CASE STUDIES OF COEFFICIEANT DIAGRAM METHOD
- PRACTICAL POLYNOMIAL DESIGN APPROACHES -

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Abstract: Coefficient Diagram Method (CDM) is one of polynomial methods in control design. Its design effectiveness mainly stems from the usage of a diagram called Coefficient Diagram. Coefficient diagram shows the coefficients of characteristics polynomial and those of numerator polynomials corresponding to sensitivity and auxiliary sensitivity function in logarithmic scale, where the abscissa is the order for the coefficients. From the shape, designer can visualize the stability, response, and robustness. A well-known difficult benchmark problem is solved to demonstrate the effectiveness of CDM.

Keywords: Control system design, Control theory, Polynomials, Benchmark examples, Stability limit.

1. INTRODUCTION

Coefficient Diagram Method (CDM) is an algebraic design approach based on polynomials and polynomial matrices (Manabe, 1998b, 2002). It has five features as follows;

(1) Polynomials and polynomial matrices are used for system representation.
(2) Characteristic polynomial and controller are simultaneously designed.
(3) Coefficient diagram is effectively utilized.
(4) The sufficient condition for stability by Lipatov constitutes the theoretical basis of CDM (Lipatov and Sokolov, 1978; Manabe, 1999).
(5) Kessler standard form (Kessler, 1960) is improved and used as the standard form of CDM.

CDM design is based on the stability index and equivalent time constant. Thus for the specified settling time, a controller of the lowest order with the narrowest bandwidth and of no-overshoot can be easily designed. CDM can be considered as "Generalized PID", because the controller can be more complex than PID, and more reliable parameter selection rules are provided. Also CDM can be considered as "Improved LQG", because the order of controller is smaller and weight selection rules are also given (Manabe, 1998a).

Ordinary design problems are effectively solved and were reported in various literatures (Hori, 1994; Manabe, 1994b, 2002). Also some more difficult problems have been solved, too. Manabe (1994a) designed the controller for a single-sensor inverted pendulum, and demonstrated its robust operation by a small toy car model with the pendulum, which was run by an inexpensive hand-made controller. Manabe (1997) worked on the ACC benchmark problem (Wie and Bernstein, 1992; Thompson, 1995). This problem requires attaining a specified settling time under the actuator input limitation for various robustness conditions. Effective trade-off was important at such problem and CDM was found to be very effective in that respect.

However, the true power of CDM will be demonstrated in the design of most difficult problems, which defy the most advanced robust control theory. There is no practical importance in designing controllers for such academic problems. But by comparing the results by advanced robust control theory and ones by CDM, the robust control theory itself will be tested and evaluated.

The purpose of this paper is to design controllers for the well-known difficult problem; the plant with a non-minimum phase zero and an unstable pole. In the design process, the relation between the robustness and the controller order will be clarified. Also the mechanism of stability-robustness trade-off, which is not well treated in robust control theory, will be explained.

Section 2 is an introduction to CDM for tutorial purpose. Section3 is the main results of this paper. Various orders of controllers are designed for different robustness specifications.

2. BASICS OF CDM

2.1 Mathematical Model

The standard block diagram of the CDM design for a single-input-single-output system is shown in Fig. 1. The extension to multi-input-multi-output can be made with proper interpretation, but it is not discussed here for simplicity. The plant equation is given as

$$A_p(s)x = u + d\,, \quad y = B_p(s)x\,, \tag{1}$$

where u, y, and d are input, output, and disturbance. The symbol x is called the basic state variable. $A_p(s)$ and $B_p(s)$ are the denominator and numerator polynomial of the plant transfer function $G_p(s)$. It will be easily seen that this expression has a direct correspondence with the control canonical form of the state-space expression, and x corresponds to the state variable of the lowest order. All the other states are expressed as the derivatives of x of high order. Controller equation is given as

$$A_c(s)u = B_a(s)y_r - B_c(s)(y+n)\,, \tag{2}$$

where y_r and n are reference input and noise on the output. $A_c(s)$ is the denominator of the controller transfer function. $B_a(s)$ and $B_c(s)$ are called the reference numerator and feedback numerator of the controller transfer function. Because the controller transfer function has two numerators, it is called two-degree-of-freedom system. This expression corresponds to the observer canonical form of the state-space expression. Elimination of y and u from Eq. (2) by Eq. (1) gives

$$P(s)x = B_a(s)y_r + A_c(s)d - B_c(s)n\,, \tag{3}$$

where $P(s)$ is the characteristic polynomial and given as

$$P(s) = A_c(s)A_p(s) + B_c(s)B_p(s)\,. \tag{4}$$

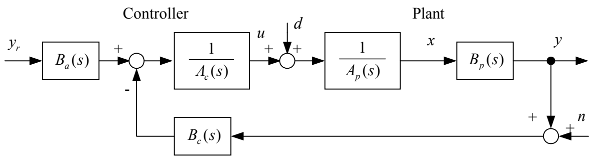


Fig. 1. Mathematical model

By Eq. (1), y and u are expressed in x as follows;

$$y = B_p(s)x\,,\ u = A_p(s) - d \tag{5}$$

2.2 Mathematical Relations

Some mathematical relations extensively used in CDM will be introduced hereafter. The characteristic polynomial is given in the following form.

$$P(s) = a_n s^n + \ldots + a_1 s + a_0 = \sum_{i=0}^{n} a_i s^i\,. \tag{6}$$

The stability index γ_i, the equivalent time constant τ, and stability limit γ_i^* are defined as follows.

$$\gamma_i = a_1^2/(a_{i+1}a_{i-1})\,, \quad i = 1 \sim n-1\,, \tag{7}$$

$$\tau = a_1/a_0\,, \tag{8}$$

$$\gamma_i^* = 1/\gamma_{i+1} + 1/\gamma_{i-1}\,,$$
$$i = 1 \sim n-1, \quad \gamma_n = \gamma_0 = \infty\,. \tag{9}$$

Also the equivalent time constant of the i-th order τ_i is defined as follows;

$$\tau_i = a_{i+1}/a_i\,, \qquad i = 1 \sim n-1\,. \tag{10}$$

Then the following relations are derived.

$$\tau_i = \tau_{i-1}/\gamma_i = \tau/(\gamma_i \cdots \gamma_2\gamma_1)\,, \tag{11}$$

$$a_i = \tau_{i-1} \ldots \tau_2\tau_1\tau \quad a_0\,, \tag{12}$$

$$= a_0\tau^i / \ (\gamma_{i-1}\gamma_{i-2}^2 \cdots \gamma_2^{i-2}\gamma_1^{i-1})\,. \tag{13}$$

The characteristic polynomial will be expressed by a_0, τ, and γ_i as follows.

$$P(s) = a_0[\{\sum_{i=2}^{n}(\prod_{j=1}^{i-1} 1/\gamma_{i-j}^{j})(\tau s)^i\} + \tau s + 1]\,. \tag{14}$$

In CDM standard form, $\gamma_1 = 2.5$ and the rest of γ_i s are all 2. Then $P(s)$ is expressed in a simple form.

$$P(s) = a_0[10^{-(n-1)} \times 2^{-(n-1)(n-6)/2}(\tau s)^n + \cdots$$
$$+0.00001(\tau s)^6 + 0.0004(\tau s)^5 + 0.008(\tau s)^4$$
$$+0.08(\tau s)^3 + 0.4(\tau s)^2 + \tau s + 1] \tag{15}$$

2.3 Coefficient Diagram

When the plant/controller polynomials are given as

$$A_p(s) = 0.25s^4 + s^3 + 2s^2 + 0.5s, \quad B_p(s) = 1,$$
$$A_c(s) = l_1 s, \quad B_c(s) = k_2 s^2 + k_1 s + k_0, \tag{16}$$
$$l_1 = 1, \quad k_2 = 1.5, \quad k_1 = 1, \quad k_0 = 0.2,$$

the characteristic polynomial is expressed as

$$P(s) = 0.25s^5 + s^4 + 2s^3 + 2s^2 + s + 0.2\,. \tag{17}$$

Then

$$a_i = [a_5 \cdots a_2\ a_1] = [0.25\ \ 1\ \ 2\ \ 2\ \ 1\ \ 0.2], \tag{18}$$

$$\gamma_i = [\gamma_4 \cdots \gamma_2\ \gamma_1] = [2\ \ 2\ \ 2\ \ 2.5], \tag{19}$$

$$\tau = 5, \tag{20}$$

$$\gamma_i^* = [\gamma_4^* \cdots \gamma_2^*\ \gamma_1^*] = [0.5\ \ 1\ \ 0.9\ \ 0.5]. \tag{21}$$

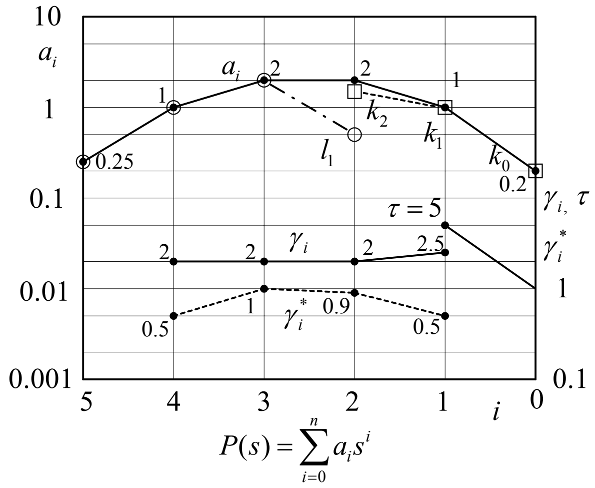


Fig. 2. Coefficient diagram

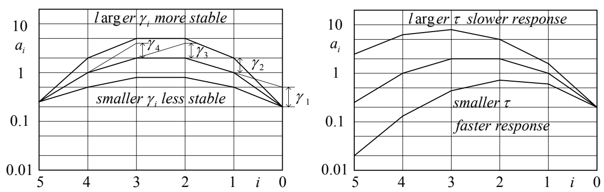


Fig. 3a. Effect of γ b. Effect of τ

The coefficient diagram is shown as in Fig. 2, where coefficient a_i is read by the left side scale, and stability index γ_i , equivalent time constant τ , and stability limit γ_i^* are read by the right side scale. The τ is expressed by a line connecting 1 to τ. The stability index γ_i can be graphically obtained (Fig. 3a). If the curvature of the a_i becomes larger (Fig. 3a), the system becomes more stable, corresponding to larger stability index γ_i. If the a_i curve is left-end down (Fig. 3b), the equivalent time constant τ is small and response is fast. The equivalent time constant τ specifies the response speed.

The coefficient diagram is also used for parameter sensitivity analysis and robustness analysis. In this example, the characteristic polynomial $P(s)$ is composed of two component polynomials; denominator polynomial $P_{l1}(s)$ and numerator polynomial $P_k(s)$.

$$P(s) = P_{l1}(s) + P_k(s) , \tag{22}$$

$$P_{l1}(s) = l_1(0.25s^5 + s^4 + 2s^3 + 0.5s^2) , \tag{23}$$

$$P_k(s) = k_2 s^2 + k_1 s + k_0 . \tag{24}$$

The auxiliary sensitivity function T(s) is expressed as

$$T(s) = P_k(s) / P(s) \tag{25}$$

Eq. (23) is shown in Fig. 2 with small circles and dash-dot lines. Eq. (24) is shown with small squares and dotted lines. Designer can visually assess the deformation of the coefficient diagram due to the parameter change of k_2, k_1, and k_0. Then he can visualize the variation of stability and response. Also from Eq. (25), it is clear that robustness can be analyzed by comparison of coefficients a_i and k_i at the coefficient diagram.

Thus the coefficient diagram indicates stability, response, and robustness (three major properties in control design) in a single diagram, enabling the designer to grasp the total picture of control system. At present, Bode diagram is used for this purpose. However coefficient diagram is more accurate and easy to use in actual design.

2.4 Stability Condition

From the Routh-Hurwitz stability criterion, the stability condition for the 3rd order is given as

$$a_2 a_1 > a_3 a_0 . \tag{26}$$

If it is expressed by stability index,

$$\gamma_2 \gamma_1 > 1 . \tag{27}$$

The stability condition for the fourth order is given as

$$a_2 > (a_1 / a_3) a_4 + (a_3 / a_1) a_0 \tag{28}$$

$$\gamma_2 > \gamma_2^* . \tag{29}$$

For the system higher than or including 5th degree, Lipatov (1978) gave the sufficient condition for stability and instability in several different forms. The conditions most suitable to CDM can be stated as follows;

"The system is stable, if all the partial 4th order polynomials are stable with the margin of 1.12. The system is unstable if some partial 3rd order polynomial is unstable."

Thus the sufficient condition for stability is given as

$$a_i > 1.12 \ [\ \frac{a_{i-1}}{a_{i+1}} a_{i+2} + \frac{a_{i+1}}{a_{i-1}} a_{i-2}] , \tag{30}$$

$$\gamma_i > 1.12 \gamma_i^* , \quad \text{for all} \quad i = 2 \sim n-2 . \tag{31}$$

The sufficient condition for instability is given as

$$a_{i+1} a_i \le a_{i+2} a_{i-1} , \tag{32}$$

$$\gamma_{i+1} \gamma_i \le 1 , \quad \text{for some} \quad i = 1 \sim n-2 . \tag{33}$$

These conditions can be graphically expressed in the coefficient diagram. Fig. 4a is a 3rd-order example. Point A is $(a_2\ a_1)^{0.5}$ and point B is $(a_3\ a_0)^{0.5}$. Thus if A is above B, the system is stable. Point C is $(\gamma_2\ \gamma_1)^{0.5}$. If it is above 1, the system is stable. Fig. 4b is a 4th-order example. Point A is obtained by drawing a line from a_4 in parallel with line $a_3 a_1$. Similarly point B is

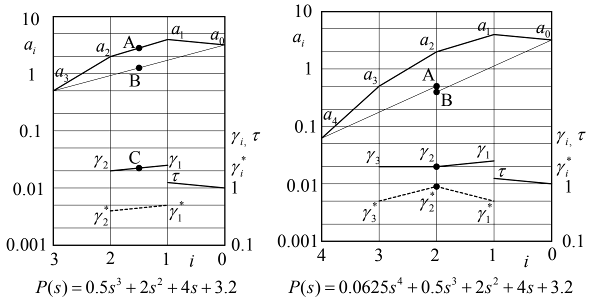


Fig. 4a. 3rd order b. 4th order

obtained by drawing a line from a_0 in parallel with line $a_3 a_1$. The stability condition is $a_2 > (A + B)$. The other condition is $\gamma_2 > \gamma_2^*$.

2.5 Design Condition

In CDM, the following stability index is recommended as the standard form.

$$\gamma_{n-1} = \cdots = \gamma_3 = \gamma_2 = 2, \quad \gamma_1 = 2.5 . \qquad (34)$$

When trade-off issue comes up, this is relaxed as follows to give the designer more design freedom.

$$\gamma_{n-1} = \gamma_2 = 2, \ \gamma_1 = 2.5,$$

$$\gamma_i > 1.5\gamma_i^*, \quad i = n-1 \sim 3 . \qquad (35)$$

Usually stability index is chosen in the following range.

$$\gamma_i = 1.5 \sim 4 . \qquad (36)$$

If all $\gamma_i \geq 1.5$, stability is guaranteed by Eq. (31). Lipatov (1978) proved that all poles are negative real, if all $\gamma_i > 4$. Then the system is overly stable. The equivalent time constant is chosen as follows;

$$\tau \simeq (1/3) \text{ of settling time.} \qquad (37)$$

3. CONTROLLER DESIGN OF NON-MINIMUM PHASE UNSTABLE PLANT

3.1 Problem Statement

The design of controller for non-minimum phase unstable plant is a cherished topic for control theorists. Although such problem is of little practical importance, it is a very good testing stone for the control theory. The following example is presented by (Henrion, et al., 2003a). He took the example from (Doyle, et al., 1992, Section 11.3).

Quote: We consider the problem of robustly stabilizing the plant

$$\frac{b(s,q)}{a(s,q)} = \frac{q(s-1)}{(s+1)(s-2)} . \qquad (38)$$

for all real gain q in the interval $[1, k_1]$. The uncertain plant polytope is therefore made of 2 vertices. In (Doyle, et al., 1992), it is shown that a robustly stabilizing controller (of arbitrarily high order) exists if and only if $k_1 < 4$. The design method proposed there is based on coprime factorisation and H-inf model matching. it is solved with the help of Navanlinna-Pick interpolation, which has the drawback of producing high-order controllers. In (Doyle, et al., 1992), a controller of eighth orders is computed for $k_1 = 3.5$. Unquote.

The problem is restated in CDM notations. The plant is given as follows;

$$A_p(s)x = u, \quad y = B_p(s)x , \qquad (39)$$

$$A_p(s) = (s+1)(s-2), \quad B_p(s) = q(s-1) .$$

the problem is to design a controller, which robustly stabilize for all real gains q in the interval $[1, q_{max}]$. The notation k_1 is changed to q_{max} in order to conform to CDM notation. In this problem, stability and robustness are of utmost importance, and performance is considered of secondary importance. The CDM design will proceeds from a simple controller to more sophisticated ones.

3.2 First Order Controller

The simplest controller, which stabilizes the plant, is first order. More precisely, it is a 1/1 order controller, where the numerator/denominator orders are 1/1. The controller is given as follows;

$$A_c(s)u = B_c(s)(y_r - y) , \qquad (40)$$

$$A_c(s) = l_1 s + l_0 ,$$

$$B_c(s) = B_{c1}(s)B_c(s), \ B_{c1}(s) = k_0, \ B_{c2}(s) = s + 1 .$$

$B_{c2}(s)$ is used to cancel the stable pole of the plant. Then the characteristic polynomial $P(s)$ becomes as follows;

$$P(s) = P_1(s)B_{c2}(s) , \qquad (41)$$

$$P_1(s) = (l_1 s + l_0)(s-2) + k_0 q(s-1) .$$

For stability, only $P_1(s)$ is important, and it will be treated as the characteristic polynomial hereafter. The coefficients are given as follows;

$$a_2 = l_1 , \qquad (42)$$

$$a_1 = -2l_1 + l_0 + k_0 q ,$$

$$a_0 = -2l_0 - k_0 q .$$

The stability condition for the second order system is that all coefficients are positive. In an ideal case, l_1 is almost 0. l_0= -1 is chosen arbitrarily. Then the system is stable for k_0 = 1 and q_{max} = 2. For relaxed condition, q_{max} = 1.99, the following parameters are selected.

$$l_1 = 0.001, \quad l_0 = -1, \quad k_0 = 1.004 . \qquad (43)$$

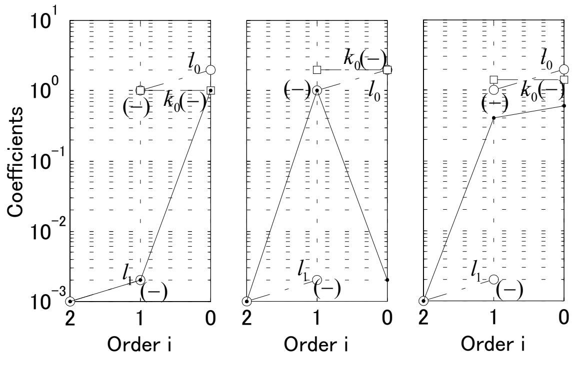
The stability is confirmed, because the coefficients fall in the following interval and are all positive.

$$a_2 = 0.001 , \qquad (44)$$

$$a_1 = -1.002 + 1.004q = [0.002, \ 0.99596] ,$$

$$a_0 = 2 - 1.004q = [0.996, \ 0.00204] .$$

The coefficient diagrams for q =1, 1.99, and 1.4 are shown in Fig. 5a, b, c. Fig. 5a suggests that the



a. $q = 1$ b. $q = 1.99$ c. $q = 1.4$

Fig. 5. Coefficient diagram, 1/1 order controller

system is very oscillatory for q =1, because of small γ_1. Fig. 5b suggests that the system is very slow, because of extremely large τ. Fig. 5c suggests that the system is fairly reasonable: a natural consequence that q is at the mid point. These figures show that a_1 and a_0 are the differences of two component polynomials related to l_0 and k_0. For this reason, $q_{\max}$ is limited to 2 in this design. The final form of the controller transfer function $G_c(s)$ is as follows;

$$G_c(s) = \frac{1.004(s+1)}{0.001s-1} \qquad (65)$$

Henrion (2003b) obtained the similar results by LMI.

3.3 Second Order Controller

By adopting a 2/2 order controller, range of q will be extended to almost 4. The controller is assumed as follows;

$$A_c(s)u = B_c(s)(y_r - y), \qquad (66)$$
$$A_c(s) = (l_1 s + l_0)(s + 2 - \varepsilon_1), \quad l_0 = -1,$$
$$B_c(s) = B_{c1}(s)B_c(s),$$
$$B_{c1}(s) = k_0(s + 1 - \varepsilon_2), \ B_{c2}(s) = s + 1.$$

Then the characteristic polynomial $P(s)$ becomes as follows;

$$P(s) = P_1(s)B_{c2}(s), \qquad (67)$$
$$P_1(s) = (l_1 s - 1)(s + 2 - \varepsilon_1)(s - 2) + k_0 q(s + 1 - \varepsilon_2)(s - 1)$$
$$= (l_1 s - 1)[s^2 - \varepsilon_1 s - (4 - 2\varepsilon_1)] + k_0 q[s^2 - \varepsilon_2 s - (1 - \varepsilon_2)].$$

As in the previous section, $P_1(s)$ will be treated as the characteristic polynomial hereafter. The coefficients are given as follows;

$$a_3 = l_1, \qquad (68)$$
$$a_2 = -\varepsilon_1 l_1 - 1 + k_0 q,$$
$$a_1 = \varepsilon_1 - (4 - 2\varepsilon_1)l_1 - k_0 q \varepsilon_2$$
$$a_0 = (4 - 2\varepsilon_1) - k_0 q(1 - \varepsilon_2).$$

In an ideal case, l_1 is close to 0. Then k_0 is almost 1 for a_2 to be marginally positive at q = 1. From the condition that a_1 and a_0 are marginally positive at $q = q_{\max}$, the following relation is derived.

$$\varepsilon_1 = 4 - q_{\max}, \quad \varepsilon_2 = \varepsilon_1 / q_{\max}. \qquad (69)$$

Thus any effort to make $q_{\max}$ close to 4, naturally makes ε_1 and ε_2 small. As the result, a_1 becomes very small, and the system becomes oscillatory with small γ_1. This deterioration is the necessary cost for larger $q_{\max}$. For $q_{\max}$ = 3.99, the following parameters are chosen.

$$\varepsilon_1 = 0.001, \ \varepsilon_2 = 0, \ l_1 = 2\times10^{-7}, \ k_0 = 1.002. \ (70)$$

The coefficients fall into the following intervals.

$$a_3 = 2\times10^{-7}, \qquad (71)$$
$$a_2 = -1 + 1.002q = [0.002, \ 2.9980],$$
$$a_1 = 0.00099920,$$
$$a_0 = 3.998 - 1.002q = [2.996, \ 0.00002].$$

Because the system is 3-rd order, the stability condition must be satisfied in addition to the condition that all coefficients are positive.

$$\gamma_2\gamma_1 = a_2 a_1 / (a_3 a_0) > 1. \qquad (72)$$

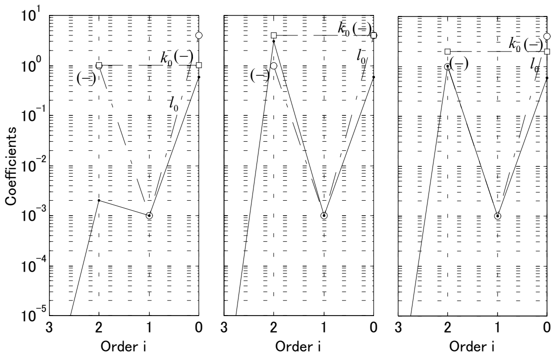
From Eq. (71), $\gamma_2\gamma_1$ is minimum at q = 1, and increases as q increases. Thus $\gamma_2\gamma_1$ falls into the following interval.

$$\gamma_2\gamma_1 = [3.3351, \ 7.4888\times10^8]. \qquad (73)$$

Thus stability for $q_{\max}$ = 3.99 is confirmed. The coefficient diagrams for q =1, 3.99, and 2 are shown in Fig. 6a, b, c. Now a_3 and a_1 are constant, and a_2 and a_0 vary by q. Compared with 1/1 order controller, a_0 allows larger variation of q and $q_{\max}$ becomes larger. The strategy taken here is to make a_3 and a_1 constant, and let a_2 and a_0 vary by q. The final form of the controller transfer function $G_c(s)$ is as follows;

$$G_c(s) = \frac{1.002(s+1)^2}{(2\times10^{-7}s - 1)(s + 1.999)}. \qquad (74)$$

The designed controller is not a good controller at all, as easily seen from the peculiar shape of the coefficient diagram. It is designed only to show $q_{\max}$ can be almost 4. Thus a second order controller with $q_{\max}$ = 3.99 is designed by CDM. Henrion, et al. (2003a) designed a 3-rd order controller with $q_{\max}$ = 3.5 by positive polynomial technique. The controller is more reasonable, because $q_{\max}$ is smaller.



a. q = 1 b. q = 3.99 c. q = 2

Fig. 6. Coefficient diagram, 2/2 order controller

3.4 Optimised First Order Controller

When ε_1 is chosen as 1 in Eq. (66), the 2/2 order controller degenerates to a 1/1 order controller.

$$A_c(s)u = B_c(s)(y_r - y), \qquad (75)$$
$$A_c(s) = l_1 s + l_0, \quad l_0 = -1,$$
$$B_c(s) = k_1 s + k_0, \quad k_0 / k_1 = 1 - \varepsilon_2.$$

The characteristic polynomial $P(s)$ becomes as follows;

$$P(s) = (l_1 s - 1)(s + 1)(s - 2) + q(k_1 s + k_0)(s - 1), \ (76)$$
$$= (l_1 s - 1)(s^2 - s - 2) + q[k_1 s^2 + (-k_1 + k_0)s - k_0].$$

The coefficients are given as follows;

$$a_3 = l_1, \qquad (77)$$
$$a_2 = -l_1 - 1 + k_1 q,$$
$$a_1 = -2l_1 + 1 + q(-k_1 + k_0),$$
$$a_0 = 2 - qk_0.$$

For the ideal case that l_1 is almost 0, k_1 = 1 and k_0 = 2/3, because ε_1 = 1 gives $q_{\max}$ = 3 and ε_2 = 1/3 by Eq.

(69). In order to make the system stable for q_{max} = 2.99, following parameters are selected.

$$l_1 = 0.0001, \quad k_1 = 1.001, \quad k_0 = 0.668 . \qquad (78)$$

The coefficients fall into the following intervals.

$$a_3 = 0.0001 , \qquad (79)$$

$$a_2 = -1.0001 + 1.001q = [0.0009, \ 1.9929] ,$$

$$a_1 = 0.9998 - 0.333q = [0.6668, \ 0.00413] ,$$

$$a_0 = 2 - 0.668q = [1.332, \ 0.00268] .$$

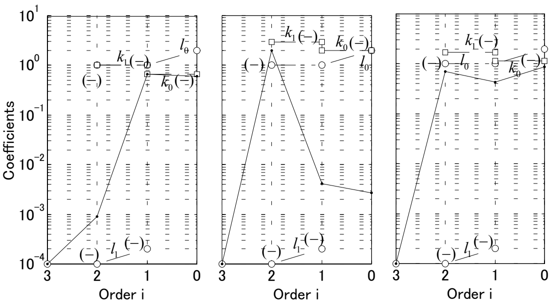
From Eq. (77), $\gamma_2\gamma_1$ is minimum at q = 1, and increases as q increases. Actually a_1/a_0 increases with increase of q. The same is true for a_2/a_3. Thus $\gamma_2\gamma_1$ falls into the following interval.

$$\gamma_2\gamma_1 = [4.5056, \ 30711] . \qquad (80)$$

The coefficient diagrams for q =1, 2.99, and 1.7 are shown in Fig. 7a, b, c. The final form of the controller transfer function $G_c(s)$ is as follows;

$$G_c(s) = \frac{1.001s + 0.668}{(0.0001s - 1)} \quad . \qquad (81)$$

Henrion, et al. (2003a) reported that a first order controller achieved q_{max} = 2.59. Their controller is very similar to this controller.



a. q = 1 b. q = 2.99 c. q = 1.7

Fig. 7. Coefficient diagram, optimised 1/1 order controller

4. CONCLUSIONS

Major conclusions in his paper are as follows;

(1) Three robustly stabilizing controllers are designed for a non-minimum phase and unstable plant. The design is made on the parameter space with clear logic and simple mathematics.

(2) The trade-off between stability and robustness is clearly expressed in a simple equation as Eq. (69). This relation will clarify the nature of the problem quite eloquently.

(3) The figurative representation of coefficient diagram and explicit formula for trade-off are the most important asset of CDM.

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