ENGINEERING METHODS OF CONTROL DESIGN FOR DISTRIBUTED PARAMETER SYSTEMS

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Abstract: In the paper engineering methods for control of distributed parameter systems given by numerical structures on complex shape 3D definition domains are presented. Controlled systems are interpreted as lumped-input/distributed-output systems. Schemes of distributed parameter control loops are arranged. A web-based control design environment from www.dpscontrol.sk will be introduced: Distributed Parameter Systems Blockset for MATLAB & Simulink – third-party MathWorks product, web service and internet monograph. - In general the paper indicates a new direction for engineering control technology, operating mainly on the time axis, to expand towards time/space coordinates of real world. *Copyright © 2005 IFAC*

Keywords: Control design; distributed parameter systems; lumped-input/distributed-output systems; time/space decomposition of dynamics and control synthesis; distributed parameter PID, algebraic, state space, robust control; web based control design environment; finite element method.

1. INTRODUCTION

In the last decade, increasing attention has been focused on the numerical dynamical analysis of diverse processes, machines, apparatuses,... as dynamical systems given over complex shape 3D definition domains. New software systems have been developed to treat these problems: ANSYS, FEMLAB, FLUENT, MODFLOW, STAR-CD, PAM-SYSTEMS,... a new discipline emerges in this sphere of applications – computational science and engineering. The fields of quantities to be studied are often monitored by camera methods or appropriate sensor fields. In accordance with systems and control theory, all these real systems are distributed parameter systems (DPS). This wide interest towards DPS dynamical analysis means serious challenge also in control of such systems... - Now, attractive 3D animations "jumping" on computer screens is a great challenge for control community to control these processes... In engineering practice, controlled DPS frequently found as lumped-input/distributed-output systems (LDS), and, controlled DPS practically always possible to arrange as LDS. The decomposition of dynamics of the controlled systems to time and space components enables to divide also the problems of identification and control synthesis into time and space tasks. In the paper, schemes for PID, algebraic, state space and robust control of DPS will be outlined. These schemes indicate also general course for using of results and tools of lumped parameter systems identification and control as well as methods of approximation and optimization for DPS control. See further e.g. adaptive or intelligent control of DPS as LDS, Hulkó (1998 - 2003). At the end of the paper, the web-based control design environment will be presented. It is arranged on www.dpscontrol.sk and includes: full downloadable demo version of Distributed Parameter Systems Blockset for MATLAB & Simulink – third-party

MathWorks product, web Service for support distributed parameter control solutions via the internet, in frame of LDScontrol and internet version of the monograph Hulkó et. al. (1998).

2. DPS - DDS - LDS

In general, DPS are systems whose state or output variables, X(x,y,z,t)/Y(x,y,z,t) are distributed variables or fields of variables, where (x,y,z) is a vector in 3D. These systems are often considered as systems whose dynamics is described by partial differential equations (PDE), Butkovskij (1965), Lions (1971), Wang (1964). In the input-output relation, PDE define distributed-input/distributed-output systems (DDS) between distributed input U(x,y,z,t) and distributed output variables Y(x,y,z,t), at initial and boundary conditions given, see Fig. 1.

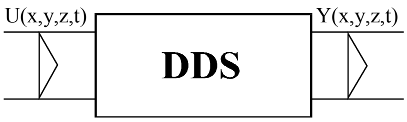

Fig. 1. Distributed-input/distributed-output system

Distributed parameter systems frequently are found in the engineering practice as LDS, see Fig. 2, having common structure according to Fig. 3. Here the DDS block is obtained as a special case of the LDS one.

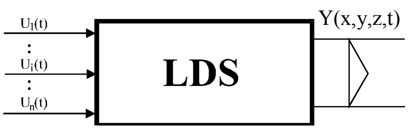

Fig. 2. Lumped-input/distributed-output system: $\{U_i(t)\}_{i=1,n}$ - lumped input variables, Y(x,y,z,t) - distributed output variable

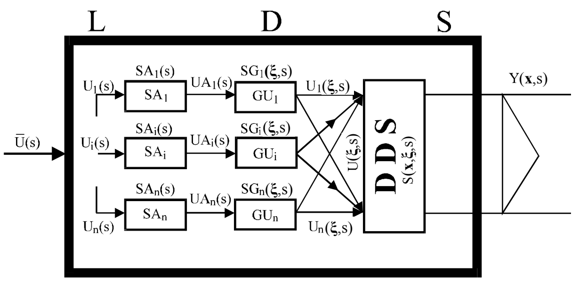

Fig. 3. Lumped-input/distributed-output system structure: $\{SA_i(s)\}_{i=1,n}$ - transfer functions of actuating gears of lumped variables $\{SA_i\}_{i=1,n}$; $\{SG_i(\xi,s)\}_{i=1,n}$ - transfer functions of generators of distributed input variables $\{GU_i\}_{i=1,n}$; $S(\mathbf{x},\xi,s)$ - transfer functions of distributed-input / distributed-output system; where $\mathbf{x}$, ξ – are vectors in 3D.

LDS offer wide possibilities for modeling, control and design of distributed parameter systems in the engineering practice.

3. BASIC CHARACTERISTICS OF LDS DYNAMICS

Let us consider LDS, distributed on the interval $\langle 0,L \rangle$, as shown in Fig. 4. Lumped discrete input variable $U_i(k)$, at unit sampling period, enters through the zero-order hold unit H_i , as $U_i(t)$, into the LDS. The output of the system will be in the form of distributed variable $Y_i(x,t)$ or $Y_i(x,k)$. At a point x_i, it will be $Y_i(x_i,t)$ or $Y_i(x_i,k)$. The discrete transfer function between $U_i(k)$ and $Y_i(x_i,k)$ is denoted as $SH_i(x_i,z)$. When the unit-step input variable is applied, in the output of the system distributed unit-step response $\mathcal{H}H_i(x,k)$ is got. Its reduced form in the steady-state is expressed as:

$$\mathcal{H}HR_i(x,\infty) = \frac{\mathcal{H}H_i(x,\infty)}{\mathcal{H}H_i(x_i,\infty)} \quad (1)$$

The reduced characteristics can be obtained from common distributed step-responses, $\{H_i(x,k)\}_i$, also. Whereas at zero-order holds $\{\mathcal{H}HR_i(x,\infty) = HR_i(x,\infty)\}_i$. This procedure enables to introduce characteristics for all lumped input - i=1,n - and distributed output variables of the studied system:

$$\{SH_i(x_i, z)\}_{i=1,n} \quad (2)$$

$$\{\mathcal{H}HR_i(x,\infty)\}_{i=1,n} \quad (3)$$

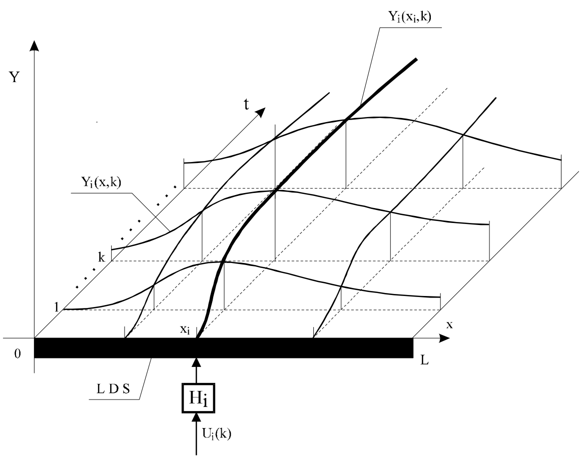

Fig. 4. Lumped-input/distributed-output system with zero-order hold unit H_i on the input

4. DISTRIBUTED PARAMETER CONTROL LOOPS

For LDS control, let us introduce distributed parameter feedback control loop, Fig. 5. Let the goal of control be to ensure the steady-state control error to be minimal, i.e. :

$$\min\left\|E(x,\infty)\right\| = \min\left\|W(x,\infty)-Y(x,\infty)\right\| = \qquad (4)$$

$$= \quad \left\|W(x,\infty)-\breve{Y}(x,\infty)\right\| = \left\|\breve{E}(x,\infty)\right\| \qquad (5)$$

where $\|.\|$ is a norm appropriately chosen.

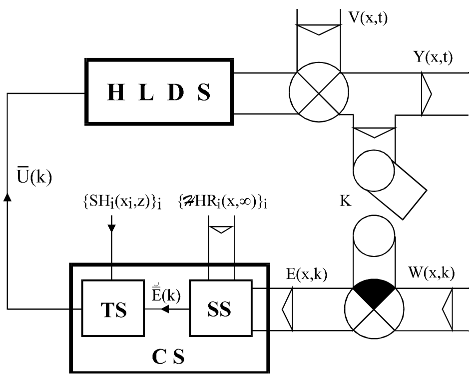

Fig. 5. Distributed parameter feedback control loop: HLDS - LDS with zero-order hold units $\{H_i\}_i$ on the input, CS - control synthesis, TS - control synthesis in time domain, SS - control synthesis in space domain, K - time/space sampling, Y(x,t) - distributed controlled variable, W(x,k) - control variable, V(x,t) - disturbance variable, E(x,k) - control error

First, the approximation problem (6) in the block of the Space Synthesis (SS) is solved on the set of reduced steady-state distributed step responses $\{\mathcal{H}\,HR_i(x,\infty)\}_{i=1,n}$, see Fig. 6.

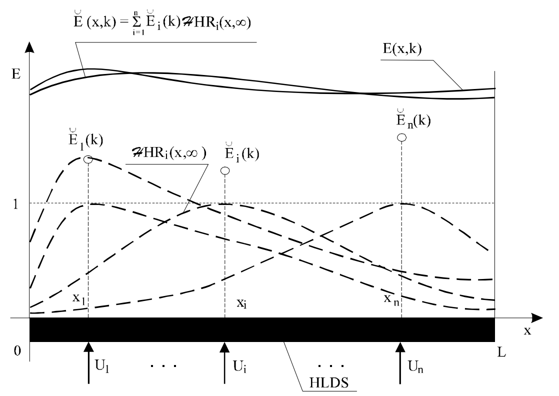

Fig. 6. Approximation problem solution

$$\min_{E_i}\left\|E(x,k)-\sum_{i=1}^{n}E_i(k)\mathcal{H}\,HR_i(x,\infty)\right\| =$$
$$=\left\|E(x,k)-\sum_{i=1}^{n}\breve{E}_i(k)\mathcal{H}\,HR_i(x,\infty)\right\| \qquad (6)$$

Further, the control errors vector $\breve{E}(k)=\{\breve{E}_i(k)\}_i$ enters the Time Synthesis block (TS), see Fig. 7.

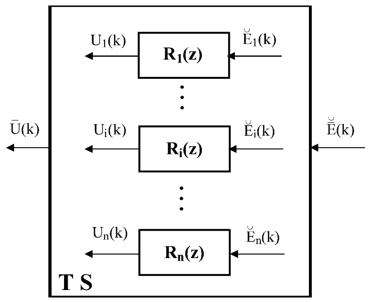

Fig. 7. Time synthesis block

Here, vector components $\{\breve{E}_i(k)\}_i$ are fed through the inputs of controllers $\{R_i(z)\}_i$ and the sequence of control variables $\overline{U}(k)$ is generated. Tuning of controllers is done - according to single components of the time part of the controlled distributed parameter system dynamics $\{SH_i(x_i,z)\}_{i=1,n}$, in single - parameter control loops $\{SH_i(x_i,z),R_i(z)\}_i$, see Fig. 8.

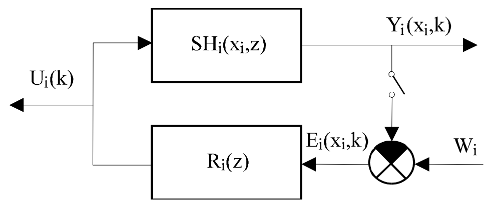

Fig. 8. The i-th single parameter control loop

During the control process, for $k\to\infty$, we get the relation

$$\min_{E_i}\left\|E(x,\infty)-\sum_{i=1}^{n}E_i(\infty)\mathcal{H}\,HR_i(x,\infty)\right\| = \left\|\breve{E}(x,\infty)\right\| \qquad (7)$$

and, thus, the control task (4), (5) is accomplished.

5. PID, ALGEBRAIC, STATE AND ROBUST CONTROL SCHEMES

The design of controllers for the block TS is realized in single-parameter control loops, see Fig. 8. For control synthesis of these loops using lumped parameter systems - PID, algebraic, state-space, robust,... adaptive or intelligent control methods offer wide possibilities, Hulkó et. al. (1998 – 2003). When the time part of the control synthesis in the block TS is realized via either PID, or algebraic controllers, then we get control loops with distributed parameters with PID, or algebraic controllers respectively, see Fig. 5. Distributed parameter control loops with state, or robust control synthesis in the time direction are shown in Fig. 9., 10. Hulkó (1998 – 2003).

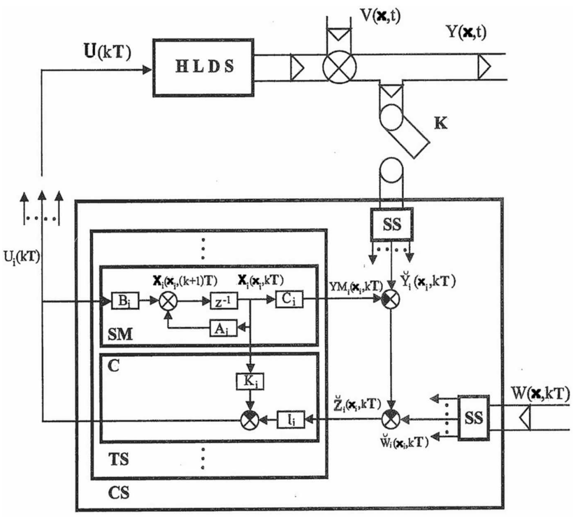

Fig. 9. Distributed parameter system of control with state controllers: HLDS - controlled system with zero-order hold units; CS - control synthesis; TS - time control synthesis; SM - state space models; C - controllers; SS - space control synthesis; K - time/space sampling; Y,W,V - distributed controlled, reference and disturbance variables; **U**(kT) - vector of control variables; $\breve{Y}_i$ ($\mathbf{x}_i$,kT), $\breve{W}_i$ ($\mathbf{x}_i$,kT) - components of vectors of controlled and reference variables; YM_i ($\mathbf{x}_i$,kT), $\breve{Z}_i$ ($\mathbf{x}_i$,kT) - components of vectors of model and modified control variables

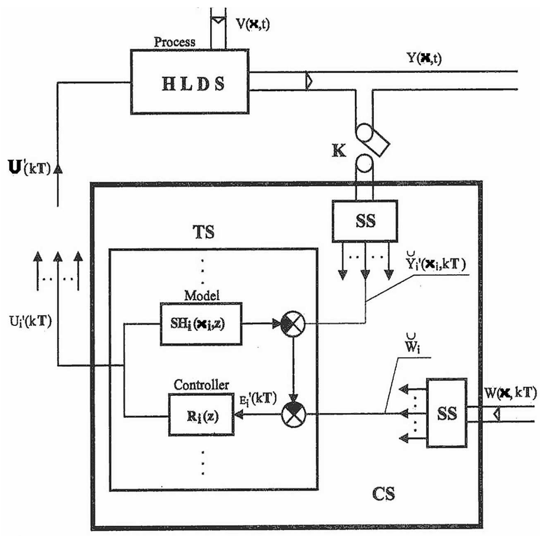

Fig. 10. Distributed parameter robust system of control: HLDS - controlled system with zero-order hold units; CS - control synthesis; TS - time control synthesis; SS - space control synthesis; $\{R_i(z)\}_i$ – controllers; $\{SH_i(\mathbf{x}_i,z)\}_i$ - controlled systems; K - time/space sampling; Y,W,V - distributed controlled, reference and disturbance variables; **U**'(kT) - vector of control variables; $\breve{Y}'(\mathbf{x}_i,kT)$, $\breve{W}_i$ - components of the vector of controlled and reference variable; $U'_i(kT)$, $E'_i(kT)$ - components of the vector of control variables and control errors.

6. WEB-BASED CONTROL DESIGN ENVIRONMENT

For support of the formulation and solution of DPS control problems in the engineering practice, the web-based control design environment was drawn up. It is arranged on www.dpscontrol.sk and includes**:** full downloadable demo version of Distributed Parameter Systems Blockset for MATLAB & Simulink – third-party MathWorks product, web Service for support distributed parameter control solutions via internet, in frame of LDScontrol – Control of Dynamical Systems Given on Complex Definition Domains and internet version of the monograph G. Hulkó et. al.: Modeling, Control and Design of Distributed Parameter Systems with Demonstrations in MATLAB.

DPS Blockset is a blockset to be run under MATLAB & Simulink for distributed parameter systems and their applications in modeling, control and design of dynamical systems given on complex 3D domains of definition. Fig. 11. The blockset features:

- Engineering methods for DPS modeling, control and design
- DPS models based on lumped-input/distributed-output systems, time/space analysis, synthesis and design tools
- Distributed parameter PID, algebraic, state space and robust control schemes
- DPS Wizard demonstrates in step-by-step operation distributed parameter control loops arrangement and setting procedures
- Suite of blocks and schemes for DPS control practically in any field of technical practice
- Interactive Control Service for support DPS control solutions via the internet

The DPS Blockset in block Show presents some typical engineering problems of DPS control from areas of technological and production processes, mechatronics and protection of the environment. Tutorial and Demos initiate users in formulation and solution of distributed parameter control problems.

The DPS Wizard in step-by-step operation demonstrates arrangement and setting procedures of distributed parameter control loops. Further suits of

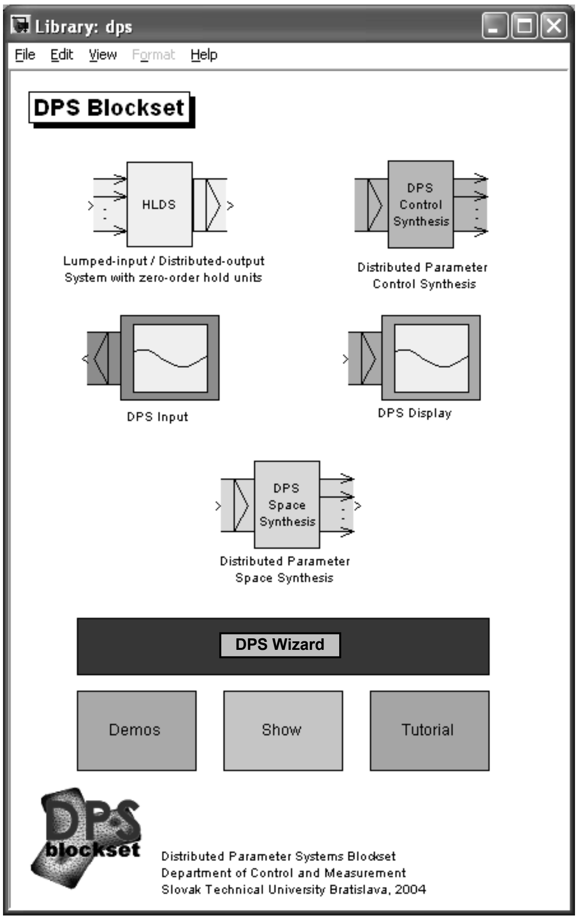

Fig. 11. DPS Blockset for MATLAB & Simulink

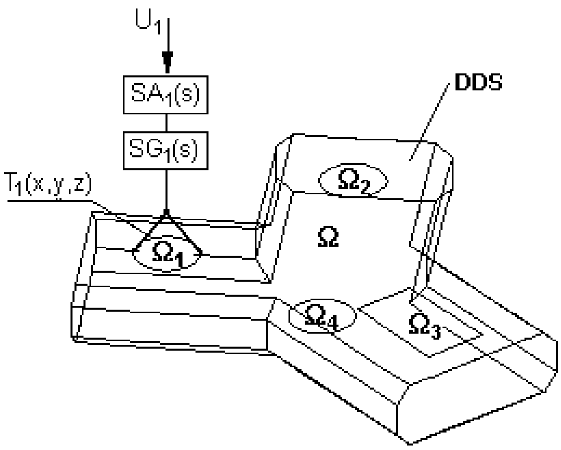

Fig. 12. Heating of 3D metal body

$\{SA_i(s)\}_i$ - actuating members with lumped input variables, $\{SG_i(s)\}_i$ - generators of distributed input variables, $\{T_i(x,y,z)\}_i$ - shaping units in space domain on $\{\Omega_i\}_i$, DDS - distributed-input/distributed-output system on the Ω, $\{U_i\}_i$ – lumped input quantities

blocks HLDS, DPS Control Synthesis,… based on time/space dynamical decomposition and synthesis offer flexible engineering methodology for DPS control. The DPS Blockset is created in open philosophy and enables solution of specified control tasks, practically in any field of technical practice. The procedure of the control design will be shown for heating of the semi-product with complex shape, see Fig. 12. First, by numerical methods of the dynamical analysis, distributed step responses of the controlled system are determined. In this case, by means of the software product FEMLAB for step inputs $\{U_i\}_i$, discrete distributed step responses $\{H_i(x_j,y_j,z_j,k)\}_{j,\,i}$ are computed in nodes $\{x_j,y_j,z_j\}_{j=1,m}$ at chosen sampling period T. The geometry model is given by the matrix of node points **M** and matrix of serial numbers of the elementary geometric formations vertexes **E**, which appears by using of the finite element method. Distributed thermal field is represented by temperatures computed at node points of the numerical net. In this case it means 2160 - components vector at node points and values between node points are computed by spline functions. After decomposition of the dynamics in the time direction, transfer functions among step inputs $\{U_i\}_i$ and corresponding partial distributed step responses $\{H_i(x_i, y_i, z_i, k)\}_{i=1,n}$ in computational points $\{x_i, y_i, z_i\}_i$, where amplitudes of partial responses reach of maximal amplitudes, are identified: $\{SH_i(x_i, y_i, z_i, z)\}_{i=1,n}$. Space components of dynamics – reduced distributed transient responses in steady-states $\{\mathcal{H}HR_i(x,y,z,\infty)\}_{i=1,n}$ are determined from distributed step responses $\{\mathcal{H}H_i(x,y,z,\infty)\}_{i=1,n}$. Using these characteristics is possible in Simulink environment to configure the distributed parameter system of control by means of blocks of the DPS Blockset, see Fig. 13. Further, in the Fig. 14 control process simulation results are presented.

Interactive Control or **Service** of LDScontrol on web site www.dpscontrol.sk offers possibilities for solution of model control problems via internet. Now let us indicate a procedure what interested person can use in solution of this demo example via internet. First, he specifies the model of geometry Ω, $\{\Omega_i\}_i$ as matrix from coordinates of mesh nodes - **M** and matrix **E** of serial numbers of vertexes of basic geometry elements of numerical scheme in counterclockwise direction. Further, he computes step responses $\{\mathbf{H}_i = [H_i(x_j,y_j,z_j,k)]_j\}_i$ caused by step inputs $\{U_i\}_i$ at chosen sampling period T. Then he chooses a reference quantity $W(x,y,z,\infty)$.

This formulation of control problem
» geometry model - **M**, **E**
» transient responses - matrices $\{\mathbf{H}_i\}_i$, $\{U_i\}_i$, T
» reference quantity - $W(x,y,z,\infty)$
he will send to the specified FTP server and results of

control process he will obtain again via internet in the form of Fig. 14.

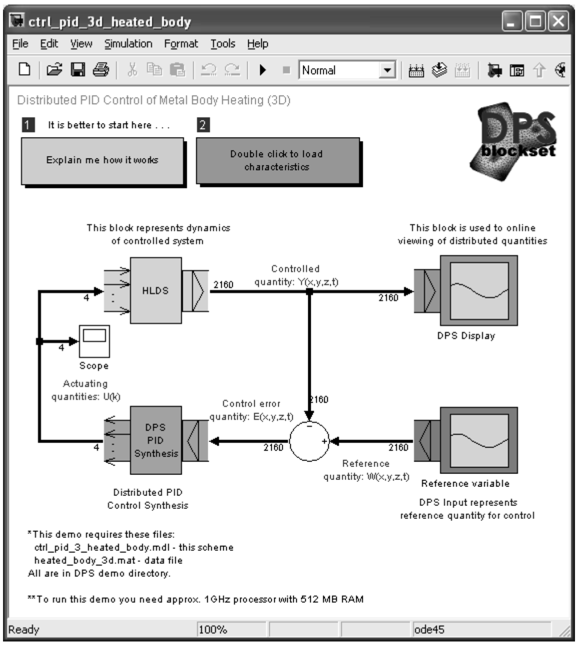

Fig. 13. Distributed parameter control loop

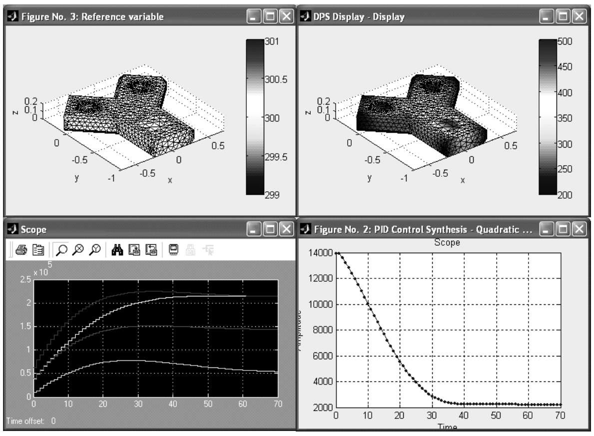

Fig. 14. Process of control. a/ W(x,y,z,∞) - distributed reference quantity; b/ Y(x,y,z,t) - distributed controlled quantity; c/ $\{U_i(t)\}_i$ - actuating variables; d/ E(t) - quadratic norm of distributed control error E(x,y,z,t)

6. CONCLUSION

Grandiose development of information technologies supports further wide-ranging distribution of diverse methods and software products for 3D numerical dynamical analysis of real systems as distributed parameter systems in any field of technical practice. Nowadays, these sophisticated dynamical analysis methods and tools are enjoying a boom.... This paper tries to react to this new important trend and challenge of engineering practice to control these systems with flexible engineering methods and tools of control design for distributed parameter systems.

Finally - in general the paper indicates a new direction for engineering control technology, operating mainly on the time axis, to expand towards time/space coordinates of real world.

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