CONTROLLABILITY OF MECHANICAL SYSTEMS WITH REGARD FOR ACTUATOR DYNAMICS

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Abstract: A series of studies on controllability of the nonlinear dynamic mechanical systems such as manipulators, aircraft, or water-craft was advanced. Earlier, controllability was considered without regard for dynamics of the mechanical system actuators. The present paper established a controllability criterion for the mechanical systems where actuator dynamics was explicitly taken into account. The established conditions for controllability have a clear physical sense. As before, the controllability conditions are related with domination of the control forces over other generalized forces of weight, environmental resistance, and so on. Now domination is required in the rate of force variation, that is, in the derivative. Stated differently, it is required that the actuator output may vary rather rapidly.

Keywords: nonlinear mechanical system, actuator dynamics, controllability criteria

1. INTRODUCTION

The Kalman controllability criterion is most known. The criterion was established for linear stationary dynamic systems. Consideration was given also to the linear systems with controls constrained in amplitude, power, or pulse (Formal'skii, 1974). Therefore these criteria may be used only for linearized nonlinear mechanical systems.

For nonlinear systems the controllability conditions "in the small" were obtained in (Krasovskii, 1968; Kirillova and Gabasov, 1971). The controllability conditions for so-called driftless systems were given in (Chow, 1939). These systems and their analogues are intensively studied in the last tens years (Brokket, 1983; Bloch *et al.*, 1990; Murray and Sastry, 1991; Cortes *et al.*, 2001).

Some mechanical systems may be reduced to the chain form - analog of driftless systems (Murray and Ortega, 1991). The conditions of reduction are sufficiently strict. Under these conditions it is difficult to take into account the external forces, which exert influence upon mechanical systems. These forces may be of the general form, i.e. are not potential, may have dissipative sense (of dry friction for example), or have nature of intensive oscillations.

Such mechanical systems were studied in (Pyatnitskii, 1996, 1997). The external forces with bounded amplitude were considered as permissible ones. Permissible control forces were bounded too. The sufficient controllability conditions were obtained. The sense of conditions was the control force domination over the disturbing external forces. Similar conditions were proved for nonholonomic mechanical systems (Matyukhin, 2004). The controllability conditions were obtained too for the class of smooth disturbing forces and for the class of smooth control forces (Matyukhin, and Pyatnitskii, 2004).

Another problem is investigated in this paper. The control forces are considered to be generated by the controlling actuators of the mechanical systems. In other words the dynamics of the controlling actuators is taken into consideration in explicit form.

1.1 Dynamic Systems under Consideration

The present paper discusses dynamic systems such as

$$\frac{d}{dt}\frac{\partial T}{\partial \dot q_i} - \frac{\partial T}{\partial q_i} = Q_i + M_i, \quad \dot M_i = F_i + u_i, \quad {\scriptstyle i=\overline{1,n}.} \tag{1}$$

The first group of equations in (1) are the well-known Lagrange equations of the second kind which describe the motions of mechanical systems. The second group includes equations describing the actuator dynamics of this system. System (1) makes use of the following standard notation: q_i and $\dot q_i$ are, respectively, the generalized system coordinate and velocities, n is the number of system degrees of freedom, $\{Q_i + M_i\}$ are the generalized forces, and $T = \frac{1}{2}\sum_{k,i=1}^{n} a_{ik}(q)\dot q_i \dot q_k$ is the system kinetic energy. The first subsystem of (1) has the following developed form:

$$\sum_{k=1}^{n} a_{ik}\ddot q_k + \sum_{k,s=1}^{n} b_{iks}\dot q_s \dot q_k = Q_i(q,\dot q,t) + M_i, \tag{2}$$

$$b_{iks}(q) = \frac{\partial a_{ik}(q)}{\partial q_s} - \frac{1}{2}\frac{\partial a_{ks}(q)}{\partial q_i}, \quad {\scriptstyle i=\overline{1,n}.}$$

The inertial properties of the mechanical system are characterized by $a_{ik}(q)$ in (2) that are related with the distribution of its masses. The components of the generalized forces $Q_i(q,\dot q,t)$ in (1) are defined by the external forces (weight, resistance, and so on) acting on the mechanical system. The system actuators develop the controlling forces M_i. In the general form, their dynamics is defined by the second group of equations [1] of (1). The values u_i are regarded as the controls of system (1).

[1] These equations may be regarded, for example, as an analog of the reduced equation $\gamma \dot M = -rM + \nu \dot q + u$ of the dc electric motor, where $\gamma > 0$ may be treated as a characteristic of the rotor inductance, r as its resistance, the function $\nu \dot q$ as the counter-electromotive force, and u as the control voltage.

We assume that for all $q_i, \dot q_k, M_j$ and $t \ge t^0$ system (1) has the following formal properties

$$|a_{ij}(q)| \le d, \quad \left|\frac{\partial a_{ij}}{\partial q_k}\right| \le d, \quad \left|\frac{\partial^2 a_{ij}}{\partial q_k \partial q_s}\right| \le d, \tag{3}$$

$$d = const > 0,$$

$$\lambda_1 \sum_{p=1}^{n} \dot q_p^2 \le T(q,\dot q) \le \lambda_2 \sum_{p=1}^{n} \dot q_p^2, \tag{4}$$

$$0 < \lambda_1 \le \lambda_2 < \infty, \quad \lambda_p = const,$$

$$|Q_i| \le h_i,$$
$$|\frac{\partial Q_i}{\partial t}| \le l_i^1, \quad |\frac{\partial Q_i}{\partial q_k}| \le l_i^2, \quad |\frac{\partial Q_i}{\partial \dot q_k}| \le l_i^3, \tag{5}$$

$$h_i, l_i^p = const \ge 0,$$

$$|F_i(q,\dot q,M,t)| \le F_i^1, \tag{6}$$

$$F_i^1 = const \ge 0,$$

$$|u_i(t)| \le L_i, \tag{7}$$

$${\scriptstyle i,k,j,s=\overline{1,n},} \quad L_i = const \ge 0.$$

The above formal assumptions seem to be sufficiently natural (Formal'skii, 1974; Pyatnitskii, 1996, 1997; Matyukhin, 2001).

1.2 Formulation of the Problem

The present paper is vectored at establishing the controllability conditions for systems of the form (1) in compliance with the following definition.

Definition 1. System (1) will be called the controllable system in the class U of controls if for arbitrary points $S^i = (q^i, \dot q^i, M^i)$ and $S^f = (q^f, \dot q^f, M^f)$ of the system state space $P = (q_1, ..., q_n, \dot q_1, ..., \dot q_n, M_1, ..., M_n)$ there exists a control $u \in U$ that drives the system from S^i to S^f in a finite time (R. Kalman).

The class U of permissible controls comprises all possible vector functions of the form $u(t) = \{u_1(t), ..., u_n(t)\}$ that satisfy constraints (7). This class is defined by the constant parameters L_i, that is, $U = U(L_i)$.

The papers (Pyatnitskii, 1996, 1997; Matyukhin, and Pyatnitskii, 2004; Matyukhin, 2004) are most close in formulation of the problem. The papers (Pyatnitskii, 1996, 1997; Matyukhin, 2004) established a controllability criterion for purely mechanical systems, that is, systems without regard for the actuator dynamics. The control forces M_i of the form $|M_i| \le H_i$ of the mechanical system (1) were directly considered as controls, and the problem of controllability was studied only in the phase space of the mechanical system $P^1 = \{q_1, ..., q_n, \dot q_1, ..., \dot q_n\}$.

In (Matyukhin and Pyatnitskii, 2004), a controllability criterion was established also for the purely mechanical systems where the constraints on the rate of increase, that is, $|M_i| \leq H_i$, $|\dot{M}_i| \leq L_i$, were also taken into consideration. The present study differs in that it explicitly takes into account the equations describing motions of the mechanical system actuators. This formulation seems to be more natural, for example, from the applied standpoint, and leads to the following distinctions.

Namely, M_i has the sense of the state of the system of differential equations (1), and the initial values $|M_i(0)| = M_i^0$ are introduced for them. In the general case, they are given, but cannot be freely assigned, for example, on the basis of the purposes of control of the mechanical system as it was admitted in (Matyukhin, and Pyatnitskii, 2004). This situation is natural from the applied point of view. Namely, the variables M_i have the sense of the generalized forces of the mechanical subsystem (1), and it is not necessary that $M_i(0)$ be, for example, small in magnitude at the initial time instant.

These considerations seem, on the one hand, natural, but on the other hand, they complicate solution of the controllability problem. For example, one has to consider this problem in a wider state space $P = \{q_1, ..., q_n, \dot{q}_1, ..., \dot{q}_n, M_1, ..., M_n\}$ of the system (1).

2. CONTROLLABILITY CRITERION OF THE SIMPLEST SYSTEM

Let us consider first the controllability criterion for the simplistic systems (1) of the form

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_i + M_i, \qquad (8)$$
$$\dot{M}_i = F_i + u_i, \quad {\scriptstyle i=\overline{1,n}}, \quad Q_i = F_i = 0.$$

In distinction to the original system (1), system (8) has no generalized forces $Q_i = 0$ and takes into account only the simplest equations of actuator motion where $F_i = 0$.

Theorem 1. Let system (8) satisfy conditions (3)-(6). Then, system (8) will be controllable in the class of controls for any numbers $L_i = const > 0$:

$$|u_i(t)| \leq L_i, \quad {\scriptstyle i=\overline{1,n}}. \qquad (9)$$

The scheme of proof of Theorem 1 is described below. The sense of Theorem 1 lies in that the mechanical system (8) can be driven to any point of the state space $P = \{q_1, ..., q_n, \dot{q}_1, ..., \dot{q}_n, M_1, ..., M_n\}$ of system (8). At that, it is of no consequence where the system was at the initial time instant. It suffices that the controls have a nonzero resource $L_i > 0$. This fact is valid for any system of the form (8) for which the above nonrestrictive constraints (3)–(6) are satisfied.

A controllability criterion having the sense of the condition for domination of the control forces M_i over the disturbing forces Q_i in the rate of growth was obtained earlier (Matyukhin and Pyatnitskii, 2001). The same sense can be rendered formally to criterion (9). Namely, there are no forces in system (8), that is, $Q_i = 0$. Therefore, by taking into consideration (8), one can rearrange (9) in

$$|\dot{M}_i| > |\dot{Q}_i|, \qquad (10)$$
$$|\dot{M}_i| \leq L_i, \; L_i > 0, \quad |\dot{Q}_i| = 0.$$

Consequently, criterion (9) in the form (10) can be regarded as the condition for domination of the control forces M_i over the disturbing forces Q_i in the growth rate.

Scheme of proof of Theorem 1. Let an arbitrary mechanical system (8) be given for which conditions (3)-(6) are valid. Given are arbitrary points $S^i = (q^i, \dot{q}^i, M^i)$ and $S^f = (q^f, \dot{q}^f, M^f)$ (Fig. 1). Let the class $u_i(t), |u_i| \leq L_i$ of controls be given where $L_i > 0$ are arbitrarily defined constants. It is desired to prove that in this class there exists a control driving system (8) from S^i to S^f in a finite time.

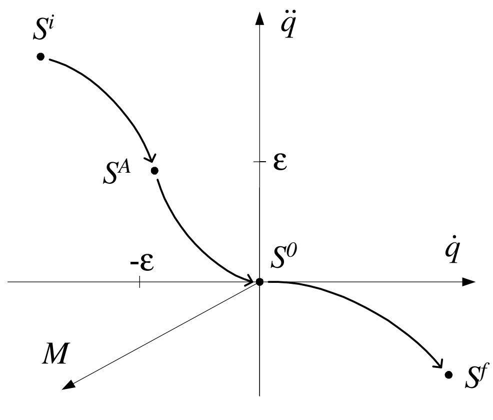

Fig. 1. Translation of system (8) from the initial point S^i to the origin S^0 and further to the final point S^f.

The proof is based on the three basic properties of the mechanical systems (8) under study. First, system motion can be retarded, that is, it is possible to provide small values of the velocities and their derivatives (at the point S^A in Fig. 1 where $|\dot{q}_i| \leq \varepsilon$, $|\ddot{q}_i| \leq \varepsilon$, $|\dddot{q}_i| \leq \varepsilon$). Second, from such a point S^A the system can be driven to the origin S^0. Third, the system can be driven from S^0 to an arbitrary given final point S^f. The possibility of retarding system (8) is substantiated by the following assertion.

Lemma 1. Let the conditions of Theorem 1 be satisfied. Then, there exists a control u^A of the class (9) that drives system (8) in a finite time from an arbitrary point S^f to a point S^A where

$$|\dot{q}_i^A| \le \varepsilon, \ |\ddot{q}_i^A| \le \varepsilon, \ |\dddot{q}_i^A| \le \varepsilon, \quad {\scriptstyle i=\overline{1,n}}. \qquad (11)$$

The concept of the proof of Lemma 1. The control u^A is as follows:

$$u_i^A = -L_i \mathrm{sign}(M_i + k\dot{q}_i), \quad k = const > 0, \quad (12)$$

with sign(x) standing for the function of sign of x. The lemma establishes that on the motion of system (8), (12)

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = M_i, \quad \dot{M}_i = u_i^A, \qquad (13)$$

relations (11) of the form

$$|\dot{q}_i(t_A)| \le \varepsilon, \ |\ddot{q}_i(t_A)| \le \varepsilon, \ |\dddot{q}_i(t_A)| \le \varepsilon, \quad {\scriptstyle i=\overline{1,n}} \qquad (14)$$

will be satisfied (at the point S^A in Fig. 1). The numbers t_A and k exist.

Namely, it is proved that system (13) enters a sliding mode of the form

$$M_i = -k\dot{q}_i, \quad {\scriptstyle i=\overline{1,n}}, \quad t \ge \tau, \qquad (15)$$

and starts moving along the trajectory of a system of the form

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = -k\dot{q}_i, \quad t \ge \tau. \qquad (16)$$

The kinetic energy of the system decreases because the relation

$$\dot{T} = \sum_{i=1}^{n} \dot{q}_i M_i$$

is true. Really,

$$\dot{T}(t) = -k\sum_{i=1}^{n} \dot{q}_i^2 \le -\frac{kT}{\lambda_2}$$

and

$$T(t) \le T(\tau) exp\frac{-k(t-\tau)}{\lambda_2}$$

are true for $t \ge \tau$. Therefore, the velocities of system (16), that is, of system (13), decrease

$$\dot{q}_i^2(t) \le T(\tau) exp\frac{-k(t-\tau)}{\lambda_2}/\lambda_1$$

in compliance with (14).

The main problem lies in substantiating existence in (12) of a coefficient $k > 0$ such that it gives rise to the mode (15). It is demonstrated that such a k can be constructed, for example, in the form

$$k = k(\Omega, S^i, S^f, L_j), \qquad (17)$$
$$\Omega = (d, \lambda_1, \lambda_2, h_j, l_j^p, n).$$

Definition 1 admits the dependence of a control like (15),(17) on the system, that is, on the parameters Ω, the initial and final points, and the parameters L_k of the control class U.

Namely, it was established that under the condition

$$|k\ddot{q}_i(t)| \le 2L_i, \quad {\scriptstyle i=\overline{1,n}} \qquad (18)$$

the system (13) enters the sliding mode (15) where

$$\tau = \max(2|\dot{q}_j(0)|/L_j). \qquad (19)$$

Inequalities (18) involve the accelerations of the system (13) for which there exist the estimates

$$2\lambda_1|\ddot{q}_i| \le \sum_{j=1}^{n}\{\sum_{k,s=1}^{n}(3d/2)|\dot{q}_k||\dot{q}_s| + |M_j|\}. \quad (20)$$

Inequalities (20) involve the velocities and control forces of system (13) for which there exist the estimates

$$|\dot{q}_i^2(t)| \le T(t)/\lambda_1, \quad t \ge 0,$$
$$|M_i(t)| \le |M_i(0)| + L_i t, \quad 0 \le t \le \tau,$$
$$|M_i(t)| \le k|\dot{q}_i(t)| \le k\sqrt{T(t)/\lambda_1}, \quad t > \tau.$$

These inequalities involve the kinetic energy of the system (13) which is bounded by

$$T(t) \le T^1, \quad t > 0,$$
$$T^1 = const(T(0), \lambda_1, L_i, \tau).$$

The above statements allow one to substantiate existence of a coefficient $k > 0$ for condition (18) under consideration.

3. CONTROLLABILITY CRITERION FOR SYSTEMS WITH GENERALIZED FORCES

This section considers system (1)

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_i(q, \dot{q}, t) + M_i, \qquad (21)$$
$$\dot{M}_i = F_i + u_i, \ {\scriptstyle i=\overline{1,n}}, F_i = 0,$$

where $F_i = 0$.

Theorem 2. Let system (21) satisfy conditions (3)-(6). Then, there exists a number $L^* > 0$ such that system (21) is controllable in the following class of controls:

$$|u_i(t)| \le L^*, \quad {\scriptstyle i=\overline{1,n}}. \qquad (22)$$

The proof is outlined below. We note that if the disturbing forces $Q_i \ne 0$ are taken into account, then in system (21) — in distinction from system (8) where $Q_i = 0$ — the minimum control resource with the constraint $L^* > 0$ is evidently insufficient. We also note that the trivial compensation of the forces Q_i in (21) is impossible. This can be readily seen if one rearranges system (21) and then condition (22) with regard for (2), respectively, in

$$u_i = \frac{d}{dt}\left\langle \frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i}\right\rangle - \dot{Q}_i$$

and

$$\left| \begin{array}{l} \sum_{k=1}^{n}(\dot a_{ik}\ddot q_k + a_{ik}\dddot q_k) + \sum_{k,s=1}^{n}(\dot b_{iks}\dot q_s\dot q_k + \\ 2b_{iks}\ddot q_s\dot q_k) + \dfrac{\partial Q_i}{\partial t} + \sum_{k=1}^{n}(\dfrac{\partial Q_i}{\partial q_k}\dot q_k + \dfrac{\partial Q_i}{\partial \dot q_k}\ddot q_k) \end{array} \right| \le L^*. \tag{23}$$

No matter how great is the predefined constant L^*, inequalities (23), obviously, cannot be satisfied. It is the case, for example, at the initial instant because the initial values $(q(0), \dot q(0), M(0))$ in system (21) are arbitrary and may be appreciably large. In this sense, the controllability conditions (22) are not trivial.

It follows from the proof of Theorem 2 that the value of the constant L^* in (22) satisfies two conditions. First, the controls $|u_i| \le L^*$ allow one to provide finite velocities and accelerations in system (21). Second, if the velocities and accelerations are finite, then the controls $|u_i| \le L^*$ admit explicit compensation of the disturbing forces Q_i.

The first condition is satisfied if the inequality

$$h^2/\nu < L^*, \tag{24}$$

$$|\dot q_i| \le \nu, \quad h = max(h_1, ..., h_n), \quad \nu = \nu(\Omega),$$

$\Omega = \Omega(d, \lambda_1, \lambda_2, h_j, l_j^p, n)$ is satisfied. Inequality (24) shows that sufficiently small generalized system velocities $|\dot q_i| \le \nu$ can be obtained only through an appreciable resource of controls L^*. We note that the theorem's assumption (5) of the form $|Q_i| \le h_i$ is significant and, according to (24), is necessary to provide finite velocities of the mechanical system.

The second condition is representable as

$$|\dot Q_i| \,|_{|\dot q_k| \le \nu,\ |\ddot q_k| \le a} < L^*, \tag{25}$$

$$\nu, a = const \ge 0$$

with regard for (23), (24). This relation is the necessary condition for the case of feasible explicit compensation of the forces Q_i at the expense of the controls $|u_i| \le L^*$. We note that the theorem's assumption (5) about boundedness of the derivatives of the forces Q_i is also essential and, according (25), allows one to compensate these forces.

We also note that the constant L^* depends on all dynamic parameters $\Omega = \Omega(d, \lambda_1, \lambda_2, h_j, l_j^p, n)$ of system (21). For example, L^* depends — in distinction to (Pyatnitskii, 1996, 1997; Matyukhin, and Pyatnitskii, 2001: Matyukhin, 2001) — on the system inertia characteristics d, λ_1, λ_2. Stated differently, if the actuator dynamics is taken into account, then the conditions for controllability of a mechanical system are defined by its inertia characteristics along with h_j, l_j^p.

Scheme of proof of Theorem 2 follows that of Theorem 1. It is based on the four basic properties of the mechanical systems (21) under consideration. The first property is the possibility of retarding of system motion when its velocity and control forces are finite (at the final point S^1 in Fig. 2 where S^i is an arbitrary initial point). The second property is the possibility of providing a small system velocities and its derivatives (point S^2). The third property is the possibility of driving the system to the origin S^0. Finally, the fourth property is the possibility of driving the system (21) from S^0 to an arbitrary final point S^f.

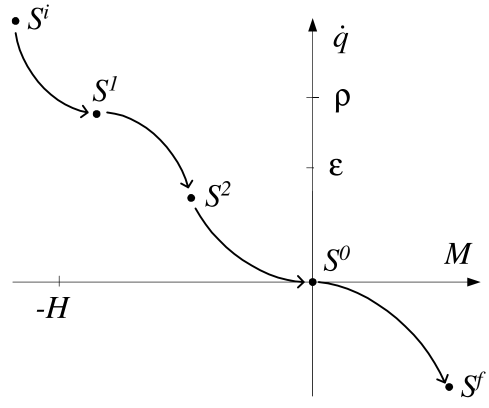

Fig. 2. Translation of system (21) from the initial point S^i to the origin S^0 and further to the final point S^f.

4. CRITERION FOR SYSTEM CONTROLLABILITY IN THE GENERAL CASE

In this section, consideration is given to system (1) in the original general form

$$\begin{array}{l} \dfrac{d}{dt}\dfrac{\partial T}{\partial \dot q_i} - \dfrac{\partial T}{\partial q_i} = Q_i(q, \dot q, t) + M_i, \\ \dot M_i = F_i(q, \dot q, M, t) + u_i, \quad {\scriptstyle i=\overline{1,n}}. \end{array} \tag{26}$$

Theorem 3. Let system (26) satisfy conditions (3)–(6). Then, there exists a number L^{**} such that system will be controllable in the following class of controls:

$$|u_i(t)| \le L^{**}, \quad {\scriptstyle i=\overline{1,n}}. \tag{27}$$

We note that in (27) the constant L^{**} satisfies the condition

$$L^{**} \ge L^* + F_i^1, \quad {\scriptstyle i=\overline{1,n}}, \quad |F_i| \le F_i^1. \tag{28}$$

Therefore, in distinction to (21) where $F_i = 0$, in system (26) the additional resource of controls must be used to counteract the disturbances $F_i(q, \dot q, M, t)$.

The proof of Theorem 3 is concerned with explicit compensation of the disturbances F_i in the actuator motion equations. Namely, for $F_i = 0$, system

(26) becomes (21) which, according to Theorem 2, is well controllable in the class (22). This means that for arbitrary points S^i and S^f of the phase space of system (21) there exists a control $u^*(t)$ driving the system from S^i to S^f in some finite time. We denote by $q^*(t)$ the trajectory of system (21) from S^i to S^f. Then, obviously, the control

$$u_i(t) = -F_i(q^*(t), \dot{q}^*(t), t) + u_i^*(t)$$

will drive system (26) from S^i to S^f in the same time (along the same trajectory $q^*(t)$). This will be the case if the class of control in system (26) obeys the condition

$$|u_i| \leq L^* + F_i^1$$

where assumption (6) of the form $|F_i(q_i^*(t), \dot{q}_i^*(t), t)| \leq F_i^1$ is taken into consideration. Therefore, system (26) turns out to be controllable in the class of controls (27).

We note that assumption $|F_i| \leq F_i^1$ of Theorem 3 is essential. Indeed, let us consider, for example, the equation of motion of actuators in (26) of the form $\dot{M} = -M + u$. Let consideration be given to the class $|u(t)| \leq L^{**}$ with $L^{**} = const > 0$. This system is uncontrollable because it cannot be driven from the point $M(0) = 0$ to, for example, the point $M = 2L^{**}$, no matter how large is the predefined number L^{**}. Therefore, in the general case the condition $|F_i| \leq F_i^1$ is necessary for controllability of system (26).

5. CONCLUSION

A criterion for controllability of mechanical systems was constructed with regard for dynamics of the mechanical system actuators. The problem was considered in the general nonlinear formulation. The criterion has a simple physical sense and can be expressed in terms of the controlling and disturbing forces of the mechanical system. For example, for the manipulation robot to be controllable it is required that the control forces dominate other generalized (weight, environmental resistance, and so on) forces. Domination is required in the rate of force variation, that is, in the derivative. In this sense, the criterion satisfies the intuitive *a priori* considerations arising when one deals with diverse problems of control of mechanical systems. Namely, the system actuators must be sufficiently fast. Only in this case it becomes possible to counteract the intensive pulsating disturbances and freely control the mechanical system.

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