \cdot \sum_{z_2'} L_2 P(z_{20}) P(f_{20})). \end{aligned}$$

If and only if $\mathrm{card}(\mathcal{Q}_v \cap \mathcal{Q}_w) = 1$ the value of s_1 is defined unambiguously and

$$\dots = \sum_{z_1'} L_1 P(z_{10}) P(f_{10}) \sum_{z_2'} L_2 P(z_{20}) P(f_{20}) \quad (15)$$

holds. This ensures the independence of the automaton states.

Corollary: A diagnostic problem for SAN can be divided into local diagnostic problems iff the automata are stochastically independent. Then $\boldsymbol{p}^c_{k_h}(f) = \boldsymbol{p}^d_{k_h}(f)$ holds.

Theorem 2. *The diagnostic problem for NAN can always be divided into local diagnostic problems. If the network contains non-measurable signals then $\mathcal{F}^c \subseteq \mathcal{F}^d$ holds.*

Proof 2. This proof will be held analogously to Proof 1. The symbols are denoted as in Fig. 2.

$$\begin{aligned} &\mathrm{Poss}(z_1(0), z_2(0), f_1(0), f_2(0), v_1(0), w_2(0)) = \\ &= \bigvee_{z_1',z_2'} \tilde{\bar{L}}(z_1', z_2', w_2, z_1, z_2, v_1, f_1, f_2) \\ &\wedge \mathrm{Poss}(z_{10}, z_{20}) \wedge \mathrm{Poss}(f_{10}, f_{20}) \\ &= \bigvee_{z_1',z_2'} \bigvee_{s_1} \bar{L}_1(z_1', s_1, z_1, v_1, f_1) \wedge \bar{L}_2(z_2', w_2, z_2, s_1, f_2) \\ &\wedge \mathrm{Poss}(z_{10}, f_{10}) \wedge \mathrm{Poss}(z_{20}, f_{20}) \\ &= \bigvee_{s_1} \left(\bigvee_{z_1'} \bar{L}_1 \wedge \mathrm{Poss}_{10} \wedge \bigvee_{z_2'} \bar{L}_2 \wedge \mathrm{Poss}_{20} \right) \\ &\leq \bigvee_{s_1} \left(\bigvee_{z_1'} \bar{L}_1 \wedge \mathrm{Poss}_{10} \wedge \bigvee_{z_2'} \bar{L}_2 \wedge \mathrm{Poss}_{20} \right) \\ &\quad \vee \left(\bigvee_{s^*,z_1'} \bar{L}_1 \wedge \mathrm{Poss}_{10} \wedge \bigvee_{s^{**} z_2'} \bar{L}_2 \wedge \mathrm{Poss}_{20} \right) \\ &\quad \text{with} \quad s^*, s^{**} \in \mathcal{N}_s, s^* \neq s^{**} \\ &= \left(\bigvee_{s_1,z_1'} \bar{L}_1 \wedge \mathrm{Poss}_{10} \right) \wedge \left(\bigvee_{s_1,z_2'} \bar{L}_2 \wedge \mathrm{Poss}_{20} \right) \\ &= \mathrm{Poss}(f_1|v_1(0)) \wedge \mathrm{Poss}(f_2|w_2(0)) \end{aligned}$$

with $\mathrm{Poss}_{10} = \mathrm{Poss}(z_{10}, f_{10})$ and $\mathrm{Poss}_{20} = \mathrm{Poss}(z_{20}, f_{20})$. This proves that the result of the centralised diagnosis is a subset of the merged results of the two local diagnosers: $\mathcal{F}^c \subseteq \mathcal{F}^d$.

[4] The superscript d marks the result of a decentralised diagnosis

Immeasurable signals lead to additional uncertainties in diagnosis. Since the goal is to have a complete result, that is to ensure that the true fault is included, all possible values of immeasurable signals have to be considered. It is impossible to ensure that the unknown signal has the same value for all connected NA, since every NA is treated by a separate diagnoser that has no information about the other diagnosers. Therefore, more signal combinations are considered during diagnosis than physically possible, leading to a larger number of spurious solutions. Since the result of the centralised diagnosis $\mathcal{F}^c$ is complete and it has been proven that $\mathcal{F}^c \subseteq \mathcal{F}^d$ holds, decentralised diagnostic result $\mathcal{F}^d$ is also complete.

The number of fault candidates can be reduced by explicitly including coupling signals in the solution and therefore treating them as dependent variables. It can also be reduced by additionally measuring coupling signals. It is the engineer's task to decide for the given system what signals may lead to a large number of fault candidates.

4.3 Complexity Considerations

The advantage of the decentralised approach becomes apparent, when analysing the computational complexity of the presented approaches. An automata network with γ automata can be represented by a table with $\sum_{i=1}^{\gamma} N_i^2 M_i R_i$ entries (worst case), where N, M, R are defined as in Section 2. A single automaton with the same behaviour as the network is represented by a table with $\prod_{i=1}^{\gamma} N_i^2 M_i R_i$ entries. Considering a network of 10 automata with 10 states, 10 inputs, and 10 outputs each, the network is represented by 10^5 and the single automaton by 10^{40} entries. A comparison with the number of grains of sand on earth (10^{24}) shows that the single automaton representation is not realisable. The same holds true for the number of calculations. The centralised diagnostic approach is of the order $O(\prod_{i=1}^{\gamma} N_i^2 C_i)$ (worst case), where C_i denotes the number of symbols of the coupling signals of automaton i. The decentralised approach is of the order $O(\sum_{i=1}^{\gamma} N_i^2 C_i)$.

5. CONCLUSION

This paper has presented an approach for modelling automata networks and a method for centralised diagnosis has been shown which does not necessitate to compose the network to a single automaton explicitly. It has been proven that automata in a nondeterministic automata network can be diagnosed separately allowing for decentralised diagnosis (Theorem 2). The result of the decentralised diagnosis is complete, meaning all candidates that explain the fault are found. However, it has also been proven that, in general, decentralised diagnosis of stochastic automata networks is not possible (Theorem 1). A criterion for testing if two automata are stochastically independent has been given.

REFERENCES

Blanke, M., M. Kinnaert, J. Lunze and M. Staroswiecki (2003). *Diagnosis and Fault-Tolerant Control*. Springer-Verlag. Berlin.

Cassandras, C. and S. Lafortune (1999). *Introduction to Discrete Event Systems*. Kluwer Academic Publishers. Boston.

Debouk, R., S. Lafortune and D. Teneketzis (2000). Coordinated decentralized protocols for failure diagnosis of discrete event systems. *Discrete Event Dynamic Systems* **10**(1-2), 33–86.

Förstner, D. and J. Lunze (2001). Fault detection of a diesel injection system by qualitative modelling. In: *Workshop on Advances in Automative Control, Karlsruhe 2001*. TU Karlsruhe.

Lamperti, G. and M. Zanella (2003). *Diagnosis of Active Systems*. first ed.. Kluwer Academic Publishers. Dordrecht, Netherlands.

Lee, A.E. and P. Varaiya (2003). *Structure and Interpretation of Signals and Systems*. Addison Wesley. Boston.

Lunze, J. and J. Neidig (2003). A new approach to distributed diagnosis using stochastic automata networks. In: *Proceedings of the Workshop on Advanced Control and Diagnosis (ACD)*. Duisburg, Germany. pp. 211–216.

Lunze, J. and J. Schröder (2003). Fault diagnosis of stochastic automata networks. In: *Safeprocess*. Washington. pp. 825–830.

Pache, Jens (2004). Analyse von stochastischen Automatennetzen. Diplomarbeit. Ruhr-Universität Bochum, Lehrstuhl für Automatisierungstechnik und Prozessinformatik.

Pencolé, Y. (2000). Decentralized diagnoser approach: Application to telecommunication networks. In: *Proceedings of the Eleventh International Workshop on Principles of Diagnosis (DX'00)*. Morelia, Mexico.

Plateau, B. and K. Atif (1991). Stochastic automata network for modeling parallel systems. *IEEE Transactions on Software Engineering* **17**(10), 1093–1108.

Schröder, J. (2003). *Modelling, State Observation and Diagnosis of Quantised Systems*. Springer-Verlag. Berlin.

Su, R. and W. M. Wonham (2004). A model of component consistency in distributed diagnosis. In: *Proceedings of the Fourteenth International Workshop on Principles of Diagnosis (DX'04)*. Carcassonne, France. pp. 63–68.