A HIERACHICAL HYBRID METHOD FOR SIMULTANEOUS LOCALIZATION AND MAPPING

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Abstract: In robotics literature, most of existing Simultaneous Localization and Mapping (SLAM) algorithms are limited by the size and type of the environments they can handle. A few methods can cope with large scale environments. In this paper, we propose a novel hierarchical hybrid method for SLAM in large scale and cyclic environments: *locally* solve SLAM by Maximum Likelihood (ML) with occupancy grid map, and *globally* by Extended Kalman Filter (EKF) with feature-based map. Experiments validated on Pioneer 2DX mobile robot demonstrate the capabilities and the robustness of our proposed algorithm.

Keywords: Autonomous Mobile Robot (AMR), Simultaneous Localization and Mapping (SLAM), Maximum Likelihood (ML), Extended Kalman Filter (EKF), Occupancy grid map, Feature-based map.

1. INTRODUCTION

Simultaneous Localization and Mapping (SLAM) is an essential capability for Autonomous Mobile Robot (AMRs) to explore unknown environments, and has been attracted immense attention in the literature in the past decades since Smith, *et al.* (1990) introduced this problem. As Newman (1999) defined, "The SLAM problem asks if it is possible for an autonomous vehicle to start in an unknown location in an unknown environment and then to incrementally build a map of this environment while simultaneously using this map to compute absolute vehicle location", which is an *inference* problem. The web site of the SLAM summer school 2002 (Christensen, 2002) provides a comprehensive coverage of the key topics and state of the art in SLAM. The dominant approach to SLAM problem is based on Extended Kalman Filter (EKF) to estimate the joint posterior distribution over the maps and the robot poses (Dissanayake, *et al.* 2001). However, EKF approach inherently requires Gaussian posteriors, which consequently needs expensive computation especially in large scale environments with a large number of features. Another more important limitation is weak to solve data association problem. In SLAM literature four major paradigms for environment representation are widely used: direct approach, feature-based map, occupancy grid-based map and topological map. Direct method (Lu, *et al.* 1997) represents the physical environment using raw data points without extracting predefined features. However, the feature based map (Ip, *et al.* 2004) compresses raw data into predefined features. Occupancy grid-based (Elfes, 1989) map is generated from stochastic estimates of the occupancy state of an object in a given cell. It is rather easy to construct and maintain it, whereas topological map (Choset, *et al.* 2001) is just graph-like spatial representation. For an unknown environment, it is usually difficult to represent the world by only one method. For example, predefined feature model is hard to get before hand because we do not know what type objects will be in the surroundings, especially harder in dynamic environments; it is a very hard computation load for occupancy grid-based method and direct method to map a large scale environment, and also some inconsistency will happen in cyclic environments. Topological map is an abstract and top level on previous methods.

Recently, Bosse and Newman, *et al.* (2004) proposed a hybrid metrical/topological approach which they called Atlas to solve SLAM in large-scale cyclic environments. In this paper, we also propose a hierarchical hybrid method to overcome the limitations of above methods. Our proposed method hierarchically incorporates two filtering methods of Maximum Likelihood (ML) and EKF, and two representation methods of occupancy grid-based map and feature-based map. We employ ML to solve SLAM basically, thanks to its simplicity and fast computation, and adopt occupancy grid map to represent the environment. However, just as the earlier discussion, grid-based approach does not provide a mechanism for loop closing and also is suffered from too much storage and computation load for large scale environments. So, we *locally* solve SLAM by ML with occupancy grid map, and *globally* solve it by EKF with feature-based map where feature is local grid map with 3-Degree of Freedom (3-DOF) state. EKF feature-based algorithm can smoothly solve the loop closing problem, which is a well-known point in the SLAM literature.

This paper is organized as follows. In the following section, we will present the problem statement of SLAM from Bayesian perspective, with a theoretical Bayesian formulation. In Section 3, we will describe our algorithm in detail. Section 4 presents some preliminary experimental results to demonstrate the capabilities and the robustness of our approach. Conclusion will come into Section 5.

2. THE SLAM PROBLEM

SLAM is referred to as the ability of an AMR to incrementally extract the surrounding features for estimating its pose in an unknown location and unknown environment. It involves simultaneously estimating positions of newly perceived landmarks and the location of the robot itself while incrementally building a map, which is an *inference* problem. Fig. 1 illustrates the generative Dynamic Bayesian Network (DBN) that underlines the rich corpus of SLAM literature (Montemerlo, *et al.* 2002). In particular, we denote the discrete time index by the variable t, odometry reading from t-1 to t by u_t, sensor measurement at t by z_t, true location of the robot by x_t, the map by m. And the following sets refer to data leading up to time t.

$$\begin{cases} u_{0:t} \equiv \{u_0, u_1, ..., u_t\} = \{u_{0:t-1}, u_t\} \\ z_{0:t} \equiv \{z_0, z_1, ..., z_t\} = \{z_{0:t-1}, z_t\} \\ x_{0:t} \equiv \{x_0, x_1, ..., x_t\} = \{x_{0:t-1}, x_t\} \end{cases} \tag{1}$$

Then SLAM can be formulated as a recursive Bayesian filtering problem based on Markov assumption (shown in Eq.2). That is, given the knowledge of current states, the future is independent of the past. In particular, it implies that the posterior estimate $p(x_t \mid z_{0:t}, u_{0:t})$ is sufficient statistics for the past data, with regards to the prediction. This assumption holds true especially for static environments which is another general assumption in the SLAM literature. Thrun (2001) has presented detailed derivations for SLAM.

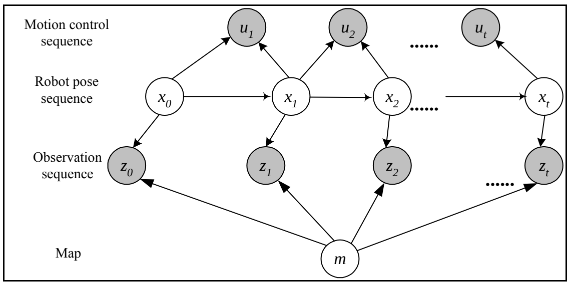

Fig. 1. A DBN for SLAM: Shaded circles denote explicit states and clear circles denote hidden or implicit states which should be inferenced from explicit ones.

$$\begin{aligned} & p(x_t, m \mid z_{0:t}, u_{0:t}) \\ & \overset{Bayes}{\propto} p(z_t \mid x_t, m, z_{0:t-1}, u_{0:t})\, p(x_t, m \mid z_{0:t-1}, u_{0:t}) \\ & \overset{Markov}{\propto} p(z_t \mid x_t, m)\, p(x_t, m \mid z_{0:t-1}, u_{0:t}) \\ & = p(z_t \mid x_t, m) \int p(x_t \mid u_t, x_{t-1}) p(x_{t-1}, m \mid z_{0:t-1}, u_{0:t-1}) dx_{t-1} \end{aligned} \tag{2}$$

We also typically term $p(z_t \mid x_t, m)$ in Eq.2 as measurement model which specifies the likelihood of the observation z for every possible location x; and term $p(x_t \mid u_t, x_{t-1})$ as motion model which defines the likelihood that the robot is at x_t given previous location x_{t-1} and motion control u_t.

We use SICK LMS 200 in our work. Just as shown in Fig. 2, the spot spacing of SICK LMS 200 is smaller than the spot diameter for an angular resolution of 0.5 degree. This means that footprints of consecutive measurements overlap each other. In spite of its high accuracy of measurement, it is still over optimistic to neglect the measurement noise. Therefore, we assume that the errors in range and bearing can be modelled as a Gaussian uncorrelated white sequence with constant variance respectively.

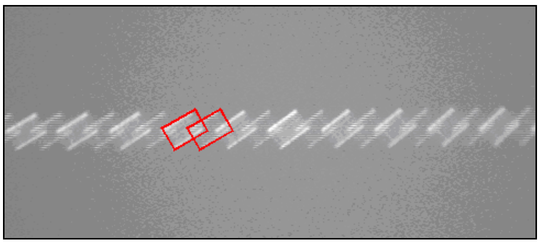

Fig. 2. Footprint of the measurement from SICK LMS200: A red rectangle indicates a footprint of one measurement point. We borrow this photo from Wang (2004) with the author's permission.

The wheel encoders in our Pioneer 2DX robot are used to measure wheel rotation and steering orientation. Position errors grow with drift, bias or slippage, and also will accumulate over time as integration errors. Therefore, we employ small motion method, which assumes that only a small error occurs for a short-distant travel, to model robot kinematics, shown in Fig. 3. Thus we also model motion noise as a zero-mean multivariate Gaussian.

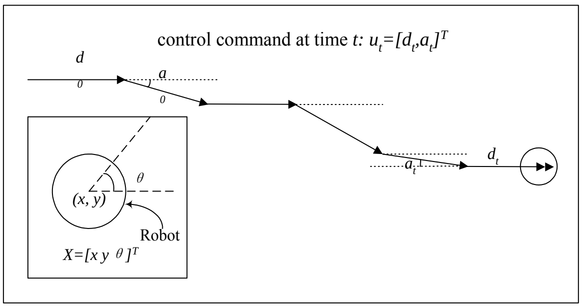

Fig. 3. Graphical representation for small motion with control command.

3. LOCAL ML WITH OCCUPANCY GRID MAP AND GLOBAL EKF WITH FEATURE-BASED MAP

To overcome the drawbacks of expensive computation and storage and inconsistency for large and cyclic environments as earlier discussion, and to find an online SLAM algorithm for such environments, we propose to *locally* solve SLAM by ML with occupancy grid map and *globally* solve it by EKF with feature-based map.

3.1. Local ML with Occupancy Grid Map

ML Estimation The idea and implementation of ML is simple, thus meeting the online computing requirements: Given a sensor measurement and odometry reading, determine the most likely pose. Then append the pose and build the map. Particularly for SLAM (2), that means to maximize the marginal likelihoods of pose and map given previous pose and map. We just use following function (3) to update the map, and in practice, we employ occupancy grid map to implement it, which we will discuss later.

$$\hat{m}(x_{0:t}, z_{0:t}) = \arg\max\ p(m \mid x_{0:t}, z_{0:t}) \qquad (3)$$

So, when the map available, SLAM problem (2) can be reduced further at each time step, with additional assumption that previous step pose $\hat{x}_{t-1}$ is known too.

$$\begin{aligned} & p(x_t, \hat{m}(x_{0:t}, z_{0:t}) \mid z_{0:t}, u_{0:t}) \\ &= p(z_t \mid x_t, \hat{m}(x_{0:t}, z_{0:t})) \\ &\quad \bullet \int p(x_t \mid u_t, x_{t-1})\, p(x_{t-1}, \hat{m}(x_{0:t-1}, z_{0:t-1}) \mid z_{0:t-1}, u_{0:t-1})\, dx_{t-1} \\ &= p(z_t \mid x_t, \hat{m}(x_{0:t}, z_{0:t}))\, p(x_t \mid u_t, \hat{x}_{t-1}) \end{aligned} \qquad (4)$$

Now, what is left to do is to calculate the *t*-th pose by ML estimate.

$$\hat{x}_t = \arg\max_{x_t} p(z_t \mid x_t, \hat{m}(x_{0:t}, z_{0:t}))\, p(x_t \mid u_t, \hat{x}_{t-1}) \qquad (5)$$

To calculate (5) is only a complex mathematics exercise. We omit the detailed information here.

Occupancy Grid Map As discussed previously, we apply occupancy grid map method to calculate (3). So here we establish the standard occupancy grid map approach (Elfes, 1989). As the name suggests, occupancy grid maps usually are represented by two-dimensional grids and generate probabilistic maps. Let $m_{x\,y}$ denote the occupancy of the grid cell at <*x*, *y*> in the map *m*. Occupancy is a binary variable: Either the cell is occupied or it is free. The problem, thus, is to calculate a posterior over a set of binary variables, each of which is a single numerical probability $p(m_{x,y} \mid x_{0:t}, z_{0:t})$. Then, we apply Bayes filters to calculate these posteriors. For computational reasons, it is common practice to calculate the so-called *log-odds*:

$$\begin{aligned} l_{x,y}^T &= \log \frac{p(m_{x,y} \mid x_{0:t}, z_{0:t})}{1 - p(m_{x,y} \mid x_{0:t}, z_{0:t})} \\ &= \log \frac{p(m_{x,y} \mid x_t, z_t)}{1 - p(m_{x,y} \mid x_t, z_t)} + \log \frac{p(m_{x,y})}{1 - p(m_{x,y})} + \log \frac{p(m_{x,y} \mid x_{0:t-1}, z_{0:t-1})}{1 - p(m_{x,y} \mid x_{0:t-1}, z_{0:t-1})} \\ &= \log \frac{p(m_{x,y} \mid x_t, z_t)}{1 - p(m_{x,y} \mid x_t, z_t)} + \log \frac{p(m_{x,y})}{1 - p(m_{x,y})} + l_{x,y}^{T-1} \end{aligned} \qquad (6)$$

$$l_{x,y}^0 = \log \frac{p(m_{x,y})}{1 - p(m_{x,y})} \qquad (7)$$

Obviously, the occupancy grid mapping algorithm is recursive with the initialization (7). So, to compute the desired probability only requires two entities: *occupancy prior* $p(m_{x,y})$ and *inverse sensor model* $p(m_{x,y} \mid x_t, z_t)$ which specifies the probability that a grid cell $m_{x\,y}$ is occupied based on a single sensor measurement z_t taken at location x_t. When log-odd is available, we need to recover the desired posterior from the calculated log-odd:

$$p(m_{x,y} \mid x_{0:t}, z_{0:t}) = 1 - [1 + e^{l_{x,y}^T}]^{-1} \qquad (8)$$

3.2. Global EKF with Feature-based Map

As we mentioned repeatedly in the previous sections, occupancy grid-based approach does not provide a mechanism for loop closing and also is suffered from too much storage and computation load for large scale environments. So, we build occupancy grid map locally, and treat each local grid map as a 3 Degree of Freedom (3-DOF) feature state represented by the gravity and orientation of the local map, then employ EKF to update these features globally. The advantage of this hierarchical scheme is to overcome the storage and inconsistency problems. To consistently close a large cyclic map, we must recognize the already visited place, specifically, we must know whether current local grid map is in a pre-visited place or not. To do so, we adopt covariance increasing (Li, 1998) when there is an inconsistency in the global map (illustrated in Fig. 4).

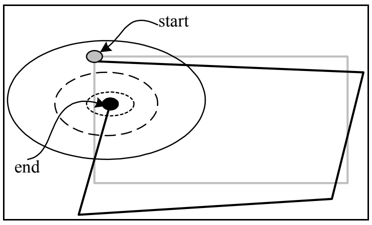

Fig. 4. Covariance increasing activated when inconsistency is detected.

EKF feature-based map is a dominant method in SLAM literature in the past decades. The overall algorithm is summarized as follows.

Algorithm *EKF*

1. Initialization step

- Initialize the mean square error covariance $P_{0|0}$, predict the position $x_{0|0}$, state noise covariance model Q_0 and measurement noise covariance model R_0.

2. Prediction step

- $$\hat{x}_{t|t-1} = E[x_t \mid z_{1:t-1}]$$
$$\approx E[f_t(\hat{x}_{t-1|t-1}, u_t, t) + (x_{t-1} - \hat{x}_{t-1|t-1})\nabla F_t + o[(x_{t-1} - \hat{x}_{t-1|t-1})^2] + \upsilon_t \mid z_{1:t-1}]$$
$$= f_t(\hat{x}_{t-1|t-1}, u_t, t)$$

- $$P_{t|t-1} = E[(x_t - \hat{x}_{t|t-1})(x_t - \hat{x}_{t|t-1})^T \mid z_{1:t-1}]$$
$$= \nabla F_t P_{t-1|t-1} \nabla F_t^T + Q_t$$

where:

$\nabla F_t = \nabla f_t(x_{t-1})|_{\hat{x}_{t-1|t-1}}$: Jacobian matrix of state transition model with respect to robot state

Q_t: state noise covariance at time t

3. Update step

- Innovation:

$$v_t \equiv z_t - \hat{z}_{t|t-1} \equiv z_t - E[z_t \mid z_{1:t-1}] = z_t - h_t(\hat{x}_{t|t-1})$$

- Innovation covariance:

$$S_t \equiv E[v_t v_t^T] = \nabla H_t P_{t|t-1} \nabla H_t^T + R_t$$

- Kalman gain:

$$K_t = P_{t|t-1} \nabla H_t^T S_t^{-1}$$

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t(z_t - h_t(\hat{x}_{t|t-1}))$$

$$P_{t|t} = P_{t|t-1} - K_t S_t K_t^T$$

where:

$\nabla H_t = \nabla h_t(x_t)|_{\hat{x}_{t|t-1}}$: Jacobian matrix of measurement model with respect to robot state

R_t: measurement noise covariance at time t

4. EXPERIMENTAL RESULTS

We have validated the proposed algorithm on a Pioneer 2DX mobile robot equipped with SICK LMS200 (shown in Fig. 5) and further experiments are still on-going now. But the preliminary results also demonstrate the capabilities and the robustness of our proposed algorithm.

We first performed our experiments in an indoor structured environment. The hand-measured world

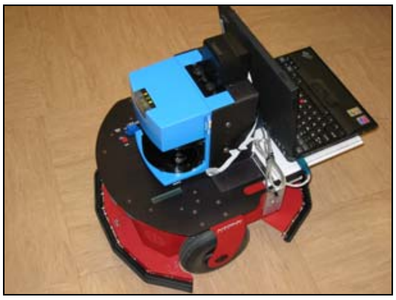

Fig. 5. Experimental platform: Pioneer 2DX mobile robot with SICK LMS200.

model is shown in Fig. 6. And the map generated by raw laser scan by using direct method is shown in Fig. 7. As seen from Fig. 7, there is obvious inconsistency among the map, especially when the robot moves into the pre-visited places such as corners of the map. Therefore we need to design some methods to overcome such inconsistency, which is just the main task in our work.

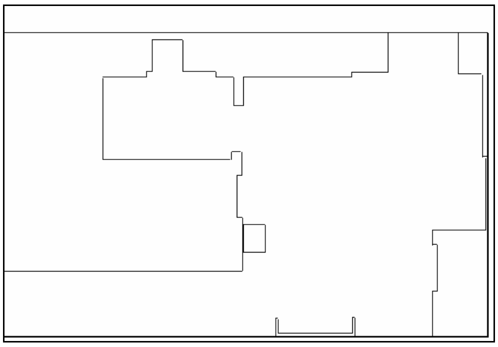

Fig. 6. The exact hand-measured world model

Fig. 7. Map generated by raw laser scan.

Then we basically use ML with occupancy grid map to model the same environment. The resulting map is shown in Fig. 8. In Fig. 8, the green place denotes unknown region, the gray place denotes free region, the dark place denotes the objects, and the red line just denotes the robot trajectory. It is easy to see from Fig. 8 that the inconsistency problem is well solved, even robot frequently moves into pre-visited region.

In Fig. 9, we also show four local grid maps how to work in global EKF level in detail in the whole mapping procedure. Map denotations in Fig. 9 are same as Fig.8.

5. CONCLUSION

In this paper, we propose a hierarchical hybrid method to solve SLAM problem. In our algorithm, we basically employ ML with occupancy grid map to solve SLAM in local level. Because there will be some inconsistency when working in cyclic environments and also the much expensive storage and computation load for occupancy grid map for large scale world, we apply EKF feature-based map to update the resulting map in global level. The practical experiments demonstrate that our approach can run in real-time. And it is easy to scale to large environments just modify the size and resolution of the grid map.

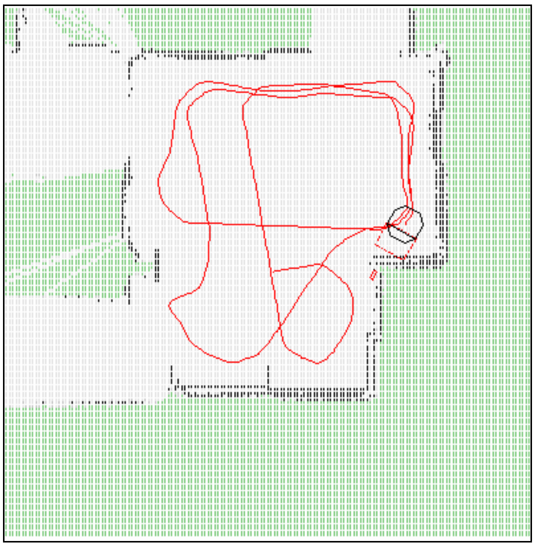

Fig. 8. Map generated by the proposed algorithm.

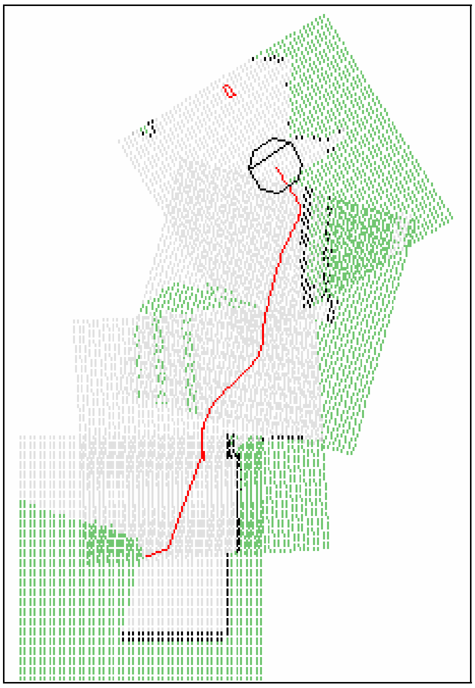

Fig. 9. Four local grid maps during whole mapping procedure.

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