MINIMUM TIME CONTROL USING STRAIGHT TRANSFER FOR A ROTARY CRANE

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Abstract: This paper provides a feedforward control method for controlling the load vibration of a rotary crane using a straight transfer transformation (STT) model. The parameters of the model were geometrically derived. The minimum time control problem was solved for the revised STT model by means of both clipping-off technique for the constraints of control inputs amplitude and the Bisection Method. Preshaping control method is also discussed for comparison with proposed minimum time control. Finally, proposed control method using the STT model was demonstrated to be effective in eliminating the influence of centrifugal force through simulation and experiments.

Keywords: Rotary crane, Modelling, Nonlinear optimum control, Vibration control, Minimum time control, Preshaping control,

1. INTRODUCTION

The fundamental motions of a rotary crane are rotation, boom luffing and load hoisting. Using a crane, a load can be transferred to any place within a limited region. With this advantage, rotary cranes are extensively applied for transferring loads in factories, construction sites, harbors and so on. **Fig.1** shows a laboratory model of a rotary crane. One major drawback to rotary cranes is the uncontrolled movement of the load. Because centrifugal force is produced by the rotary motion, the load sways easily.

Many studies attempting to control the sway of rotary crane loads have been published. Sakawa and Nakazumi applied an open-loop scheme with feedback control scheme to allow the sway of the load to decay at the end point of transfer(Sakawa and Nakazumi, 1985). Stefan et al. presented a feedback method to control the load rotation(Stefan and Eberhard, 2000). Takaki and Nishimura designed a gain-scheduled H_∞ compensator based on the LMI for the length of the load rope to control the sway(Takaki, 1998). But these papers did not consider the condition of simultaneous rotary motion and boom luffing motion. Using a

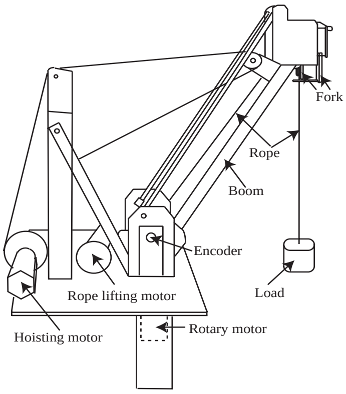


Fig. 1. A laboratory rotary crane built in this paper

rotary crane, Yamazaki et al. presented the idea of a straight transfer without centrifugal force that makes the crane tip and the load move on a straight line in the X-Y coordinates by simultaneous rotation and boom luffing(Yamazaki and Hisamura, 1978). Although straight transfer is limited to working spaces within 180 degrees, it is obvious that straight transfer by a rotary crane uses less space to transfer a load from starting point to end point than rotational transfer.

In recent years, the authors have also studied control of the rotary crane(Shen and Terashima, 2002; Shen and Yano, 2003). Authors present a general optimal control method of a rotary crane using straight transfer by controlling the rotation, boom's luffing and load's hoisting simultaneously, and a straight transfer transformation (STT) model was derived. Using the Davidon-Fletcher-Powell (DFP) method for the STT model considering the requisite change in rope length, the actual control inputs for a rotary crane were obtained.

Based on (Shen and Yano, 2003), this paper presents the minimum time control for STT by means of both clipping-off technique for the constraints of control inputs amplitude and the Bisection Method. The minimum time control can reduce the transfer time and well control the vibration angle of the load, even if the length of rope changed. Further, the present method is compared with Preshaping control method effective in practical application and evaluated.

2. MINIMUM TIME CONTROL

In (Shen and Yano, 2003), the STT model was derived as follows:

$$\ddot{\psi} = u_\psi \tag{1}$$

$$\ddot{\xi} = -\frac{g}{l}\sin\xi + \frac{\ddot{\tilde{x}}'}{l}\cos\xi - \frac{\ddot{\tilde{z}}'}{l}\sin\xi - 2\frac{\dot{l}\dot{\xi}}{l} \tag{2}$$

$$\ddot{l} = \frac{\ddot{\tilde{z}}' + \ddot{\xi}l\sin\xi + \dot{\xi}^2 l\cos\xi + 2\dot{l}\dot{\xi}\sin\xi}{\cos\xi} \tag{3}$$

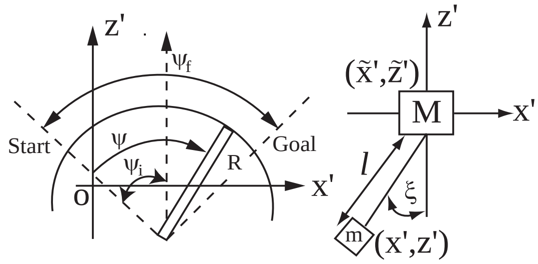


Fig. 2. Straight transfer transformation model

$$\ddot{\tilde{x}}' = R\left\{\ddot{\psi}\cos(\psi-\psi_i) - \dot{\psi}^2\sin(\psi-\psi_i)\right\} \tag{4}$$

$$\ddot{\tilde{z}}' = -R\left\{\ddot{\psi}\sin(\psi-\psi_i) + \dot{\psi}^2\cos(\psi-\psi_i)\right\} \tag{5}$$

In addition to the suppression control of residual vibration, it is hoped to safely transfer the load as fast as possible while satisfying the restricted conditions of the actual apparatus. Minimum time control is one of the effective methods to satisfy such requirements. Here, the case in which the rope length is varied will be also discussed. For this case, the transfer process can be divided into three steps: an accelerating motion step, a uniform motion step and a decelerating motion step. If the STT model is directly used to calculate the acceleration $\ddot{\psi}$, the velocity of the imaginary boom will be constant during the uniform motion, but at this time, the acceleration of the load in x' direction (see **Fig.2**) is not zero, and there appears some sway. In order to keep the swing angle of the load during the uniform motion at zero, the independent variable $\ddot{\tilde{x}}'$ is proposed instead of $\ddot{\psi}$ as the control input, and the STT model can be easily deformed to the following type:

$$\ddot{\tilde{x}}' = u_x \tag{6}$$

$$\ddot{\psi} = \frac{\frac{u_x}{R} - \dot{\psi}^2\sin(\psi_i-\psi)}{\cos(\psi_i-\psi)} \tag{7}$$

$$\ddot{\tilde{z}}' = \frac{u_x\sin(\psi_i-\psi) - R\dot{\psi}^2}{\cos(\psi_i-\psi)} \tag{8}$$

$$\ddot{\xi} = -\frac{g}{l}\sin\xi + \frac{\ddot{\tilde{x}}'}{l}\cos\xi - \frac{\ddot{\tilde{z}}'}{l}\sin\xi - 2\frac{\dot{l}\dot{\xi}}{l} \tag{9}$$

$$\ddot{l} = \frac{\ddot{\tilde{z}}' + \ddot{\xi}l\sin\xi + \dot{\xi}^2 l\cos\xi + 2\dot{l}\dot{\xi}\sin\xi}{\cos\xi} \tag{10}$$

Here u_x presents the acceleration of the load in x' direction.

Transformation from virtual control input u_x in Eq.(6) obtained for STT model to control inputs $\ddot{\theta}(t), \ddot{\phi}(t)$ and $\ddot{l}(t)$ in absolute coordinates of rotary crane can be easily achieved by coordinate-transformation, which is in detail described in

(Shen and Yano, 2003). Here, in real rotary crane as show in **Fig.1**, $\ddot{\theta}(t), \ddot{\phi}(t)$ and $\ddot{l}(t)$ are acceleration of rotary angle, luffing angle and rope length, respectively. Further, actual control inputs in experiments are u_θ, u_ϕ and u_l, where respectively, input voltages to motor driver to produce $\ddot{\theta}(t), \ddot{\phi}(t)$ and $\ddot{l}(t)$, and its relationship between u_θ and $\ddot{\theta}$, u_ϕ and $\ddot{\phi}$, u_l and $\ddot{l}$ are expressed by second-order lag differential equation(Shen and Yano, 2003). Now the problem is to obtain the acceleration input u_x, when the rotary crane starts, while maintaining u_x at zero during the uniform motion. Then, if the swing angle is suppressed at zero at the end of acceleration interval, there is no residual vibration during the constant velocity interval. Considering the restricted conditions for the actual crane, the maximum value of u_x is selected to be $u_{x_{max}} = 1[m/s^2]$, and the maximum velocity of the load is selected to be $v_{x_{max}} = 0.17[m/s]$ by simulation analysis. Then this problem can be reconfigured as follows:

For the constrained magnitude $|u_x| \leq 1[m/s^2]$, with the terminal condition

$$[\dot{x}_{t_f} \;\; \xi_{t_f} \;\; \dot{\xi}_{t_f}]^T = [0.17 \;\; 0 \;\; 0]^T \qquad (11)$$

The index function is chosen to be

$$J = x(t_f)^T W x(t_f), \qquad (12)$$

where $x(t_f) = [v - \dot{x}_{t_f} \;\; 0 - \xi_{t_f} \;\; 0 - \dot{\xi}_{t_f}]^T$, $W = diag[10^5 \;\; 10^5 \;\; 10^5]$

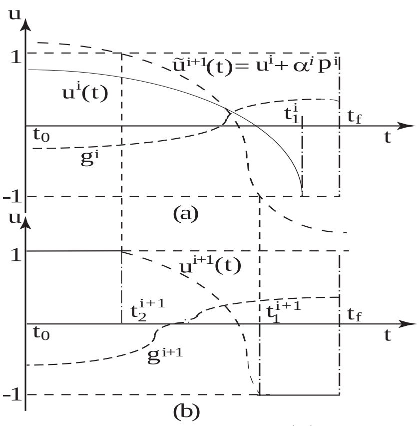


Fig. 3. Clipping-off process: (a) ith iteration, $S^i = [t_1^i, t_f]$; (b) (i+1)th iteration,$S^{i+1} = \{[t_0, t_2^{i+1}], [t_1^{i+1}, t_f]\}$

Here, t_f is the end time of acceleration interval. Then, the problem is to find the optimum input u_x such that t_f becomes minimum time under the constraints of control input's amplitude.

This is a well-known problem as two-point boundary problem. In general, it is impossible to solve this problem analytically except for a simple case. The input shape of minimum time control is Bang-Bang Type. There are many numerical methods to solve the nonlinear optimal control. One of the effective methods is a conjugate-gradient method. For the fixed end time control, conjugate gradient method is a powerful way using DFP method, or FR (Fletcher-Reeves) method. Then, in this paper, in order to find the minimum time control, we extended the method of DFP for the fixed end time problem to the minimum time control problem by reducing the end time. Thus, the benefit of this method enables us to solve both problem of the fixed end time and the minimum time control in the same framework. Now, control input constraints problem can be resolved by clipping-off gradient algorithms (Quintana and Davision, 1974), where the FR method is used as an optimization technique.

Fig. 3 depicts the clipping-off process: given an estimate u^i for the optimal u^*, compute the gradient g^i of u, and by means of this gradient, determine a new estimate $\tilde{u}^{i+1}$, which, when it is clipped off at the upper and lower bounds, gives u^{i+1}. This means that a gradient technique is used to compute only the ' unsaturated ' segments of u^{i+1}, while the Maximum Principle is applied in finding the 'saturated' segments of u^{i+1}. This method is easily extended to the case of DFP. But using DFP method, only **step 7**(Shen and Yano, 2003), that is to say, a new d_i, needs to be calculated, and the other steps are the same. Here, the new d_i is calculated as follows:

$$\mathbf{d}_i = -\mathbf{g}_i$$

$$-\beta_i \sum_{k=0}^{i-1} \left[\frac{(\mathbf{s}_k, \mathbf{g}_i)}{(\mathbf{y}_k, \mathbf{y}_k)} \mathbf{s}_k - \frac{(\mathbf{B}_k \mathbf{y}_k, \mathbf{g}_i)}{\mathbf{y}_k, \mathbf{B}_k \mathbf{y}_k} \mathbf{B}_k \mathbf{y}_k \right]$$

, and

$$\beta_i = \begin{bmatrix} 1, & I_2 > 0 \;\; or \;\; I_2 < 0 \;\; but \;\; I_1 > -I_2 \\ \gamma\beta_m, & I_2 < 0 \;\; or \;\; I_1 \leq -I_2 \\ 0, & I_2 = 0 \end{bmatrix} \qquad (13)$$

, where

$$I_1 = \int_{U_i} g_i g_i dt$$

$$I_2 = \int_{U_i} g_i \sum_{k=0}^{i-1} \left[\frac{(\mathbf{s}_k, \mathbf{g}_i)}{(\mathbf{y}_k, \mathbf{y}_k)} \mathbf{s}_k - \frac{(\mathbf{B}_k \mathbf{y}_k, \mathbf{g}_i)}{\mathbf{y}_k, \mathbf{B}_k \mathbf{y}_k} \mathbf{B}_k \mathbf{y}_k \right] dt$$

and $0 < \gamma < 1$, $\beta_m = -\frac{I_1}{I_2}$. U_i presents the non-optimal saturation area.

Eq.(13) is straightforwardly proved in the following way.

For all values of u_i and J_{u_i}, the time area in which $\mid u_i \mid > 1$ and $u_i J_{u_i} < 0$ is defined as the optimal saturation area S_i, and the remaining time area is defined as the non-optimal saturation area U_i. Then,

$$\delta J_i = \int_{U_i} \left.\frac{\partial H}{\partial u}\right|_i \delta u_i dt \tag{14}$$

, where

$$\delta u_i = \begin{cases} \alpha_i d_i, \; t \in U_i \\ 0, \quad\;\; t \in S_i \end{cases} \tag{15}$$

, and

$$\left.\frac{\partial H}{\partial u}\right|_i = g_i \tag{16}$$

Then, substituting Eq.(15) and Eq.(16) into Eq.(14), it follows that

$$\delta J_i = \alpha_i \int_{U_i} g_i d_i dt, \tag{17}$$

and then it follows:

$$\delta J_i = -\alpha_i [I_1 + \beta_i I_2] \tag{18}$$

According to Eq.(13), the following result can be obtained:

$$J(u_{i+1}) < J(u_i) \;\; i = 0, 1, 2, \cdots, \tag{19}$$

which means using β_i to calculate the searching direction $\mathbf{d}_i$, and $u_{i+1} = u_i + \alpha_i \mathbf{d}_i$ can be calculated, and the DFP algorithm converges to a calculation of the performance index. This completes the proof.

Here, in this paper, the minimum time t_f can be searched by the Bisection Method. The algorithm of the Bisection Method is:

[Procedure of finding Minimum Time by Bisection Method]

(Step1) Give an arbitrary initial number of sampling N_0, and the minimum time $t_f = N_0 \times \Delta t$, here $\Delta t = 0.01s$ represents the sampling period. And also time step length h to renew the end time.

(Step2) Within $N_c = 500$, (N_c is the limited calculating number), calculate the index function $J(N_i \Delta t)$. If the terminal condition $J < \epsilon$ is satisfied, then let $N_{i+1} = N_i - \frac{h}{2}(i = 0, 1, \cdots)$, and then, the end time should be shorten; otherwise, let $N_{i+1} = N_i + \frac{h}{2}$, in which case the end time should be prolonged. Here $int(\frac{h}{2}) \rightarrow h$.

(Step3) If $h \neq 1$, then let $i = i + 1$, goto **Step2**; if $h = 1$ and the terminal condition is satisfied, then $N_i \Delta t$ is the optimum solution, otherwise, the previous calculating result $N_{i-1} \Delta t$ is the optimum solution(the minimum time).

(**Remark**) The initial value of N_0 and h is given as follows:

$$N_0 = c \times \frac{v_{x_{max}}}{u_{x_{max}}} \times \frac{1}{\Delta t} = c \times \frac{0.17}{1} \times \frac{1}{0.01},$$

and

$$h = 100.$$

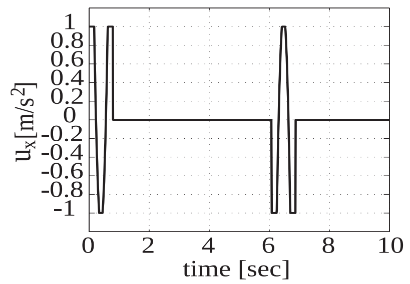
With the experiment condition, initial position of the crane tip is $(x_i, y_i, z_i) = (0.73, 0, 1.05)m$, and the final position is $(x_f, y_f, z_f) = (0, 0.73, 1.05)m$, the factor c is selected to be 3.

3. SIMULATION AND EXPERIMENTAL RESULTS

Table 2 shows the calculating result for each step by Bisection Method. Here, i presents the calculation step by Bisection Method, t_f presents the minimum acceleration (or deceleration) time. N_c presents the calculation number by DFP method, t_c presents the calculation time, and J presents the index function. The computer for calculation is Endeavor(R)4(3.20GHz).

Table 1. Calculating result of t_f using proposed Method.

i	$t_f[s]$	N_c	J	$t_c[s]$	h
0	0.51	100	226	3.1	100
1	1.01	42	4.8×10^{-3}	0.8	50
2	0.76	100	4.72	3.7	25
3	0.89	55	4×10^{-3}	1.1	13
4	0.82	100	0.036	3.6	7
5	0.86	83	3.4×10^{-3}	1.2	4
6	0.84	95	4.9×10^{-3}	1.6	2
7	0.83	100	4.9×10^{-3}	2.9	1



$t_f = 0.82s$(Minimum time control)

Fig. 4. The acceleration of the load in x' direction

Fig.4(a) shows the acceleration input of minimum time control, and **Fig.5** shows the simulation and experimental results.

All of the results of the experiment agree with the simulation results, and the deceleration input is obtained from $-u_x$ simply. The optimal computing time is 18s. If the position model(Shen and

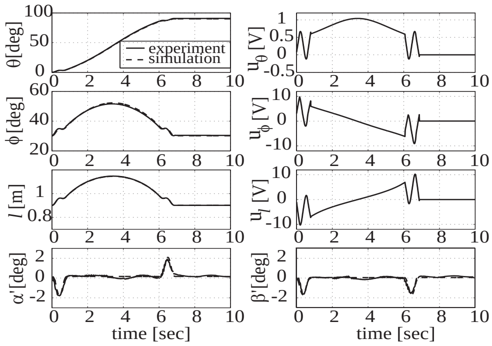


Fig. 5. The simulation and experimental result with the minimum time control($t_{all} = 6.88s$)

Yano, 2003) is used, it will take the time of about 25 times to compute, compared with the case of using STT model. Thus, the proposed method is effective.

4. COMPARISON OF THE PROPOSED MINIMUM TIME CONTROL WITH PRESHAPING CONTROL

In order to compare with the proposed method, Preshaping control effective in real application is introduced in this section. The input shaping method was developed by Neil Singer and Warren Seering(Singer and Seering, 1990). The general principle is to superpose two input responses so that the input responses cancel the vibrations after a short time delay.

In order to get a linear model for the vibration angle, for Eq. (4), if ξ is very small, $\sin\xi \approx \xi$, $\cos\xi \approx 1$, and omit last two parts, then

$$\ddot{\xi} = \frac{u_{x0}}{l_0} - \frac{g}{l_0}\xi \tag{20}$$

Here, l_0 presents the standard rope length, and u_{x0} presents the standard acceleration input. If it is defined that $\omega_n = \sqrt{\frac{g}{l_0}}$ and $K = \frac{1}{g}$, the transfer function of Eq.(20) is as follows:

$$G(s) = \frac{\xi(s)}{u_{x0}(s)} = \frac{K\omega_n^2}{s^2 + \omega_n^2} \tag{21}$$

Here, the damping factor ζ is zero. If the standard rope length is chosen to be $l_0 = 0.9m$, then resonant frequency $\omega_n = \sqrt{\frac{9.8}{0.9}} = 3.3$rad/s. Then the input time T and the input amplitude K_m can be calculated as follows:

$$\begin{cases} T = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.95s \\ K_m = exp(\dfrac{-\zeta\pi}{\sqrt{1-\zeta^2}}) = 1 \end{cases} \tag{22}$$

Considering that $u_{x_{max}} = 1[m/s^2]$, $v_{x_{max}} = 0.17[m/s]$, the acceleration or deceleration time is $t_a = t_1 + T = \frac{v_{x_{max}}}{u_{x_{max}}} + T = 0.17 + 0.95 = 1.12s$, and the uniform time is $t_u = \frac{S}{u_{x_{max}} t_1} - T = \frac{1.03}{1\times 0.17} - 0.95 = 5.14s$, and the whole transfer time is $t_{all} = t_u + 2t_a = 7.4s$.

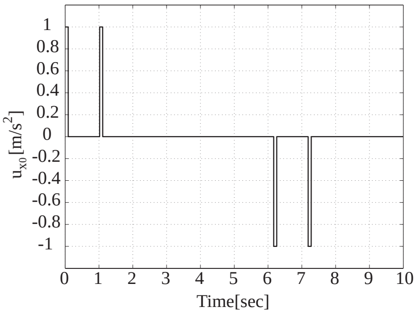


Fig. 6. Preshaping input

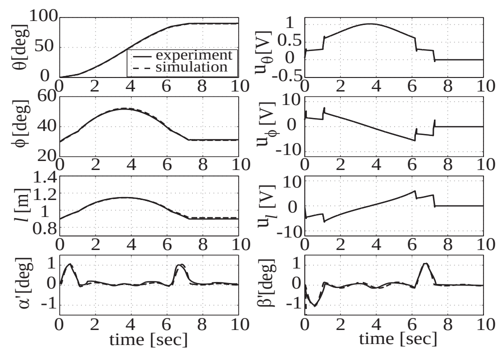


Fig. 7. Experimental result by Preshaping input with the change of rope length

Fig. 4 shows that acceleration input u_{x0} was calculated from Preshaping method, and Fig. 7 shows the simulation and experimental results considering the change of rope length, and it is clear that the vibration occurs even if during the uniform transfer. In order to eliminate this vibration, the acceleration input must be re-calculated, and the principle will be introduced as follows, which is proposed by Toyohara, et al in (Toyohara and Shimotsu, 2003).

Considering the change of the rope length, Eq.(4) can be changed as follows:

$$\ddot{\xi} = \frac{u_x}{l} - \frac{g}{l}\xi - 2\frac{\dot{l}\dot{\xi}}{l} \tag{23}$$

Letting Eq.(23) be equal to Eq.(20), that is to say

$$\frac{u_x}{l} - \frac{g}{l}\xi - 2\frac{\dot{l}\dot{\xi}}{l} = \frac{u_{x0}}{l_0} - \frac{g}{l_0}\xi \tag{24}$$

Namely, this means that the vibration angle is the same as that without changing the rope length. From Eq.(24), the new acceleration input can be obtained as follows:

$$u_x = \frac{l}{l_0} + (1 - \frac{l}{l_0})g\xi + 2\dot{l}\dot{\xi} \tag{25}$$

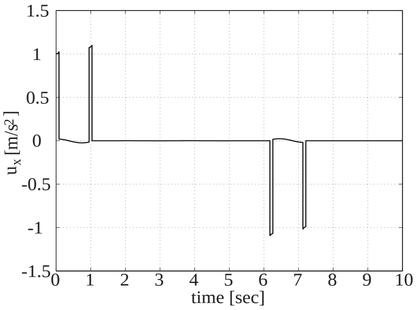


Fig. 8. Preshaping input considering the change of rope length

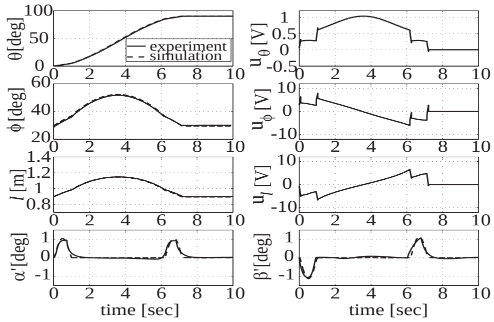


Fig. 9. Experimental result by Preshaping input considering the change of rope length

Fig. 8 shows the new acceleration input, and Fig. 9 shows the simulation and the experimental results using the new input. The vibration during the uniform transfer disappears.

Advantage aspect of Preshaping control is that the acceleration input can be calculated simply and immediately. Then the effect on eliminating the vibration angle is also good, but comparing to the proposed minimum time control, the transfer time by Preshaping control is longer. In the case that the transfer route is previously fixed, the minimum time control input is calculated offline, and the transfer efficiency will be excellent compared with the Preshaping control.

5. CONCLUSION

The main results and contributions of this paper are summarized as follows:

(1) In order to eliminate the effect of the centrifugal force, three motions of rotation, boom luffing and rope hoisting are simultaneously being moved, so that the load is transferred by a straight line and STT model is derived.

(2) In order to keep the vibration angle zero during the uniform velocity transfer, a revised STT model based on STT model is derived. Using this model, not only the minimum time problem, but also the Preshaping control problem can be resolved simply.

(3) Utilizing the revised STT model, the minimum time control problem of a load in the straight transfer with rope length change could be solved by using both of DFP method with clipping-off algorithm and Bisection method.

(4) Comparing with Preshaping control, it is demonstrated through experiments that the proposed minimum time controls method not only controls the residual vibration angle easily, but also can reduce the whole transfer time.

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