SIMULTANEOUS CONTROL OF GRASP/MANIPULATION AND CONTACT POINTS WITH ROLLING CONTACT

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Abstract: In this paper, for simultaneous control of grasp/manipulation and contact points by a two-fingered robot hand with the pure rolling contact, we provide an entire treatment of the system equations including motion and force constraint, which consist of the generalized coordinates and the contact coordinates. In contrast to most previous studies where specified degrees of freedom (DOF) of fingers are consiered, we provide a general treatment of the system for *any* DOF of the fingers. Utilizing the results, a control design method which achieves the simultaneous control is proposed.

Keywords: multi-fingered robot hand, grasp and manipulation, nonholonomy of rolling

1. INTRODUCTION

Recently, control of grasp and manipulation of an object by a multi-fingered robot hand has been studied by many researchers. In the control of grasp and manipulation, the contact points between the fingers and the object can be changed simultaneously by utilizing the nonholonomy of rolling. However, since the system equations consist of the contact coordinates for the contact points as well as the generalized coordinates, the simultaneous control of the grasp/manipulation and the contact coordinates is somewhat involved. The control problem has been studied from two separated viewpoints. On the one hand, for the dynamical model of the robot hand and the object, the tracking control of the object motion and the internal force has been considered (Cole *et al.*, 1989; Sarkar and Yun, 1997). On the other hand, for the simple kinematic model of the contact coordinates, the regulation of the contact coordinates has been considered (Li and Canny, 1990; Bicchi and Marigo, 2002). To achieve the simultaneous control of the object motion/internal force and the contact coordinates, more detailed analysis of the relationship between the generalized coordinates and the contact coordinates is required.

In this paper, for the simultaneous control by a two-fingered robot hand with the pure rolling contact, we provide an entire treatment of the system equations, which consist of the generalized coordinates and the contact coordinates. In contrast to the most previous studies (Cole *et al.*, 1989; Sarkar and Yun, 1997) which consider specified degrees of freedom (DOF) of the fingers, we provide a general treatment of the system for *any* DOF of the fingers. Utilizing the results, a control design method which achieves the simultaneous control is proposed.

2. MODELING

2.1 System Configuration

In this paper, we consider two fingertips grasping an object shown in **Fig. 1**. The pair of two fingertips is a simplified model of a two-fingered robot hand, each finger of which has m_i DOF ($0 \leq m_i \leq 6$). The contact point between each finger and the object is single. In the following, the number of the fingers and the contact points is described by $i = 1, 2$. Arguments of vectors and matrices are described explicitly only when they appear first time, and will be omitted in the sequel for notational simplicity. In this study, we make the following assumptions.

Assumption 1. The surfaces of each finger and the object are the regular surfaces (Murray *et al.*, 1994). Therefore, contact points on the surfaces of each finger and the object can be described by $\boldsymbol{c}(\boldsymbol{\alpha}) \in \mathbb{R}^3$, where $\boldsymbol{c}(\cdot) : \mathbb{R}^2 \mapsto \mathbb{R}^3$ is a local

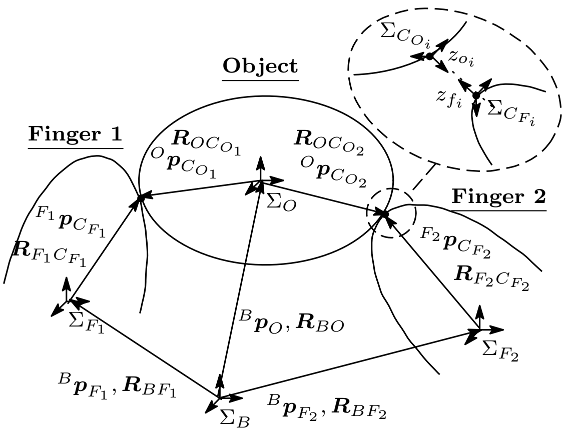

Fig. 1. Two fingertips grasping an object.

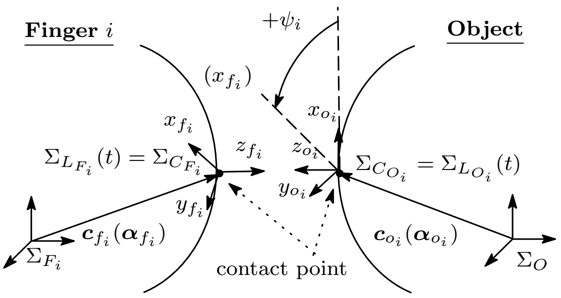

Fig. 2. Contact coordinates of ith contact point.

orthogonal chart and $\boldsymbol{\alpha} \in \mathbb{R}^2$ is local coordinates.

Assumption 2. The frictional forces at each contact point follow the *Coulomb's law*. The contact force applied to the object by each finger is composed of translational forces and a moment about the contact normal.

Assumption 3. The constraint at each contact point is described by the pure rolling contact. The forces generated by the constraint do not work on the system (*d'Alembert's principle*).

In **Fig. 1**, Σ_B is the reference coordinate frame. Σ_{F_i} and Σ_O are the coordinate frames fixed to the ith finger and the object, respectively. The configuration of Σ_{F_i} relative to Σ_B is represented by the position vector ${}^{B}\boldsymbol{p}_{F_i}(\boldsymbol{\theta}_{F_i}) \in \mathbb{R}^3$ and the rotation matrix $\boldsymbol{R}_{BF_i}(\boldsymbol{\theta}_{F_i}) \in \mathbb{R}^{3\times 3}$ where $\boldsymbol{\theta}_{F_i} \in \mathbb{R}^{m_i}$ represents the generalized coordinates of the ith finger, and $\boldsymbol{\theta}_F := [\ \boldsymbol{\theta}_{F_1}^{\mathrm{T}} \quad \boldsymbol{\theta}_{F_2}^{\mathrm{T}}\ ]^{\mathrm{T}} \in \mathbb{R}^m$, $m := m_1 + m_2$. Similarly, the configuration of Σ_O relative to Σ_B is represented by ${}^{B}\boldsymbol{p}_O \in \mathbb{R}^3$ and $\boldsymbol{R}_{BO}({}^{B}\boldsymbol{\phi}_O) \in \mathbb{R}^{3\times 3}$ where ${}^{B}\boldsymbol{\phi}_O$ represents the local parameterization of $\boldsymbol{R}_{BO}$ and $\boldsymbol{x}_O := [\ {}^{B}\boldsymbol{p}_O^{\mathrm{T}} \quad {}^{B}\boldsymbol{\phi}_O^{\mathrm{T}}\ ]^{\mathrm{T}} \in \mathbb{R}^6$. Note that the DOF of the generalized coordinates is $(m+6)$. In the dashed area, $\Sigma_{C_{F_i}}$ and $\Sigma_{C_{O_i}}$ are the coordinate frames attached on the surfaces of the ith finger and the object with the origins at the ith contact point. The z_{f_i}- and z_{o_i}-axes of the frames are outward and normal to the surfaces of the ith finger and the object, respectively. The configuration of $\Sigma_{C_{F_i}}$ relative to Σ_{F_i} is represented by the position vector ${}^{F_i}\boldsymbol{p}_{C_{F_i}} \in \mathbb{R}^3$ and the rotation matrix $\boldsymbol{R}_{F_iC_{F_i}} \in \mathbb{R}^{3\times 3}$. Similarly, the configuration of $\Sigma_{C_{O_i}}$ relative to Σ_O is represented by ${}^{O}\boldsymbol{p}_{C_{O_i}} \in \mathbb{R}^3$ and $\boldsymbol{R}_{OC_{O_i}} \in \mathbb{R}^{3\times 3}$.

Figure 2 shows the neighborhood of the ith contact point, where the ith finger and the object are depicted separately. $\Sigma_{L_{F_i}}(t)$ and $\Sigma_{L_{O_i}}(t)$ are the local frames fixed relative to Σ_{F_i} and Σ_O, respectively, which coincide at time t with $\Sigma_{C_{F_i}}$ and $\Sigma_{C_{O_i}}$. From Assumption 1, ${}^{F_i}\boldsymbol{p}_{C_{F_i}}$ and ${}^{O}\boldsymbol{p}_{C_{O_i}}$ in **Fig.1** can be described as ${}^{F_i}\boldsymbol{p}_{C_{F_i}} := \boldsymbol{c}_{f_i}(\boldsymbol{\alpha}_{f_i})$ and ${}^{O}\boldsymbol{p}_{C_{O_i}} := \boldsymbol{c}_{o_i}(\boldsymbol{\alpha}_{o_i})$ respectively, where $\boldsymbol{c}_{f_i}(\cdot), \boldsymbol{c}_{o_i}(\cdot) : \mathbb{R}^2 \mapsto \mathbb{R}^3$ are local orthogonal charts and $\boldsymbol{\alpha}_{f_i} \in \mathbb{R}^2$, $\boldsymbol{\alpha}_{o_i} \in \mathbb{R}^2$ are local coordinates. In addition, let ψ_i be the angle between the x- axes of $\Sigma_{C_{F_i}}$ and $\Sigma_{C_{O_i}}$ as shown in **Fig. 2**, then the configuration of the contact points is described by $\boldsymbol{\eta} := [\ \boldsymbol{\eta}_1^{\mathrm{T}} \quad \boldsymbol{\eta}_2^{\mathrm{T}}\ ]^{\mathrm{T}} \in \mathbb{R}^{10}$, where $\boldsymbol{\eta}_i := [\ \boldsymbol{\alpha}_{f_i}^{\mathrm{T}} \quad \boldsymbol{\alpha}_{o_i}^{\mathrm{T}} \quad \psi_i]^{\mathrm{T}} \in \mathbb{R}^5$ is called the contact coordinates for the contact point (Montana, 1988).

$\boldsymbol{\tau} := [\ \boldsymbol{\tau}_1^{\mathrm{T}} \quad \boldsymbol{\tau}_2^{\mathrm{T}}\ ]^{\mathrm{T}} \in \mathbb{R}^m$ describes the input to the fingers, where $\boldsymbol{\tau}_i \in \mathbb{R}^{m_i}$ is the force/torque applied to $\boldsymbol{\theta}_{F_i}$. From Assumption 2, the contact force is described by ${}^{C}\boldsymbol{F}_C := [\ {}^{C}\boldsymbol{F}_{C_1}^{\mathrm{T}} \quad {}^{C}\boldsymbol{F}_{C_2}^{\mathrm{T}}\ ]^{\mathrm{T}} \in \mathbb{R}^8$, where ${}^{C}\boldsymbol{F}_{C_i} \in \mathbb{R}^4$ is the contact force applied to the object by the ith finger.

2.2 Contact Kinematics

At the ith contact point, the following equations hold (Murray and Sastry, 1990):

$$ {}^{B}\boldsymbol{p}_{F_i} + \boldsymbol{R}_{BF_i}{}^{F_i}\boldsymbol{p}_{C_{F_i}} = {}^{B}\boldsymbol{p}_O + \boldsymbol{R}_{BO}{}^{O}\boldsymbol{p}_{C_{O_i}} \quad (1)$$

$$ \boldsymbol{R}_{BF_i}\boldsymbol{R}_{F_iC_{F_i}} = \boldsymbol{R}_{BO}\boldsymbol{R}_{OC_{O_i}}\boldsymbol{R}_{C_{O_i}C_{F_i}}. \quad (2)$$

Eq. (1) requires that the position vectors of the contact frames $\Sigma_{C_{F_i}}$ and $\Sigma_{C_{O_i}}$ with respect to the reference frame Σ_B coincide with each other. Eq. (2) requires that the contact normals and the tangent planes at the origins of $\Sigma_{C_{F_i}}$ and $\Sigma_{C_{O_i}}$ coincide with each other. Eqs. (1) and (2) relate the contact coordinates $\boldsymbol{\eta}_i$ to the generalized coordinates $(\boldsymbol{\theta}_{F_i}, \boldsymbol{x}_O)$.

Let $\boldsymbol{V}_{C_i} := [\, v_{Cx_i} \ v_{Cy_i} \ v_{Cz_i} \ \omega_{Cx_i} \ \omega_{Cy_i} \ \omega_{Cz_i}\,]^{\mathrm{T}} \in \mathbb{R}^6$ be the velocity of $\Sigma_{L_{F_i}}$ relative to $\Sigma_{L_{O_i}}$ seen from $\Sigma_{L_{F_i}}$. Differentiating (1) and (2) with respect to time t yields the motion of the contact coordinates $\dot{\boldsymbol{\eta}}_i$ as a function of the relative motion $\boldsymbol{V}_{C_i}$ (Murray and Sastry, 1990; Montana, 1988):

$$ \dot{\boldsymbol{\eta}}_i = \boldsymbol{H}_i(\boldsymbol{\eta}_i)\boldsymbol{V}_{C_i} \quad (3)$$

$$ 0 = v_{Cz_i}, \quad (4)$$

where

$$ \boldsymbol{H}_i := \begin{bmatrix} -\boldsymbol{M}_{gf_i}^{-1}\boldsymbol{K}_{R_i}^{-1}\tilde{\boldsymbol{K}}_{go_i} & \boldsymbol{0} & \boldsymbol{M}_{gf_i}^{-1}\boldsymbol{K}_{R_i}^{-1}\boldsymbol{E} & \boldsymbol{0} \\ \hline \boldsymbol{M}_{go_i}^{-1}\boldsymbol{R}_{\psi_i}\boldsymbol{K}_{R_i}^{-1}\boldsymbol{K}_{gf_i} & \boldsymbol{0} & \boldsymbol{M}_{go_i}^{-1}\boldsymbol{R}_{\psi_i}\boldsymbol{K}_{R_i}^{-1}\boldsymbol{E} & \boldsymbol{0} \\ \hline \begin{matrix} -\boldsymbol{T}_{gf_i}\boldsymbol{K}_{R_i}^{-1}\tilde{\boldsymbol{K}}_{go_i} \\ +\boldsymbol{T}_{go_i}\boldsymbol{R}_{\psi_i}\boldsymbol{K}_{R_i}^{-1}\boldsymbol{K}_{gf_i} \end{matrix} & 0 & \begin{matrix} \boldsymbol{T}_{gf_i}\boldsymbol{K}_{R_i}^{-1}\boldsymbol{E} \\ +\boldsymbol{T}_{go_i}\boldsymbol{R}_{\psi_i}\boldsymbol{K}_{R_i}^{-1}\boldsymbol{E} \end{matrix} & 1 \end{bmatrix} \quad (5)$$

$$ \boldsymbol{K}_{R_i} := \boldsymbol{K}_{gf_i} + \tilde{\boldsymbol{K}}_{go_i}, \tilde{\boldsymbol{K}}_{go_i} := \boldsymbol{R}_{\psi_i}\boldsymbol{K}_{go_i}\boldsymbol{R}_{\psi_i} \quad (6)$$

$$ \boldsymbol{E} := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \boldsymbol{R}_{\psi_i} := \begin{bmatrix} \cos\psi_i & -\sin\psi_i \\ -\sin\psi_i & -\cos\psi_i \end{bmatrix}. \quad (7)$$

$\boldsymbol{M}_{gf_i}$, $\boldsymbol{M}_{go_i} \in \mathbb{R}^{2\times 2}$, $\boldsymbol{K}_{gf_i}$, $\boldsymbol{K}_{go_i} \in \mathbb{R}^{2\times 2}$ and $\boldsymbol{T}_{gf_i}$, $\boldsymbol{T}_{go_i} \in \mathbb{R}^{1\times 2}$ are the geometric parameters defined by using $\boldsymbol{c}_{f_i}$ and $\boldsymbol{c}_{o_i}$. $\boldsymbol{K}_{R_i} \in \mathbb{R}^{2\times 2}$ is called the relative curvature form. $\boldsymbol{R}_{\psi_i} \in \mathbb{R}^{2\times 2}$ is the rotation matrix of the x- and y-axes of $\Sigma_{C_{F_i}}$ relative to the x- and y-axes of $\Sigma_{C_{O_i}}$. In addition, $\boldsymbol{V}_{C_i}$ is given by

$$\boldsymbol{V}_{C_i} = \boldsymbol{D}_{J_{F_i}}(\boldsymbol{\theta}_{F_i}, \boldsymbol{\eta}_i)\dot{\boldsymbol{\theta}}_{F_i} - \boldsymbol{D}_{T_{O_i}}(\boldsymbol{x}_O, \boldsymbol{\eta}_i)\dot{\boldsymbol{x}}_O, \quad (8)$$

where

$$\boldsymbol{D}_{J_{F_i}} := \boldsymbol{D}_{F_i}\boldsymbol{J}_{F_i}(\boldsymbol{\theta}_{F_i}), \boldsymbol{D}_{T_{O_i}} := \boldsymbol{D}_{O_i}\boldsymbol{T}_O(\boldsymbol{x}_O) \quad (9)$$

$$\boldsymbol{D}_{F_i} := \begin{bmatrix} \boldsymbol{R}_{BC_{F_i}}^{\mathrm{T}} & -\boldsymbol{R}_{BC_{F_i}}^{\mathrm{T}}(\boldsymbol{R}_{BF_i}{}^{F_i}\boldsymbol{p}_{C_{F_i}})^{\wedge} \\ \boldsymbol{0}_{3\times 3} & \boldsymbol{R}_{BC_{F_i}}^{\mathrm{T}} \end{bmatrix} \quad (10)$$

$$\boldsymbol{D}_{O_i} := \begin{bmatrix} \boldsymbol{R}_{BC_{F_i}}^{\mathrm{T}} & -\boldsymbol{R}_{BC_{F_i}}^{\mathrm{T}}(\boldsymbol{R}_{BO}{}^{O}\boldsymbol{p}_{C_{O_i}})^{\wedge} \\ \boldsymbol{0}_{3\times 3} & \boldsymbol{R}_{BC_{F_i}}^{\mathrm{T}} \end{bmatrix}. \quad (11)$$

$\boldsymbol{R}_{BC_{F_i}}$ is the rotation matrix of $\Sigma_{C_{F_i}}$ relative to Σ_B. $(\cdot)^{\wedge}$ stands for the skew-symmetric matrix equivalent to a vector product. $\boldsymbol{J}_{F_i}(\boldsymbol{\theta}_{F_i}) \in \mathbb{R}^{6\times m_i}$ and $\boldsymbol{T}_O(\boldsymbol{x}_O) \in \mathbb{R}^{6\times 6}$ are the transformation matrix from $\dot{\boldsymbol{\theta}}_{F_i}$ to $[\ {}^B\dot{\boldsymbol{p}}_{F_i}^{\mathrm{T}} \ \ {}^B\boldsymbol{\omega}_{F_i}^{\mathrm{T}}\ ]^{\mathrm{T}}$ and the one from $\dot{\boldsymbol{x}}_O$ to $[\ {}^B\dot{\boldsymbol{p}}_O^{\mathrm{T}} \ \ {}^B\boldsymbol{\omega}_O^{\mathrm{T}}\ ]^{\mathrm{T}}$ respectively, where ${}^B\boldsymbol{\omega}_{F_i} \in \mathbb{R}^3$ and ${}^B\boldsymbol{\omega}_O \in \mathbb{R}^3$ are the rotational velocities of Σ_{F_i} and Σ_O relative to Σ_B respectively. In this study, we assume that $\boldsymbol{J}_{F_i}$ and $\boldsymbol{T}_O$ are full column rank and nonsingular respectively. Note that $\boldsymbol{J}_{F_i}$ is determined by the link mechanics of the ith finger.

Combining (8) and (3), we get

$$\dot{\boldsymbol{\eta}}_i = \boldsymbol{H}_i(\boldsymbol{D}_{J_{F_i}}\dot{\boldsymbol{\theta}}_{F_i} - \boldsymbol{D}_{T_{O_i}}\dot{\boldsymbol{x}}_O). \quad (12)$$

Eq. (12) relates the velocities of the contact coordinates $\dot{\boldsymbol{\eta}}_i$ to those of the generalized coordinates $(\dot{\boldsymbol{\theta}}_{F_i}, \dot{\boldsymbol{x}}_O)$.

2.3 Dynamical Equations

Since the constraint at the ith contact point is the pure rolling contact from Assumption 3, the constraint is expressed by (Murray *et al.*, 1994; Montana, 1988)

$$\boldsymbol{B}_C^{\mathrm{T}}\boldsymbol{V}_{C_i} = \boldsymbol{0}, \quad \boldsymbol{B}_C := \begin{bmatrix} \boldsymbol{I}_3 & \boldsymbol{0}_{3\times 1} \\ \boldsymbol{0}_{3\times 3} & \boldsymbol{e} \end{bmatrix}, \quad (13)$$

where $\boldsymbol{e} := [0\ 0\ 1]^{\mathrm{T}}$. Therefore, substituting (8) into (13) yields the motion constraint on the generalized coordinates described by

<u>Motion Constraint</u>

$$\boldsymbol{A}_F(\boldsymbol{\theta}_F, \boldsymbol{\eta})\dot{\boldsymbol{\theta}}_F - \boldsymbol{A}_O(\boldsymbol{x}_O, \boldsymbol{\eta})\dot{\boldsymbol{x}}_O = \boldsymbol{0}, \quad (14)$$

where

$$\boldsymbol{A}_F := \begin{bmatrix} \boldsymbol{B}_C^{\mathrm{T}}\boldsymbol{D}_{J_{F_1}} & \boldsymbol{0}_{4\times m_2} \\ \boldsymbol{0}_{4\times m_1} & \boldsymbol{B}_C^{\mathrm{T}}\boldsymbol{D}_{J_{F_2}} \end{bmatrix}, \boldsymbol{A}_O := \begin{bmatrix} \boldsymbol{B}_C^{\mathrm{T}}\boldsymbol{D}_{T_{O_1}} \\ \boldsymbol{B}_C^{\mathrm{T}}\boldsymbol{D}_{T_{O_2}} \end{bmatrix}. \quad (15)$$

The motion constraint (14) will be realized by applying appropriate contact forces. The condition is characterized as follows:

<u>Constraint on Contact Force</u>

$${}^C\boldsymbol{F}_C \in FC, \quad FC := FC_1 \times FC_2 \subset \mathbb{R}^8, \quad (16)$$

where FC_i describes the set of forces which lies in the friction cone at ith contact point (Murray *et al.*, 1994).

From Assumption 3, the equations of motion of the fingers and the object are derived from the Lagrange equations with the constraint (14) as follows:

<u>Equations of Motion</u>

$$\boldsymbol{M}_F(\boldsymbol{\theta}_F)\ddot{\boldsymbol{\theta}}_F + \boldsymbol{C}_F(\boldsymbol{\theta}_F, \dot{\boldsymbol{\theta}}_F)\dot{\boldsymbol{\theta}}_F + \boldsymbol{N}_F(\boldsymbol{\theta}_F) = \boldsymbol{\tau} - \boldsymbol{A}_F^{\mathrm{T}}(\boldsymbol{\theta}_F, \boldsymbol{\eta})^C\boldsymbol{F}_C \quad (17)$$

$$\boldsymbol{M}_O(\boldsymbol{x}_O)\ddot{\boldsymbol{x}}_O + \boldsymbol{C}_O(\boldsymbol{x}_O, \dot{\boldsymbol{x}}_O)\dot{\boldsymbol{x}}_O + \boldsymbol{N}_O(\boldsymbol{x}_O) = {}^B\boldsymbol{F}_O = \boldsymbol{A}_O^{\mathrm{T}}(\boldsymbol{x}_O, \boldsymbol{\eta})^C\boldsymbol{F}_C, \quad (18)$$

where $\boldsymbol{M}_F > 0 \in \mathbb{R}^{m\times m}$, $\boldsymbol{M}_O > 0 \in \mathbb{R}^{6\times 6}$ are the generalized inertia matrices, $\boldsymbol{C}_F \in \mathbb{R}^{m\times m}$, $\boldsymbol{C}_O \in \mathbb{R}^{6\times 6}$ are the Coriolis matrices, and $\boldsymbol{N}_F \in \mathbb{R}^m$, $\boldsymbol{N}_O \in \mathbb{R}^6$ are the gravity terms. Note that the contact force ${}^C\boldsymbol{F}_C$ plays the role of the Lagrange multipliers, and ${}^B\boldsymbol{F}_O(:= \boldsymbol{A}_O^{\mathrm{T}}{}^C\boldsymbol{F}_C)$ is the resultant force applied to Σ_O by ${}^C\boldsymbol{F}_C$.

3. SYSTEM ANALYSIS

3.1 Properties of Motion Constraint

In this subsection, we clarify properties of the motion constraint on the generalized coordinates (14) by associating it with constraints on the contact coordinates. Consider the following conditions:

$$\boldsymbol{A}_{\eta_i}(\boldsymbol{\eta}_i)\dot{\boldsymbol{\eta}}_i = \boldsymbol{0}, \quad \boldsymbol{A}_{\eta_i} \in \mathbb{R}^{3\times 5} \quad (i = 1, 2) \quad (19)$$

$$\boldsymbol{b}_C^{\mathrm{T}}(\boldsymbol{D}_{J_{F_i}}\dot{\boldsymbol{\theta}}_{F_i} - \boldsymbol{D}_{T_{O_i}}\dot{\boldsymbol{x}}_O) = 0 \quad (i = 1, 2), \quad (20)$$

where

$$\boldsymbol{A}_{\eta_i} := \begin{bmatrix} -\boldsymbol{M}_{gf_i} & \boldsymbol{R}_{\psi_i}\boldsymbol{M}_{go_i} & \begin{matrix} 0 \\ 0 \end{matrix} \\ -\boldsymbol{T}_{gf_i}\boldsymbol{M}_{gf_i} & -\boldsymbol{T}_{go_i}\boldsymbol{M}_{go_i} & 1 \end{bmatrix} \quad (21)$$

$$\boldsymbol{b}_C := [\ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ ]^{\mathrm{T}}. \quad (22)$$

The following theorem holds.

Theorem 1.
(**i**) Suppose (12) holds. The motion constraint (14) is equivalent to (19) and (20).
(**ii**) Suppose $\boldsymbol{K}_{R_i}$ defined by (6) is full rank and $\boldsymbol{c}_{f_i}$ is not the specular image (Marigo and Bicchi, 2000) of $\boldsymbol{c}_{o_i}$. The constraints on the contact coordinates (19) are the maximal nonholonomic constraints and the constraints on the generalized coordinates (20) are the holonomic constraints.

Proof: **(i)** Eq. (14) is represented as

$$\boldsymbol{B}_C^{\mathrm{T}}(\boldsymbol{D}_{J_{F_i}}\dot{\boldsymbol{\theta}}_{F_i} - \boldsymbol{D}_{T_{O_i}}\dot{\boldsymbol{x}}_O) = \boldsymbol{0}. \quad (23)$$

Substituting (12) into (19) and combining the resultant equation with (20) lead to

$$\tilde{\boldsymbol{B}}_C^{\mathrm{T}}(\boldsymbol{D}_{J_{F_i}}\dot{\boldsymbol{\theta}}_{F_i} - \boldsymbol{D}_{T_{O_i}}\dot{\boldsymbol{x}}_O) = \boldsymbol{0}, \tilde{\boldsymbol{B}}_C^{\mathrm{T}} := \begin{bmatrix} \boldsymbol{A}_{\eta_i}\boldsymbol{H}_i \\ \boldsymbol{b}_C^{\mathrm{T}} \end{bmatrix}. \quad (24)$$

Therefore, in order to clarify that (23) and (24) are equivalent, it is enough to show that a nonsingular matrix $\boldsymbol{E}_4 \in \mathbb{R}^{4\times 4}$ exists such that $\boldsymbol{E}_4\tilde{\boldsymbol{B}}_C^{\mathrm{T}} = \boldsymbol{B}_C^{\mathrm{T}}$. This is immediate since by using (21), (5), (6) and $\boldsymbol{R}_\psi = \boldsymbol{R}_\psi^{\mathrm{T}} = \boldsymbol{R}_\psi^{-1}$, we get

$$\tilde{\boldsymbol{B}}_C^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}. \quad (25)$$

(ii) Since $\boldsymbol{c}_{f_i}$ is not the specular image of $\boldsymbol{c}_{o_i}$, (19) are the maximal nonholonomic constraints (Marigo and Bicchi, 2000). Consider the constraint $h_{Cz_i}(\boldsymbol{\theta}_{F_i}, \boldsymbol{x}_O, \boldsymbol{\eta}_i) = 0$ where

$$h_{Cz_i} := \boldsymbol{e}^{\mathrm{T}} \boldsymbol{R}_{BCF_i}^{\mathrm{T}} \Big[{}^{B}\boldsymbol{p}_{F_i} + \boldsymbol{R}_{BF_i} {}^{F_i}\boldsymbol{p}_{C_{F_i}} - ({}^{B}\boldsymbol{p}_O + \boldsymbol{R}_{BO} {}^{O}\boldsymbol{p}_{C_{O_i}}) \Big]. \quad (26)$$

We can show that the differential of (26) is equivalent to the left-hand of (20) by using the facts: the contact condition (1); the property of the rotation matrix $\dot{\boldsymbol{R}}_{BO} {}^{O}\boldsymbol{p}_{C_{O_i}} = -(\boldsymbol{R}_{BO} {}^{O}\boldsymbol{p}_{C_{O_i}})^{\wedge B}\boldsymbol{\omega}_O$ and $\dot{\boldsymbol{R}}_{BF_i} {}^{F_i}\boldsymbol{p}_{C_{F_i}} = -(\boldsymbol{R}_{BF_i} {}^{F_i}\boldsymbol{p}_{C_{F_i}})^{\wedge B}\boldsymbol{\omega}_{F_i}$; third elements of $\boldsymbol{R}_{F_iC_{F_i}}^{\mathrm{T}} {}^{F_i}\dot{\boldsymbol{p}}_{C_{F_i}}$ and $\boldsymbol{R}_{OC_{O_i}}^{\mathrm{T}} {}^{O}\dot{\boldsymbol{p}}_{C_{O_i}}$ are zero (Murray *et al.*, 1994). Therefore, (20) is the holonomic constraints. ∎

From Theorem 1, the system has 6 nonholonomic constraints and 2 holonomic constraints. Therefore, the position of the DOF of the system can be $(m + 4)$ under a certain condition, which is shown in the next subsection.

3.2 Degrees of Freedom of System

In this subsection, we clarify the DOF of the velocity and the position of the system.

Firstly, since the motion constraint (19) consists of 8 constraint equations of the velocity, the DOF of the velocity of the system is $(m - 2)$.

Secondly, consider the DOF of the position. A general solution of the constraints (19) with respect to $\dot{\boldsymbol{\eta}}_i$ is given by

$$\dot{\boldsymbol{\eta}}_i = \boldsymbol{A}_{\eta_i}^{\perp}(\boldsymbol{\eta}_i)\boldsymbol{\omega}_{C_i}, \boldsymbol{A}_{\eta_i}\boldsymbol{A}_{\eta_i}^{\perp} = \boldsymbol{0} \quad (i = 1, 2), \quad (27)$$

where $\boldsymbol{A}_{\eta_i}^{\perp} \in \mathbb{R}^{5\times 2}$ consists of the 4th and 5th columns of $\boldsymbol{H}_i$ of (5) and $\boldsymbol{\omega}_{C_i} := [\, \omega_{Cx_i} \;\; \omega_{Cy_i} \,]^{\mathrm{T}} \in \mathbb{R}^2$ is the rolling velocity. Therefore, from the property of the maximal nonholonomic constraints (19), the DOF of the position depends whether the rolling velocity $\boldsymbol{\omega}_C := [\, \boldsymbol{\omega}_{C_1}^{\mathrm{T}} \;\; \boldsymbol{\omega}_{C_2}^{\mathrm{T}} \,]^{\mathrm{T}} \in \mathbb{R}^4$ can be generated from the $(m - 2)$ DOF of the velocity or not. The following theorem holds.

Theorem 2. The relation between $(\dot{\boldsymbol{\theta}}_F, \dot{\boldsymbol{x}}_O)$ and $\boldsymbol{\omega}_C$ is given by

$$\boldsymbol{\mathcal{A}}(\boldsymbol{\theta}_F, \boldsymbol{x}_O, \boldsymbol{\eta}) \begin{bmatrix} \dot{\boldsymbol{\theta}}_F \\ \dot{\boldsymbol{x}}_O \end{bmatrix} = \overline{\boldsymbol{A}}_{\omega_C}\boldsymbol{\omega}_C, \quad (28)$$

where

$$\boldsymbol{\mathcal{A}}(\boldsymbol{\theta}_F, \boldsymbol{x}_O, \boldsymbol{\eta}) := \begin{bmatrix} \overline{\boldsymbol{A}}_F & -\overline{\boldsymbol{A}}_O \end{bmatrix} \in \mathbb{R}^{12\times(m+6)} \quad (29)$$

$$\overline{\boldsymbol{A}}_F := \begin{bmatrix} \overline{\boldsymbol{B}}_C^{\mathrm{T}}\boldsymbol{D}_{J_{F_1}} & \boldsymbol{0}_{6\times m_2} \\ \boldsymbol{0}_{6\times m_1} & \overline{\boldsymbol{B}}_C^{\mathrm{T}}\boldsymbol{D}_{J_{F_2}} \end{bmatrix}, \overline{\boldsymbol{A}}_O := \begin{bmatrix} \overline{\boldsymbol{B}}_C^{\mathrm{T}}\boldsymbol{D}_{T_{O_1}} \\ \overline{\boldsymbol{B}}_C^{\mathrm{T}}\boldsymbol{D}_{T_{O_2}} \end{bmatrix} \quad (30)$$

$$\overline{\boldsymbol{A}}_{\omega_C} := \begin{bmatrix} \overline{\boldsymbol{B}}_C^{\mathrm{T}}\boldsymbol{K}_{B_C^{\mathrm{T}}} & \boldsymbol{0}_{6\times 2} \\ \boldsymbol{0}_{6\times 2} & \overline{\boldsymbol{B}}_C^{\mathrm{T}}\boldsymbol{K}_{B_C^{\mathrm{T}}} \end{bmatrix} \quad (31)$$

$$\overline{\boldsymbol{B}}_C := \begin{bmatrix} \boldsymbol{B}_C & \boldsymbol{K}_{B_C^{\mathrm{T}}} \end{bmatrix}, \boldsymbol{K}_{B_C^{\mathrm{T}}} := [\boldsymbol{0}_{2\times 3} \;\; \boldsymbol{I}_2 \;\; \boldsymbol{0}_{2\times 1}]^{\mathrm{T}}. \quad (32)$$

Furthermore, $\boldsymbol{\omega}_C$ is generated from $(\dot{\boldsymbol{\theta}}_F, \dot{\boldsymbol{x}}_O)$ iff $\boldsymbol{\mathcal{A}}$ of (29) is full row rank.

Proof: From the definition $\boldsymbol{\omega}_{C_i} := [\, \omega_{Cx_i} \;\; \omega_{Cy_i} \,]^{\mathrm{T}}$,

$$\boldsymbol{\omega}_{C_i} = \boldsymbol{K}_{B_C^{\mathrm{T}}}^{\mathrm{T}}\boldsymbol{V}_{C_i}. \quad (33)$$

Therefore, by substituting (8) into (33), $\boldsymbol{\omega}_{C_i}$ is expressed by $(\dot{\boldsymbol{\theta}}_{F_i}, \dot{\boldsymbol{x}}_O)$ as

$$\boldsymbol{\omega}_{C_i} = \boldsymbol{K}_{B_C^{\mathrm{T}}}^{\mathrm{T}}\boldsymbol{D}_{J_{F_i}}\dot{\boldsymbol{\theta}}_{F_i} - \boldsymbol{K}_{B_C^{\mathrm{T}}}^{\mathrm{T}}\boldsymbol{D}_{T_{O_i}}\dot{\boldsymbol{x}}_O. \quad (34)$$

Combining (34) and (14), we get (28). For the proof of the latter part, notice that (28) can be interpreted as the simultaneous linear equations with respect to $[\, \dot{\boldsymbol{\theta}}_F^{\mathrm{T}} \;\; \dot{\boldsymbol{x}}_O^{\mathrm{T}} \,]^{\mathrm{T}}$. Since $\overline{\boldsymbol{B}}_C^{\mathrm{T}}\boldsymbol{K}_{B_C^{\mathrm{T}}} = [\, \boldsymbol{0}_{2\times 4} \;\; \boldsymbol{I}_2 \,]^{\mathrm{T}}$ from (32) and (33), $\overline{\boldsymbol{A}}_{\omega_C}$ of (31) is full column rank. Therefore, (28) can be solved with respect to $[\, \dot{\boldsymbol{\theta}}_F^{\mathrm{T}} \;\; \dot{\boldsymbol{x}}_O^{\mathrm{T}} \,]^{\mathrm{T}}$ for arbitrary $\boldsymbol{\omega}_C$ iff $\boldsymbol{\mathcal{A}} \in \mathbb{R}^{12\times(m+6)}$ of (28) is full row rank. This fact proves the claim. ∎

From Theorem 2, $\boldsymbol{\omega}_C \in \mathbb{R}^4$ can be generated from the $(m - 2)$ DOF of the velocity iff the number of the DOF of the fingers, m, is greater than or equal to 6 and $\boldsymbol{\mathcal{A}}$ is full row rank. In that case, the contact coordinates $\boldsymbol{\eta} \in \mathbb{R}^{10}$ can be regulated by $\boldsymbol{\omega}_C$ under (27). Combining the rest of the DOF of the velocity except $\boldsymbol{\omega}_C$ with 10, the DOF of the position is $10 + (m - 6) = (m + 4)$.

4. CONTROL DESIGN

In this section, we consider the control design to achieve the control of the $(m + 4)$ DOF of the position of the system.

4.1 Control Objectives

Consider the following control objectives:

(A) To make the contact force ${}^{C}\boldsymbol{F}_C$ lie in the friction cone FC.
(B) To make the rolling velocity $\boldsymbol{\omega}_C \in \mathbb{R}^4$ follow a desired trajectory.
(C) To make the fingers/object motion $\boldsymbol{v}_N \in \mathbb{R}^{(m-6)}$ follow a desired trajectory, where $\boldsymbol{v}_N$ causes no effect on the rolling velocity $\boldsymbol{\omega}_C$.

The control objective (A) represents that the fingers do not slip, and (B) and (C) represent the control of the $(m - 2)$ DOF of the velocity of the system. Since the contact coordinates $\boldsymbol{\eta} \in \mathbb{R}^{10}$ can be regulated by making $\boldsymbol{\omega}_C$ follow appropriate trajectory with nonholonomy of rolling, all of the $(m + 4)$ DOF of the position of the system can be controlled. One such trajectory has been proposed by (Nakashima *et al.*, 2002). These control variables $(\boldsymbol{\eta}, \boldsymbol{v}_N) \in \mathbb{R}^{(m+4)}$ can be associated with 12 variables of $(\boldsymbol{\theta}_F, \boldsymbol{x}_O, \boldsymbol{\eta}) \in \mathbb{R}^{(m+16)}$ except the control variables since we have 12 equations of (1) and (2) $(i = 1, 2)$. Note that (2) gives only 3 equations because it relates the 2 rotation matrices. Therefore, it can be realized that we control the control variables such that the 12 variables are regulated to desired target points.

To realize the control objectives, we make the following assumptions:

Assumption 4.
(i) $\boldsymbol{A}_O^{\mathrm{T}} \in \mathbb{R}^{6\times 8}$ is full row rank.
(ii) There exists an internal force $\boldsymbol{F}_N \in \mathbb{R}^8$ such that $\boldsymbol{F}_N \in \mathcal{N}(\boldsymbol{A}_O^{\mathrm{T}})$ and $\boldsymbol{F}_N \in \mathrm{Int}(FC)$.

Assumption 5.
(i) $\boldsymbol{A}_F^{\mathrm{T}} \in \mathbb{R}^{m\times 8}$ is maximal full rank.
(ii) $\mathcal{N}(\boldsymbol{A}_O^{\mathrm{T}}) \subset \mathcal{R}((\boldsymbol{A}_F^{\mathrm{T}})^{+})$.
Assumption 6. $\boldsymbol{\mathcal{A}}$ is full row rank.

$\mathrm{Int}(FC)$ represents the interior of the friction cone, $\mathcal{N}(\cdot)$ represents the kernel, $\mathcal{R}(\cdot)$ represents the range of value and $(\cdot)^{+}$ represents the pseudo inverse matrix. Assumption 4 corresponds to the Force Closure (Murray *et al.*, 1994) in the robotics literatures. Assumption 5 guarantees that the internal force can be generated by the inputs $\boldsymbol{\tau}$. Assumption 6 guarantees that $\boldsymbol{\omega}$ can be generated by the $(m-2)$ DOF of the velocity.

4.2 Expression of Contact Force

In this subsection, we give an explicit relationship between the contact force and the internal force, which is effective to achieve the control objective (A). Consider a decomposition of ${}^{C}\boldsymbol{F}_C$ as

$$ {}^{C}\boldsymbol{F}_C = (\boldsymbol{A}_O^{\mathrm{T}})^{+\,B}\boldsymbol{F}_O + \boldsymbol{K}_{A_O^{\mathrm{T}}}(\boldsymbol{x}_O,\boldsymbol{\eta})\boldsymbol{f}_N, \tag{35} $$

where

$$ \boldsymbol{K}_{A_O^{\mathrm{T}}} := [\ \boldsymbol{k}_1 \ \ \boldsymbol{k}_2\ ] \in \mathbb{R}^{8\times 2} $$
$$ = \left[\begin{array}{c|c} \boldsymbol{R}_{BC_{F_1}}^{\mathrm{T}}\,{}^{B}\boldsymbol{e}_{12} & \boldsymbol{R}_{BC_{F_1}}^{\mathrm{T}}({}^{B}\boldsymbol{p}_{C_{O_{12}}}^{\wedge})^{+}\boldsymbol{\tau}_N \\ 0 & \dfrac{1}{({}^{B}\boldsymbol{p}_{C_{O_{12}}})^{\mathrm{T}\,B}\boldsymbol{e}_{1z}} \\ \boldsymbol{R}_{BC_{F_2}}^{\mathrm{T}}\,{}^{B}\boldsymbol{e}_{21} & -\boldsymbol{R}_{BC_{F_2}}^{\mathrm{T}}({}^{B}\boldsymbol{p}_{C_{O_{12}}}^{\wedge})^{+}\boldsymbol{\tau}_N \\ 0 & \dfrac{-1}{({}^{B}\boldsymbol{p}_{C_{O_{12}}})^{\mathrm{T}\,B}\boldsymbol{e}_{2z}} \end{array}\right] \tag{36} $$

$$ \boldsymbol{\tau}_N := \frac{{}^{B}\boldsymbol{e}_{1z}}{({}^{B}\boldsymbol{p}_{C_{O_{12}}})^{\mathrm{T}\,B}\boldsymbol{e}_{1z}} + \frac{-{}^{B}\boldsymbol{e}_{2z}}{({}^{B}\boldsymbol{p}_{C_{O_{12}}})^{\mathrm{T}\,B}\boldsymbol{e}_{2z}} \tag{37} $$

$$ {}^{B}\boldsymbol{p}_{C_{O_{12}}} := \boldsymbol{R}_{BO}({}^{O}\boldsymbol{p}_{C_{O_2}} - {}^{O}\boldsymbol{p}_{C_{O_1}}) \tag{38} $$

$$ {}^{B}\boldsymbol{e}_{zi} := \boldsymbol{R}_{BC_{F_i}}\boldsymbol{e}, \quad \boldsymbol{e} := [\ 0\ \ 0\ \ 1\ ]^{\mathrm{T}}. \tag{39} $$

${}^{B}\boldsymbol{e}_{ij} \in \mathbb{R}^3 (i,j = 1,2, i \neq j)$ is the unit vector from the contact point i to j (${}^{B}\boldsymbol{e}_{12} = -{}^{B}\boldsymbol{e}_{21}$). $\boldsymbol{\tau}_N$ is the moment produced by the moments about ${}^{B}\boldsymbol{e}_{1z}$ and ${}^{B}\boldsymbol{e}_{2z}$. ${}^{B}\boldsymbol{p}_{C_{O_{12}}}$ is the vector from the contact point 2 to 1 and ${}^{B}\boldsymbol{e}_{iz} \in \mathbb{R}^3$ is the unit vector in direction of z_{f_i}-axis of $\Sigma_{C_{F_i}}$. Note that ${}^{B}\boldsymbol{e}_{ij} /\!\!/ {}^{B}\boldsymbol{p}_{C_{O_{12}}}$. The following lemma holds.

Lemma 1. Consider $\boldsymbol{K}_{A_O^{\mathrm{T}}}$ defined by (36)–(39). The following equations hold.

$$ (\mathbf{i}):\ \boldsymbol{A}_O^{\mathrm{T}}\boldsymbol{K}_{A_O^{\mathrm{T}}} = \boldsymbol{0} \tag{40} $$

$$ (\mathbf{ii}):\ \boldsymbol{k}_1^{\mathrm{T}}\boldsymbol{k}_2 = 0 \tag{41} $$

Proof: **(i)** Combining (11), (15) and (36), and noting that ${}^{B}\boldsymbol{p}_{C_{O_{12}}}^{\wedge}\,{}^{B}\boldsymbol{e}_{12} = \boldsymbol{0}$ from ${}^{B}\boldsymbol{e}_{12} /\!\!/ {}^{B}\boldsymbol{p}_{C_{O_{12}}}$, we get $\boldsymbol{A}_O^{\mathrm{T}}\boldsymbol{k}_1 = \boldsymbol{0}$. Similarly, we can easily confirm that the upper 3 elements of $\boldsymbol{A}_O^{\mathrm{T}}\boldsymbol{k}_2$ is zero. With $\boldsymbol{\tau}_N$ defined by (37), the lower 3 elements of $\boldsymbol{A}_O^{\mathrm{T}}\boldsymbol{k}_2$ result in $(\boldsymbol{I}_3 - {}^{B}\boldsymbol{p}_{C_{O_{12}}}^{\wedge}({}^{B}\boldsymbol{p}_{C_{O_{12}}}^{\wedge})^{+})\boldsymbol{\tau}_N$. Since $\boldsymbol{\tau}_N^{\mathrm{T}\,B}\boldsymbol{p}_{C_{O_{12}}} = 0$ from (37), $\boldsymbol{\tau}_N$ can be represented as $\boldsymbol{\tau}_N = ({}^{B}\boldsymbol{p}_{C_{O_{12}}}^{\wedge})\boldsymbol{z}, \boldsymbol{z} \in \mathbb{R}^3$. Noting ${}^{B}\boldsymbol{p}_{C_{O_{12}}}^{\wedge}({}^{B}\boldsymbol{p}_{C_{O_{12}}}^{\wedge})^{+\,B}\boldsymbol{p}_{C_{O_{12}}}^{\wedge} = {}^{B}\boldsymbol{p}_{C_{O_{12}}}^{\wedge}$, we can confirm that $(\boldsymbol{I}_3 - {}^{B}\boldsymbol{p}_{C_{O_{12}}}^{\wedge}({}^{B}\boldsymbol{p}_{C_{O_{12}}}^{\wedge})^{+})\boldsymbol{\tau}_N = \boldsymbol{0}$.

(ii) Noting that $({}^{B}\boldsymbol{p}_{C_{O_{12}}}^{\wedge})^{+} = \dfrac{({}^{B}\boldsymbol{p}_{C_{O_{12}}}^{\wedge})^{\mathrm{T}}}{\|{}^{B}\boldsymbol{p}_{C_{O_{12}}}\|^2}$ and $({}^{B}\boldsymbol{e}_{12} - {}^{B}\boldsymbol{e}_{21}) /\!\!/ {}^{B}\boldsymbol{p}_{C_{O_{12}}}$, we get $\boldsymbol{k}_1^{\mathrm{T}}\boldsymbol{k}_2 = 0$. ∎

From Lemma 1 and the property of the pseudo inverse matrix $\boldsymbol{A}_O^{\mathrm{T}}(\boldsymbol{A}^{\mathrm{T}})^{+} = \boldsymbol{I}_6$, we can confirm that (35) is a general solution of ${}^{B}\boldsymbol{F}_O := \boldsymbol{A}_O^{\mathrm{T}\,C}\boldsymbol{F}_C$ (See (18)). In addition, $\boldsymbol{f}_N := [\ f_{N_1} \ \ f_{N_2}\ ]^{\mathrm{T}} \in \mathbb{R}^2$ produces the internal forces $\boldsymbol{k}_1 f_{N_1}$ and $\boldsymbol{k}_2 f_{N_2}$ which cause no effect on Σ_O, and they are independent each other. Physically, from the observation of the elements of $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$, f_{N_1} represents the magnitude of the translational forces in the directions of ${}^{B}\boldsymbol{e}_{12}$ and ${}^{B}\boldsymbol{e}_{21}$, and f_{N_2} represents the magnitude of the moments about $\boldsymbol{\tau}_N$ and $-\boldsymbol{\tau}_N$. From the property of the friction cone of the soft-finger contact (Murray *et al.*, 1994), the control objective (A) is achieved by controlling f_{N_1} appropriately.

4.3 Expression of Finger and Object Motion

In this subsection, we give an explicit relationship between the velocity of the generalized coordinates $(\dot{\boldsymbol{\theta}}_F, \dot{\boldsymbol{x}}_O)$ and $(\boldsymbol{\omega}_C, \boldsymbol{v}_N)$, which is effective to achieve the control objectives (B) and (C). Consider a decomposition of $[\ \dot{\boldsymbol{\theta}}_F^{\mathrm{T}} \ \ \dot{\boldsymbol{x}}_O^{\mathrm{T}}\ ]^{\mathrm{T}}$ as

$$ \begin{bmatrix} \dot{\boldsymbol{\theta}}_F \\ \dot{\boldsymbol{x}}_O \end{bmatrix} = \boldsymbol{\mathcal{A}}^{-}\overline{\boldsymbol{A}}_{\omega_C}\boldsymbol{\omega}_C + \boldsymbol{K}_{\mathcal{A}}(\boldsymbol{\theta}_F,\boldsymbol{x}_O,\boldsymbol{\eta})\boldsymbol{v}_N, \tag{42} $$

where

$$ \boldsymbol{\mathcal{A}}^{-} := \boldsymbol{\mathcal{A}}^{+} - \boldsymbol{K}_{\mathcal{A}}\boldsymbol{T}_{\mathcal{A}}^{\mathrm{T}}\boldsymbol{\mathcal{A}}^{+} \in \mathbb{R}^{(m+6)\times 12} \tag{43} $$

$$ \boldsymbol{K}_{\mathcal{A}} := \overline{\boldsymbol{\mathcal{A}}}^{-1}\begin{bmatrix} \boldsymbol{0}_{12\times(m-6)} \\ \boldsymbol{I}_{(m-6)} \end{bmatrix} \in \mathbb{R}^{(m+6)\times(m-6)} \tag{44} $$

$$ \overline{\boldsymbol{\mathcal{A}}} := \begin{bmatrix} \boldsymbol{\mathcal{A}} \\ \boldsymbol{T}_{\mathcal{A}}^{\mathrm{T}} \end{bmatrix}, \boldsymbol{T}_{\mathcal{A}} \in \mathbb{R}^{(m+6)\times(m-6)}. \tag{45} $$

$\boldsymbol{T}_{\mathcal{A}}$ is an arbitrary matrix, which make $\overline{\boldsymbol{\mathcal{A}}}$ nonsingular. The following lemma holds.

Lemma 2. Consider $\boldsymbol{\mathcal{A}}^{-}$ and $\boldsymbol{K}_{\mathcal{A}}$ defined by (43)–(45). Suppose that $\boldsymbol{T}_{\mathcal{A}}$ is chosen to make $\overline{\boldsymbol{\mathcal{A}}}$ of (45) nonsingular. The following equations hold.

$$ (\mathbf{i}): \boldsymbol{\mathcal{A}}\boldsymbol{\mathcal{A}}^{-} = \boldsymbol{I}_{12}, \boldsymbol{\mathcal{A}}\boldsymbol{K}_{\mathcal{A}} = \boldsymbol{0} \tag{46} $$

$$ (\mathbf{ii}): \boldsymbol{v}_N = \boldsymbol{T}_{\mathcal{A}}^{\mathrm{T}}[\ \dot{\boldsymbol{\theta}}_F^{\mathrm{T}} \ \ \dot{\boldsymbol{x}}_O^{\mathrm{T}}\ ]^{\mathrm{T}} \tag{47} $$

Proof: **(i)** Noting that (43), $\boldsymbol{\mathcal{A}}\boldsymbol{\mathcal{A}}^{+} = \boldsymbol{I}_{12}$ and $\boldsymbol{\mathcal{A}} = [\ \boldsymbol{I}_{12} \ \ \boldsymbol{0}_{12\times(m-6)}\ ]\,\overline{\boldsymbol{\mathcal{A}}}$ from (45), we get (46).

(ii) Premultiplying (42) by $\overline{\boldsymbol{\mathcal{A}}}$ and noting (43)–(45) and $\boldsymbol{\mathcal{A}}\boldsymbol{\mathcal{A}}^{+} = \boldsymbol{I}_{12}$,

$$ \begin{bmatrix} \boldsymbol{\mathcal{A}} \\ \boldsymbol{T}_{\mathcal{A}}^{\mathrm{T}} \end{bmatrix}\begin{bmatrix} \dot{\boldsymbol{\theta}}_F \\ \dot{\boldsymbol{x}}_O \end{bmatrix} = \overline{\boldsymbol{\mathcal{A}}}\boldsymbol{\mathcal{A}}^{-}\overline{\boldsymbol{A}}_{\omega_C}\boldsymbol{\omega}_C + \begin{bmatrix} \boldsymbol{0}_{12\times(m-6)} \\ \boldsymbol{I}_{(m-6)} \end{bmatrix}\boldsymbol{v}_N $$
$$ = \left\{ \begin{bmatrix} \boldsymbol{\mathcal{A}} \\ \boldsymbol{T}_{\mathcal{A}}^{\mathrm{T}} \end{bmatrix}\boldsymbol{\mathcal{A}}^{+} - \begin{bmatrix} \boldsymbol{0}_{12\times(m-6)} \\ \boldsymbol{I}_{(m-6)} \end{bmatrix}\boldsymbol{T}_{\mathcal{A}}^{\mathrm{T}}\boldsymbol{\mathcal{A}}^{+} \right\}\overline{\boldsymbol{A}}_{\omega_C}\boldsymbol{\omega}_{\boldsymbol{C}} $$
$$ + \begin{bmatrix} \boldsymbol{0}_{12\times(m-6)} \\ \boldsymbol{I}_{(m-6)} \end{bmatrix}\boldsymbol{v}_N $$

$$= \begin{bmatrix} \boldsymbol{I}_{12} \\ \boldsymbol{0}_{(m-6)\times 12} \end{bmatrix} \overline{\boldsymbol{A}}_{\omega_C}\boldsymbol{\omega}_C + \begin{bmatrix} \boldsymbol{0}_{12\times(m-6)} \\ \boldsymbol{I}_{(m-6)} \end{bmatrix} \boldsymbol{v}_N. \quad (48)$$

It is clear that the lower $(m-6)$ rows of (48) means (47) (The upper 12 rows of (48) corresponds to (28).). ∎

From Lemma 2, we can confirm that (42) is a general solution of (28), and $\boldsymbol{v}_N \in \mathbb{R}^{(m-6)}$ can be related to the velocity of the generalized coordinates $(\dot{\boldsymbol{\theta}}_F, \dot{\boldsymbol{x}}_O)$. If $\boldsymbol{T}_{\mathcal{A}}$ is especially chosen as a constant matrix, from (47), $\int \boldsymbol{v}_N dt$ can be directly associated with the *position* of the generalized coordinates $(\boldsymbol{\theta}_F, \boldsymbol{x}_O)$. Therefore, the $(m-6)$ DOF of the position can be directly controlled by specifying a target point of $\int \boldsymbol{v}_N dt$.

4.4 Linearizing Compensator

In this subsection, we propose a linearizing compensator for $\boldsymbol{\omega}_C \in \mathbb{R}^4$, $\boldsymbol{v}_N \in \mathbb{R}^{m-6}$ and $\boldsymbol{f}_N \in \mathbb{R}^2$. A controller for the linearized system, which achieves the control objectives (A), (B) and (C), can be easily designed from the linear control theory. A linearizing compensator is given by

$$\boldsymbol{\tau} = \widehat{\boldsymbol{M}} \begin{bmatrix} \boldsymbol{u}_{\omega_C} \\ \boldsymbol{u}_{v_N} \end{bmatrix} + \widehat{\boldsymbol{C}} \begin{bmatrix} \boldsymbol{\omega}_C \\ \boldsymbol{v}_N \end{bmatrix} + \overline{\boldsymbol{N}} + \boldsymbol{A}_F^{\mathrm{T}} \boldsymbol{K}_{A_O^{\mathrm{T}}} \boldsymbol{u}_{f_N}, \quad (49)$$

where

$$\widehat{\boldsymbol{M}} := \overline{\boldsymbol{M}}\boldsymbol{S}, \quad \widehat{\boldsymbol{C}} := \overline{\boldsymbol{M}}\dot{\boldsymbol{S}} + \overline{\boldsymbol{C}}\boldsymbol{S} \quad (50)$$

$$\overline{\boldsymbol{M}} := \left[\, \boldsymbol{M}_F \;\; \boldsymbol{A}_F^{\mathrm{T}}(\boldsymbol{A}_O^{\mathrm{T}})^{+}\boldsymbol{M}_O \,\right] \quad (51)$$

$$\overline{\boldsymbol{C}} := \left[\, \boldsymbol{C}_F \;\; \boldsymbol{A}_F^{\mathrm{T}}(\boldsymbol{A}_O^{\mathrm{T}})^{+}\boldsymbol{C}_O \,\right]$$

$$\overline{\boldsymbol{N}} := \boldsymbol{N}_F + \boldsymbol{A}_F^{\mathrm{T}}(\boldsymbol{A}_O^{\mathrm{T}})^{+}\boldsymbol{N}_O$$

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{S}_F \\ \boldsymbol{S}_O \end{bmatrix} := \left[\, \boldsymbol{\mathcal{A}}^{-}\overline{\boldsymbol{A}}_{\omega_C} \;\; \boldsymbol{K}_{\mathcal{A}} \,\right] \quad (52)$$

$\boldsymbol{S}_F \in \mathbb{R}^{m\times(m-2)}$, $\boldsymbol{S}_O \in \mathbb{R}^{6\times(m-2)}$. $\boldsymbol{u}_{\omega_C} \in \mathbb{R}^4$, $\boldsymbol{u}_{v_N} \in \mathbb{R}^{(m-6)}$ and $\boldsymbol{u}_{f_N} \in \mathbb{R}^2$ are the new inputs for $\boldsymbol{\omega}_C$, $\boldsymbol{v}_N$ and $\boldsymbol{f}_N$ respectively. The following theorem holds.

Theorem 3. Consider the system (17) and (18) with the motion constraint (14). By the controller (49), the system is linearized as

$$\dot{\boldsymbol{\omega}}_C = \boldsymbol{u}_{\omega_C}, \quad \dot{\boldsymbol{v}}_N = \boldsymbol{u}_{v_N}, \quad \boldsymbol{f}_N = \boldsymbol{u}_{f_N}. \quad (53)$$

Proof: Combining (17), (18), (35) and (42), and substituting (49) into the resultant equation, we get the closed loop system given by

$$\widehat{\boldsymbol{M}} \begin{bmatrix} \dot{\boldsymbol{\omega}}_C - \boldsymbol{u}_{\omega_C} \\ \dot{\boldsymbol{v}}_N - \boldsymbol{u}_{v_N} \end{bmatrix} + \boldsymbol{A}_F^{\mathrm{T}}\boldsymbol{K}_{A_O^{\mathrm{T}}}(\boldsymbol{f}_N - \boldsymbol{u}_{f_N}) = \boldsymbol{0}. \quad (54)$$

Noting that (42) is the solution of (14) since (28) includes (14), we get $\boldsymbol{A}_F\boldsymbol{S}_F = \boldsymbol{A}_O\boldsymbol{S}_O$ from (52). Premultiplying (54) by $\boldsymbol{S}_F^{\mathrm{T}}$ and noting that (50), (51), $\boldsymbol{S}_F^{\mathrm{T}}\boldsymbol{A}_F^{\mathrm{T}} = \boldsymbol{S}_O^{\mathrm{T}}\boldsymbol{A}_O^{\mathrm{T}}$, $\boldsymbol{A}_O^{\mathrm{T}}(\boldsymbol{A}_O^{\mathrm{T}})^{+} = \boldsymbol{I}_6$ and $\boldsymbol{A}_O^{\mathrm{T}}\boldsymbol{K}_{A_O^{\mathrm{T}}} = \boldsymbol{0}$, we get

$$\left[\, \boldsymbol{S}_F^{\mathrm{T}} \;\; \boldsymbol{S}_O^{\mathrm{T}} \,\right] \begin{bmatrix} \boldsymbol{M}_F & \boldsymbol{0}_{m\times 6} \\ \boldsymbol{0}_{6\times m} & \boldsymbol{M}_O \end{bmatrix} \begin{bmatrix} \boldsymbol{S}_F \\ \boldsymbol{S}_O \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\omega}}_C - \boldsymbol{u}_{\omega_C} \\ \dot{\boldsymbol{v}}_N - \boldsymbol{u}_{v_N} \end{bmatrix} = \boldsymbol{0}.$$

Since the coefficient matrix of the above equation is nonsingular from $\boldsymbol{M}_F > 0$ and $\boldsymbol{M}_O > 0$, we get $\dot{\boldsymbol{\omega}}_C - \boldsymbol{u}_{\omega_C} = \boldsymbol{0}$ and $\dot{\boldsymbol{v}}_N - \boldsymbol{u}_{v_N} = \boldsymbol{0}$. Next, substituting these results into (54), we get $\boldsymbol{A}_F^{\mathrm{T}}\boldsymbol{K}_{A_O^{\mathrm{T}}}(\boldsymbol{f}_N - \boldsymbol{u}_{f_N}) = \boldsymbol{0}$. Note that the following equation holds (MacLane and Birkoff, 1967):

$$\dim(\mathcal{R}(\boldsymbol{A}_F^{\mathrm{T}}\boldsymbol{K}_{A_O^{\mathrm{T}}})) + \dim(\mathcal{N}(\boldsymbol{A}_F^{\mathrm{T}}) \cap \mathcal{R}(\boldsymbol{K}_{A_O^{\mathrm{T}}})) = \dim(\mathcal{R}(\boldsymbol{K}_{A_O^{\mathrm{T}}})), \quad (55)$$

where $\dim(\cdot)$ describe the dimension and $\dim(\mathcal{R}(\cdot)) = \mathrm{rank}(\cdot)$. Since $\mathcal{N}(\boldsymbol{A}_O^{\mathrm{T}}) \subset \mathcal{R}\big((\boldsymbol{A}_F^{\mathrm{T}})^{+}\big)$ from Assumption 5 (ii) and $\mathbb{R}^m = \mathcal{R}\big((\boldsymbol{A}_F^{\mathrm{T}})^{+}\big) \oplus \mathcal{N}(\boldsymbol{A}_F^{\mathrm{T}})$, $\mathcal{N}(\boldsymbol{A}_F^{\mathrm{T}}) \cap \mathcal{N}(\boldsymbol{A}_O^{\mathrm{T}}) = \emptyset$ holds ($\emptyset$ is the empty set). Therefore, since $\dim(\mathcal{N}(\boldsymbol{A}_F^{\mathrm{T}}) \cap \mathcal{R}(\boldsymbol{K}_{A_O^{\mathrm{T}}})) = 0$ from $\mathcal{R}(\boldsymbol{K}_{A_O^{\mathrm{T}}}) = \mathcal{N}(\boldsymbol{A}_O^{\mathrm{T}})$, $\dim(\mathcal{R}(\boldsymbol{A}_F^{\mathrm{T}}\boldsymbol{K}_{A_O^{\mathrm{T}}})) = \dim(\mathcal{R}(\boldsymbol{K}_{A_O^{\mathrm{T}}})) = 2$ from (55). Since $\boldsymbol{A}_F^{\mathrm{T}}\boldsymbol{K}_{A_O^{\mathrm{T}}} \in \mathbb{R}^{m\times 2}$ is full column rank, we get $\boldsymbol{f}_N - \boldsymbol{u}_{f_N} = \boldsymbol{0}$. ∎

5. CONCLUSION

In this paper, for the simultaneous control of the object motion/internal force and the contact coordinates by a two-fingered robot hand with the pure rolling contact, we provided the entire treatment of the system equations including the motion and force constraint, which consist of the generalized coordinates and the contact coordinates. We considered the general treatment of the system for *any* DOF of the fingers. Utilizing the results, the control design method which achieves the simultaneous control was proposed.

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