ROBUST STABILIZATION OF SINGULAR STOCHASTIC SYSTEMS WITH DELAYS

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Abstract: This paper deals with a class of continuous-time uncertain singular linear systems with Markovian jump parameters and time delays. Sufficient conditions on stochastic stability and stochastic stabilizability are developed. A design algorithm for a state-feedback controller which guarantees that the closed-loop dynamic will be regular, impulse free and robustly stochastically stable is proposed in terms of the solutions to a set of coupled linear matrix inequalities.

Keywords: Jump linear systems, linear matrix inequality (LMI), singular systems, state feedback, stochastic stability, stochastic stabilizability.

1. INTRODUCTION

In the past decades, there have been considerable research efforts on the study of singular systems. This is due to the extensive applications of singular systems in many practical systems, such as circuits boundary control systems, chemical processes, and other areas (Dai, 1989; Kumar and Daoutidis, 1995). Singular systems are also referred to as descriptor systems, implicit systems, generalized state-space systems, differential-algebraic systems or semi-state systems (Dai, 1989). A great number of fundamental notions and results in control and systems theory based on state-space approach have been successfully extended to singular systems; see, e.g., (Takaba *et al.*, 1995; Verghese *et al.*, 1981; Xu *et al.*, 2002*a*; Fridman, 2001; Lan and Huang, 2003; Xu *et al.*, 2002*b*), and the references therein.

Recently, a class of stochastic systems driven by continuous-time Markov chains has been used to model many practical systems, where random failures and repairs and sudden environment changes may occur. For more detail, we refer the reader to (Boukas and Liu, 2002), (Mariton, 1991) and the references therein. This motivates the study of Markovian jump systems. For example, sufficient conditions on stochastic stability and stabilization for such systems were reported in (Feng *et*

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al., 1992; Ji and Chizeck, 1990; Mao, 1999; Boukas and Hang, 1999; Boukas and Liu, 2001*b*) via different approaches. The H_∞ control problem was investigated in (de Souza and Fragoso, 1996; Shi and Boukas, 1997), where sufficient conditions for the solvability of this problem were proposed. When time delays appear in a Markovian jump system, the results on stability analysis and H_∞ control were reported in (Boukas *et al.*, 2001), (Cao and Lam, 2000), and (Boukas and Liu, 2001*a*) for different types of time delays. For more detail on Markovian jumping systems with time delay, we refer the reader to (Boukas and Liu, 2002) and the references therein. However, up to date singular systems with Markovian jump parameters and time delays has not yet been fully investigated.

This paper is concerned with the problems of robust stability analysis and robust stabilization for uncertain singular Markovian jump systems with time delays in the system state. In terms of a set of coupled linear matrix inequalities, we present first a sufficient condition, which guarantees regularity, absence of impulses and robust stochastic stability of such systems. Based on this, a sufficient condition for the existence of the state-feedback controller ensuring regularity, absence of impulses and robust stochastic stability is proposed.

Notation: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "^{T}" denotes matrix transposition and the notation $X \geq Y$(respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrices with compatible dimensions. $\mathrm{E}(\cdot)$ denotes the expectation operator with respective to some probability measure P. $\|\cdot\|$ denotes the Euclidean norm for vectors.

2. PROBLEM STATEMENT

Consider the following class of uncertain singular Markovian jump linear systems defined on a complete probability space $(\Omega, \mathcal{F}, \mathrm{P})$:

$$\begin{aligned} E\dot{x}(t) &= A(r_t,t)x(t) + A_1(r_t,t)x(t-h) \\ &\quad + B(r_t,t)u(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the control input, $h > 0$ is a constant, and represents the time-delay in the system state. The mode switching process is assumed to be governed by a continuous-time discrete-state Markov process $\{r_t : t \geq 0\}$ taking values in a finite state space $\mathcal{S} \triangleq \{1, 2, \ldots, s\}$ and having the mode transition probabilities

$$\begin{aligned} &\Pr(r_{t+\Delta} = j | r_t = i) \\ &= \begin{cases} \pi_{ij}\Delta + o(\Delta), & \text{if } j \neq i \\ 1 + \pi_{ii}\Delta + o(\Delta), & \text{if } j = i \end{cases} \end{aligned} \tag{2}$$

where $\Delta > 0$, and $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0$, and $\pi_{ij} \geq 0$ $(i, j \in \mathcal{S}, j \neq i)$ denotes the switching rate from mode i to mode j and $\pi_{ii} \triangleq -\sum_{j=1,j\neq i}^{s} \pi_{ij}$. The matrix $\Pi \triangleq (\pi_{ij}) \in \mathbb{R}^{s\times s}$ is known as the mode transition rate matrix. The initial condition of the system is specified as $(r_0, \phi(\cdot))$, where $r_0 \in \mathcal{S}$ is the initial mode state and $\phi(\cdot)$ is the initial functional such that $x(s) = \phi(s), s \in [-h, 0]$.

For each mode $i \in \mathcal{S}$, the system matrices $A_i(t) \triangleq A(r_t = i, t)$, $A_{1i}(t) \triangleq A_1(r_t = i, t)$ and $B_i(t) \triangleq B(r_t = i, t)$ are assumed to have the norm-bounded uncertainties of the form

$$\begin{aligned} &\left[A_i(t)\ A_{1i}(t)\ B_i(t)\right] \\ &= \left[A_i\ A_{1i}\ B_i\right] + E_iF_i(t)\left[H_{ai}\ H_{a1i}\ H_{bi}\right] \end{aligned} \tag{3}$$

where matrices A_i, A_{1i}, B_i, E_i, H_{ai}, H_{a1i} and H_{bi} are known constant real matrices with appropriate dimensions, while $F_i(t)$ denotes the uncertainties in these system matrices and satisfies $F_i^T(t)F_i(t) \leq I$ for all $i \in \mathcal{S}$. The uncertainties that satisfy the conditions (3) are referred to as admissible uncertainties. The matrix E with $0 \leq \mathrm{rank}E \leq n$ may be singular.

Definition 1. (Dai, 1989)

i. The nominal singular Markovian jump linear system of uncertain system (1) with $u(t) \equiv 0$ is said to be regular if $\det(\lambda E - A_i)$ is not identically zero for each mode $i \in \mathcal{S}$.
ii. The nominal singular Markovian jump linear system of uncertain system (1) with $u(t) \equiv 0$ is said to be impulse free if $\deg(\det(\lambda E - A_i)) = \mathrm{rank}(E)$ for each mode $i \in \mathcal{S}$.

For more details on other properties and the existence of the solution of system (1), we refer the reader to (Xu *et al.*, 2002*b*), and the references therein. In general, regularity is often a sufficient condition for the analysis and the synthesis of singular systems.

For uncertain system (1), we have the following definitions.

Definition 2. Uncertain system (1) with $u(t) \equiv 0$ is said to be robustly stochastically stable if there exists a constant $T(r_0, \phi(\cdot))$ such that

$$\mathrm{E}\left(\int_0^\infty \|x(t)\|^2 dt \mid r_0, \phi(\cdot)\right) \leq T(r_0, \phi(\cdot))$$

holds over all admissible uncertainties (3).

In this paper, we are interested in the design of a state-feedback control law

$$u(t) = K(r_t)x(t) \tag{4}$$

where $K_i \triangleq K(r_t = i) \in \mathbb{R}^{m\times n}$ is the controller to be determined for each $i \in \mathcal{S}$.

Substituting the controller (4) into the uncertain system (1), we obtain the closed-loop system

$$E\dot{x}(t) = A_{cl}(r_t, t)x(t) + A_1(r_t, t)x(t-h) \quad (5)$$

where $A_{cli}(t) = (A_i + B_iK_i) + E_iF_i(t)(H_{ai} + H_{bi}K_i)$ for all $i \in \mathcal{S}$.

Definition 3. Uncertain singular Markovian jump linear system (1) is said to be robustly stabilizable in the stochastic sense if there exists a control law (4) such that the closed-loop system (5) is robustly stochastically stable.

This paper studies the stochastic stability and the stochastic stabilizability of the class of the uncertain singular Markovian jump linear systems (1). Our goal is to design a state-feedback controller (4) guaranteing that the closed-loop (5) is regular, impulse free and robustly stochastically stable. In the rest of this paper, we will assume that all the required assumptions are satisfied, i.e. the complete access to the system mode and state. Our methodology in this paper will be mainly based on the Lyapunov theory and some algebraic results. The conditions we will develop here will be in terms of the solutions to coupled linear matrix inequalities that can be easily solved using LMI control toolbox.

Lemma 1. (Boukas *et al.*, 2004) The nominal system of the uncertain singular Markovian jump system (1) with $u(t) \equiv 0$ is regular, impulse free and stochastically stable if there exist matrices $P_i \in \mathbb{R}^{n\times n}$, $i \in \mathcal{S}$, and a matrix $Q \in \mathbb{R}^{n\times n}$ with $Q = Q^T > 0$ such that the coupled LMIs

$$E^TP_i = P_i^TE \geq 0$$

$$\begin{bmatrix} \begin{pmatrix} P_i^TA_i + A_i^TP_i \\ +Q + \sum_{j=1}^{s} \pi_{ij}E^TP_j \end{pmatrix} & P_i^TA_{1i} \\ A_{1i}^TP_i & -Q \end{bmatrix} < 0$$

hold for each $i \in \mathcal{S}$.

Lemma 2. Given real matrices Q, E and H of appropriate dimensions with $Q = Q^T$, then

$$Q + EF(t)H + (EF(t)H)^T < 0$$

holds for all $F(t)$ satisfying $F^T(t)F(t) \leq I$ if and only if there exists some real number $\lambda > 0$ such that

$$Q + \lambda H^TH + \frac{1}{\lambda}EE^T < 0$$

3. MAIN RESULTS

In this section, we will develop results that assure that the uncertain system (1) with $u(t) \equiv 0$ is regular, impulse free and robustly stochastically stable. We will also design a state feedback controller of the form (4) that guarantees the same goal.

The result which guarantees that the uncertain singular system (1) with $u(t) \equiv 0$ is regular, impulse free and robustly stochastically stable is summarized by the following result.

Theorem 1. The uncertain singular Markovian jump linear system (1) with $u(t) \equiv 0$ is regular, impulse free and robustly stochastically stable if there exist matrices $P_i \in \mathbb{R}^{n\times n}$, positive scalars $\lambda_i > 0$, $i \in \mathcal{S}$, and a matrix $Q \in \mathbb{R}^{n\times n}$ with $Q = Q^T > 0$ such that the coupled LMIs

$$E^TP_i = P_i^TE \geq 0 \quad (6)$$

$$\begin{bmatrix} Q_{1i} & P_i^TA_{1i} + \lambda_iH_{ai}^TH_{a1i} & P_i^TE_i \\ A_{1i}^TP_i + \lambda_iH_{a1i}^TH_{ai} & -Q + \lambda_iH_{a1i}^TH_{a1i} & 0 \\ E_i^TP_i & 0 & -\lambda_iI \end{bmatrix} < 0 \quad (7)$$

hold for all $i \in \mathcal{S}$, where

$$Q_{1i} = P_i^TA_i + A_i^TP_i + Q + \sum_{j=1}^{s} \pi_{ij}E^TP_j + \lambda_iH_{ai}^TH_{ai}$$

PROOF. Based on Lemma 1, the uncertain system (1) with $u(t) \equiv 0$ is regular, impulse free and robustly stochastically stable if there exist matrices $P_i \in \mathbb{R}^{n\times n}$, $i \in \mathcal{S}$, and a matrix $Q \in \mathbb{R}^{n\times n}$ with $Q = Q^T > 0$ such that both LMI (6) and

$$\begin{bmatrix} \begin{pmatrix} P_i^TA_i(t) + A_i^T(t)P_i \\ +Q + \sum_{j=1}^{s} \pi_{ij}E^TP_j \end{pmatrix} & P_i^TA_{1i}(t) \\ A_{1i}^T(t)P_i & -Q \end{bmatrix} < 0$$

hold for each $i \in \mathcal{S}$. Noting that the form of the uncertainties (3), the above inequality can be rewritten as

$$\begin{bmatrix} \begin{pmatrix} P_i^TA_i + A_i^TP_i \\ +Q + \sum_{j=1}^{s} \pi_{ij}E^TP_j \end{pmatrix} & P_i^TA_{1i} \\ A_{1i}^TP_i & -Q \end{bmatrix} + \begin{bmatrix} P_i^TE_i \\ 0 \end{bmatrix} F_i(t) \begin{bmatrix} H_{ai} & H_{a1i} \end{bmatrix} + \begin{bmatrix} H_{ai}^T \\ H_{a1i}^T \end{bmatrix} F_i^T(t) \begin{bmatrix} E_i^TP_i & 0 \end{bmatrix} < 0$$

According to Lemma 2, the above inequality holds for all $F_i(t)$ satisfying $F_i^T(t)F_i(t) \leq I$ if and only if there exists a real number $\lambda_i > 0$ such that

$$\begin{bmatrix} \begin{pmatrix} P_i^T A_i + A_i^T P_i \\ +Q + \sum_{j=1}^{s} \pi_{ij} E^T P_j \end{pmatrix} & P_i^T A_{1i} \\ A_{1i}^T P_i & -Q \end{bmatrix} + \frac{1}{\lambda_i} \begin{bmatrix} P_i^T E_i E_i^T P_i & 0 \\ 0 & 0 \end{bmatrix} + \lambda_i \begin{bmatrix} H_{ai}^T \\ H_{a1i}^T \end{bmatrix} \begin{bmatrix} H_{ai} & H_{a1i} \end{bmatrix} < 0$$

In view of Schur complement equivalence, the above inequality is equivalent to LMI (7). This completes the proof. □

To design the state feedback controller of the form (4) which assures that the uncertain closed-loop system (5) is regular, impulse free and robustly stochastically stable, we have the following result.

Theorem 2. Consider uncertain singular Markovian jump linear system (1), there exists a state-feedback controller (4) such that the closed-loop system (5) is regular, impulse free and robustly stochastically stable if there exist matrices $X_i \in \mathbb{R}^{n\times n}$, $W_i \in \mathbb{R}^{n\times n}$ with $W_i = W_i^T$, positive scalars $\alpha_i > 0$, $i \in \mathcal{S}$, and a matrix $Z \in \mathbb{R}^{n\times n}$ with $Z = Z^T > 0$ such that the coupled LMIs

$$0 \le (EX_i)^T = EX_i < W_i \tag{8}$$

$$\begin{bmatrix} Q_{2i} & M_{1i}^T & X_i^T & M_{2i} \\ M_{1i} & -\alpha_i + H_{ai} Z H_{ai}^T & 0 & 0 \\ X_i & 0 & -Z & 0 \\ M_{2i}^T & 0 & 0 & -Q_{3i} \end{bmatrix} < 0 \tag{9}$$

hold for each $i \in \mathcal{S}$, where

$$\begin{aligned} Q_{2i} &= (A_i X_i + B_i Y_i) + (A_i X_i + B_i Y_i)^T \\ &\quad + \pi_{ii} X_i^T E_i + \alpha_i E_i E_i^T + A_{1i} Z A_{1i}^T \\ M_{1i} &= H_{ai} X_i + H_{bi} Y_i + H_{a1i} Z A_{1i}^T \\ M_{2i} &= \left[\sqrt{\pi_{i1}} X_i^T \ \cdots \ \sqrt{\pi_{i(i-1)}} X_i^T \ \sqrt{\pi_{i(i+1)}} X_i^T \right. \\ &\quad \left. \cdots \ \sqrt{\pi_{is}} X_i^T \right] \\ Q_{3i} &= \mathrm{diag}(X_1 + X_1^T - W_1, \ldots, X_{i-1} + X_{i-1}^T - W_{i-1}, \\ &\quad X_{i+1} + X_{i+1}^T - W_{i+1}, \ldots, X_s + X_s^T - W_s) \end{aligned}$$

In this case, a stabilizing controller (4) is given by $K_i = Y_i X_i^{-1}$, $i \in \mathcal{S}$.

PROOF. Similarly to the proof of Theorem 1, the closed-loop system (5) is regular, impulse free and robustly stochastically stable if there exist matrices $P_i \in \mathbb{R}^{n\times n}$, $i \in \mathcal{S}$, and a matrix $Q \in \mathbb{R}^{n\times n}$ with $Q = Q^T > 0$ such that the coupled LMIs

$$E^T P_i = P_i^T E \ge 0 \tag{10}$$

$$\begin{bmatrix} \bar{Q}_{1i} & P_i^T A_{1i} + \lambda_i \bar{H}_{ai}^T H_{a1i} & P_i^T E_i \\ A_{1i}^T P_i + \lambda_i H_{a1i}^T \bar{H}_{ai} & -Q + \lambda_i H_{a1i}^T H_{a1i} & 0 \\ E_i^T P_i & 0 & -\lambda_i I \end{bmatrix} < 0 \tag{11}$$

hold for each $i \in \mathcal{S}$, where

$$\bar{Q}_{1i} = P_i^T \bar{A}_i + \bar{A}_i^T P_i + Q + \sum_{j=1}^{s} \pi_{ij} E^T P_j + \lambda_i \bar{H}_{ai}^T \bar{H}_{ai}$$

$$\bar{A}_i = A_i + B_i K_i$$
$$\bar{H}_{ai} = H_{ai} + H_{bi} K_i$$

Pre- and post-multiply both sides of the LMI (10) by P_i^{-T} and P_i^{-1}, respectively, and define $X_i \triangleq P_i^{-1}$, we have that LMI (10) is equivalent to

$$(EX_i)^T = EX_i \ge 0 \tag{12}$$

Pre- and post-multiply both sides of the LMI (11), respectively, by $\mathrm{diag}(P_i^{-T}, I, I)$ and $\mathrm{diag}(P_i^{-1}, I, I)$ and define $Y_i \triangleq K_i P_i^{-1}$, we have that LMI (11) is equivalent to

$$\begin{bmatrix} \tilde{Q}_{1i} & A_{1i} + \lambda_i (H_{ai} X_i + H_{bi} Y_i)^T H_{a1i} & E_i \\ * & -Q + \lambda_i H_{a1i}^T H_{a1i} & 0 \\ * & * & -\lambda_i I \end{bmatrix} < 0$$

where

$$\begin{aligned} \tilde{Q}_{1i} &= X_i^T \bar{Q}_{1i} X_i \\ &= (A_i X_i + B_i Y_i) + (A_i X_i + B_i Y_i)^T + X_i^T Q X_i \\ &\quad + \pi_{ii} X_i^T E^T + \sum_{j=1, j\ne i}^{s} \pi_{ij} X_i^T E^T X_j^{-1} X_i \\ &\quad + \lambda_i (H_{ai} X_i + H_{bi} Y_i)^T (H_{ai} X_i + H_{bi} Y_i) \end{aligned}$$

Define $\alpha_i \triangleq \frac{1}{\lambda_i}$, and in view of Schur complement equivalence, the above inequality is equivalent to

$$\begin{bmatrix} \hat{Q}_{1i} & A_{1i} & (H_{ai} X_i + H_{bi} Y_i)^T \\ A_{1i}^T & -Q & H_{a1i}^T \\ H_{ai} X_i + H_{bi} Y_i & H_{a1i} & -\alpha_i I \end{bmatrix} < 0$$

where

$$\begin{aligned} \hat{Q}_{1i} &= (A_i X_i + B_i Y_i) + (A_i X_i + B_i Y_i)^T + X_i^T Q X_i \\ &\quad + \pi_{ii} X_i^T E^T + \sum_{j=1, j\ne i}^{s} \pi_{ij} X_i^T E^T X_j^{-1} X_i \\ &\quad + \alpha_i E_i E_i^T \end{aligned}$$

Pre- and post-multiply both sides of the above inequality by

$$\begin{bmatrix} I & 0 & 0 \\ 0 & 0 & I \\ 0 & I & 0 \end{bmatrix}$$

we have

$$\begin{bmatrix} \hat{Q}_{1i} & (H_{ai} X_i + H_{bi} Y_i)^T & A_{1i} \\ H_{ai} X_i + H_{bi} Y_i & -\alpha_i I & H_{a1i} \\ A_{1i}^T & H_{a1i}^T & -Q \end{bmatrix} < 0$$

Define $Z \triangleq Q^{-1}$, the above inequality is equivalent to

$$\begin{bmatrix} \hat{Q}_{1i} + A_{1i} Z A_{1i}^T & M_{1i}^T \\ M_{1i} & -\alpha_i I + H_{a1i} Z H_{a1i}^T \end{bmatrix} < 0 \tag{13}$$

where M_{1i} is given in Theorem 2. Note that for each $j \in \mathcal{S}$, from LMIs (8) and (9), we have

$$0 \le EX_j = X_j^T E^T < W_j < X_j + X_j^T$$

On the other hand, from the inequality

$$(X_j W_j^{-\frac{1}{2}} - W_j^{\frac{1}{2}})(X_j W_j^{-\frac{1}{2}} - W_j^{\frac{1}{2}})^T \ge 0$$

we conclude that

$$X_j W_j^{-1} X_j^T \geq X_j + X_j^T - W_j > 0$$

So we have

$$E^T X_j^{-1} \leq X_j^{-T} W_j X_j^{-1} = (X_j W_j^{-1} X_j^T)^{-1} \\ \leq (X_j + X_j^T - W_j)^{-1}$$

Therefore, for any $i, j \in \mathcal{S}, j \neq i$, the following inequality holds:

$$\pi_{ij} X_i^T E^T X_j^{-1} X_i \leq \pi_{ij} X_i^T (X_j + X_j^T - W_j)^{-1} X_i$$

Hence, we obtain

$$\sum_{j=1, j\neq i}^{s} \pi_{ij} X_i^T E^T X_j^{-1} X_i \\ \leq \sum_{j=1, j\neq i}^{s} \pi_{ij} X_i^T (X_j + X_j^T - W_j)^{-1} X_i \\ = M_{2i} Q_{3i}^{-1} M_{2i}^T$$

In view of Schur complement equivalence, we conclude that inequality (13) holds if the LMI (9). This completes the proof. □

Remark 1. The results we developed in this paper extend those developed for the deterministic case on stability and stabilizability. In fact, if we have only one model, that is, $s = 1$, the results reduce to those of deterministic singular linear systems with time delays.

4. CONCLUSION

This paper dealt with the class of continuous-time uncertain singular linear systems with Markovian jumps and time-delays in the state vector. Results on stochastic stability and its robustness, and stochastic stabilizability and its robustness are developed. The LMI framework is used to establish the sufficient conditions on stability and stabilizability. The conditions we developed are delay independent. The results we developed can easily be solved using any LMI toolbox like the one of Matlab or the one of Scilab.

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