QUANTUM FEEDBACK CONTROL USING QUANTUM CLONING AND STATE RECOGNITION[1]

Daoyi Dong, Chenbin Zhang, Zonghai Chen

Department of automation, University of Science and Technology of China, Hefei, 230027, P. R. China
E-mail address: *dy_dong@263.net (D. Y. Dong); chenzh@ustc.edu.cn(Z. H. Chen)*

Abstract: A scheme of quantum feedback control with an optimal cloning machine is proposed. The design of quantum feedback control algorithms is separated into a state recognition strategy, which gives "on-off" signal to the actuator through recognizing some copies from cloning machine, and a feedback (control) strategy through feeding back the another copies of cloning machine. Precise feedback is abandoned and a compromise between information acquisition and measurement disturbance is established. The recognition process involves measurement and is destructive, however, the feedback step without measurement is preserving quantum coherence, so the scheme can perform some quantum control tasks with coherent feedback.

Keywords: quantum control; quantum system; quantum feedback control; quantum cloning; state recognition

1. INTRODUCTION

The control of quantum phenomena is an important problem that lies at the heart of several fields, including quantum computer, atomic physics and molecular chemistry (Rabitz, *et al*., 2000). Nowadays, quantum control theory is becoming a rapidly increasing research field and it expects to determine how to drive quantum mechanical systems from an initial given state to a pre-determined target state with some given time T (Solomon and Schrirmer, 2002). The current research on quantum control mainly involves controllability of quantum system (Clark, *et al*., 2003), quantum optimal control (D`Alessandro and Dahleh, 2001) and quantum feedback control (Yanagisawa and Kimra, 2003a; Yanagisawa and Kimra, 2003b; Doherty, *et al.*, 2000; Doherty, *et al*., 2001). Here, the attention will be taken to quantum feedback control and quantum cloning in quantum information technology is used to design feedback channel.

In classical control applications, feedback is a most effective strategy, and recently it is also used to quantum control. Scientists expect to obtain information about the system from the quantum system to be controlled, process the information and feed it back to the system to complete active control of quantum system in a desired way. However, a quantum system (such as electron spin, photon polarization and two-level atom) is essentially different from a classical system. The measurement on a classical system doesn't change its state, but a measurement of a quantum system will disturb its quantum state. Some quantum feedback strategies including Markovian quantum feedback (Wiseman and Milburn, 1993), Bayesian quantum feedback (Doherty and Jacobs, 1999), non-Markovian quantum feedback with time delay (Giovannetti, *et al*., 1999) and coherent quantum feedback (Lloyd,

[1]Partially supported by the Innovation Fund of the University of Science and Technology of China for Graduate Students (KD2004048).

2000) have been presented, and they have also been applied to coherence maintenance of quantum information and quantum error-correction. On the other hand, the rapid development of quantum information technology makes it possible that one applies some achievements in quantum information to quantum control theory (Nielsen and Chuang, 2000). This paper applies quantum optimal cloning machine in quantum information theory to design quantum feedback channel. Here, the problem of designing quantum feedback control algorithms is separated into a state recognition strategy (Dong and Chen, 2003) and a feedback (control) strategy. The recognition process gives actuators "on-off" signal through recognizing some copies from cloning machine and is destructive. However, the feedback process doesn't involve measurement and can preserve quantum coherence. So the scheme can perform some quantum control tasks with coherent feedback.

2. QUANTUM FEEDBACK CONTROL

Recently, scientists are greatly interested in quantum feedback control and have also proposed many feedback strategies including Markovian quantum feedback, Bayesian quantum feedback, non-Markovian quantum feedback with time delay and coherent quantum feedback. The first theoretical work on quantum feedback was presented by Yamamoto *et al.*(1986), where they treated the fluctuations of the photocurrent in negative feedback way to generate amplitude squeezed state.

In 1993, Wiseman and Milburn first presented a quantum theory of optical feedback via homodyne detection, where the homodyne photocurrent was fed back onto optical cavity to alter the dynamics of the source cavity. In this quantum feedback theory, only the current photocurrent is immediately fed back and may then be forgotten, so the master equation describing the resulting evolution is Markovian and the theory is called Markovian quantum feedback (Wiseman, *et al.*, 2002). It has been applied to generate squeezed light (Wiseman and Milburn, 1993) and stabilize the internal state of atoms (Wang and Wiseman, 2001).

Markovian quantum feedback doesn't use the previous knowledge about the system to be controlled, therefor Doherty and Jacobs presented a quantum feedback scheme using continuous state estimation in 1999. They made the best of the detailed information from measurement record, and divided quantum feedback control process into two steps: a state estimation step and a feedback control step. Since the best state estimation will use all previous measurement results, not just the lastest ones, the feedback is called Bayesian quantum feedback (Wiseman, *et al.*, 2002). Recently, Doherty *et al.* (2001) also tried to consider separately optimizing state estimation step and feedback control step. Bayesian quantum feedback has been used to cool and confine a single quantum degree of freedom (Doherty and Jacobs, 1999), switch the state of a particle in a double-well potential (Doherty, *et al.*, 2000) and control the decoherence of solid-state qubit (Ruskov and Korotkov, 2002). Comparison research shows that Bayesian quantum feedback is never inferior, and is usually superior, to Markovian quantum feedback in stabilizing the quantum state of the simplest nonlinear quantum system (Wiseman, *et al.*, 2002).

Both Markovian quantum feedback and Bayesian quantum feedback ignore the effect of feedback time delay. However, the delay can't be ignored in some situation. When nonzero feedback time delay is considered, the dynamics of system evolution exhibits strong non-Markovian. Giovannetti and co-workers first studied the non-Markovian quantum feedback with time delay in 1999. Their results show that feedback can also improve the dynamics of quantum systems for the delay time not too large.

Markovian feedback, Bayesian feedback and non-Markovian feedback with time delay all use the feedback information from measurement results, however, measurement destroys the quantum characteristics of feedback information, so feedback information becomes classical information. As a result, although the system under control is quantum system, feedback controller processes classical information and effective quantum channel in feedback loop isn't constructed, so the strategies can be called quantum control with classical feedback (Fig.1(a)). Differently, Lloyd presented a coherent quantum feedback scheme in which controller obtained quantum information, processed it and coherently fed back to the system to be controlled (Fig.1(b)). In this feedback strategy, the quantum characteristics in feedback loop aren't destroyed and the strategy can accomplish some tasks which are not possible using classical feedback (Lloyd, 2000). Recently, Ting (2002) also proposed an alternative method for quantum feedback control, where a cloning machine served to obtain the feedback signal and the output (Fig.1(c)). Ting's quantum feedback method can also perform coherent feedback control at the cost of feeding precisely back the output.

3. QUANTUM FEEDBACK CONTROL USING QUANTUM CLONING AND STATE RECOGNITION

As an essential idea in classical control, feedback is used to compensate the effects of unpredictable disturbances on a system under control, or to make automatic control possible when the initial state of the system is unknown. To control a system, one must obtain the information about the evolving system state through measurements. However, in quantum feedback control, it is impossible to extract information about the state of system through measurement without disturbing it. Moreover, measurement backaction greatly complicates the notion of quantum feedback control. Following Ting's idea, this paper renounces precise feedback

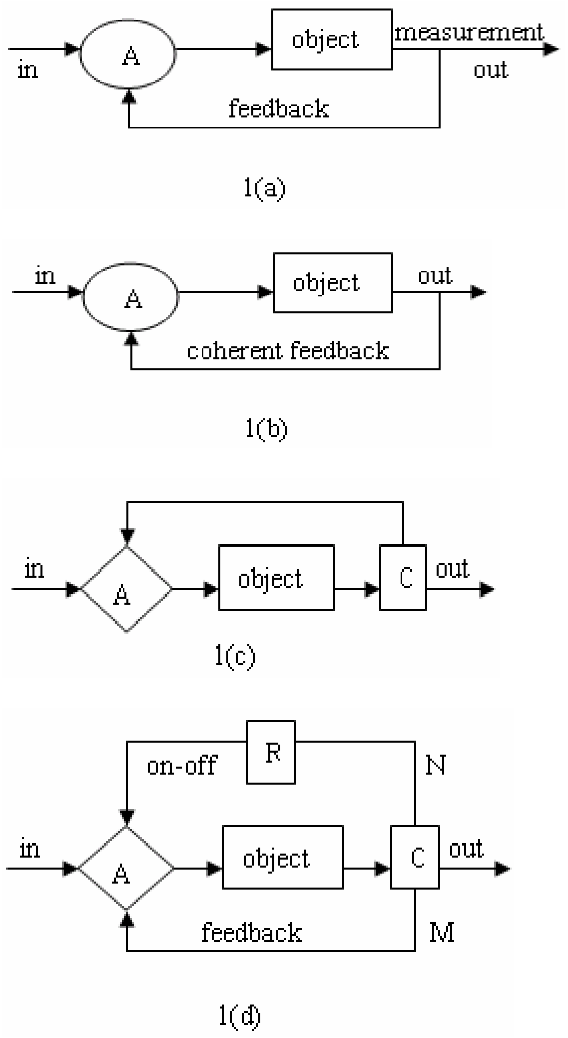

Fig. 1.(a) General quantum feedback; (b) Coherent quantum feedback; (c) Quantum feedback control using quantum cloning; (d) Quantum coherent feedback based quantum cloning and state recognition.

and adds an optimal cloning machine at the output side. Different from Ting's complete abandonment of measurement, we separate quantum feedback control design into a state recognition step involving measurements and a feedback control step without measurements (Fig.1(d)). So this scheme establishes a compromise between information acquisition and measurement disturbance. This is obviously different from traditional quantum feedback since there the information acquisition necessarily results in destroying the state of quantum system.

The general picture of quantum feedback control using quantum cloning and state recognition is as follows (Fig.1(d)). The quantum system to be controlled is called object. The actuator generates input quantum signal to drive the object and its output is sent into an optimal cloning machine (cloner). The cloner (C) inaccurately clones the state, generates (N+M+1) copies, and a copy is taken as the output of system. State recognizer (R) receives N unknown copies, makes some measurements on them and obtains some information about their states. Then, one can recognize the N copies through appropriate recognition algorithm. If the N copies are enough "good", the recognizer gives an "on" signal to the actuator, and the actuator receives another M copies from the cloner as feedback and generates new quantum signal to drive the object until the given control objective is reached. Here, the new signal of the actuator is determined by the feedback information and control objective. If the N copies are not enough "good", the recognizer will send an "off" signal to the actuator, and the actuator will not receive feedback copies from the cloner.

According no-cloning theorem (Wootters and Zurek, 1982), arbitrary unknown quantum state cannot be copied exactly, and this is also an important difference between quantum control system and classical control system. However, this doesn't exclude the possibility of approximately cloning quantum state. In fact, there is optimal universal quantum cloning machine that can approximately copy arbitrary unknown quantum state with unity probability (Gisin and Massar, 1997). Besides approximate cloning, linearly independent quantum states can also be probabilistically cloned by a general unitary reduction operation with probability less than unity (Duan and Guo, 1998). In probabilistic cloning, the machine yields faithful copies of the input state with a postselection of the measurement result. However, optimal universal quantum cloning machine can be decomposed into rotations and controlled NOTs gates and doesn't involve measurement. Here, we use approximately cloning to amplitude the output state of the object, that is to say, the output state is approximately copied, and some copies are used to obtain information through quantum measurement and other copies are used to determine feedback control.

The ideal 1→ 2 cloning process is described by the transformation:

$$|s\rangle_a |M\rangle_x \to |s\rangle_a |s\rangle_b |\tilde{M}\rangle_x \qquad (1)$$

Where $|s\rangle_a$ is the state of the original mode, $|s\rangle_b$ is the copied state, $|M\rangle_x$ is the original state of the quantum cloning machine and $|\tilde{M}\rangle_x$ is the final state of the quantum cloning machine. The whole process of quantum cloning is to produce at the output of the cloning machine two identical states $|s\rangle_a$ and $|s\rangle_b$ in the modes a and b, respectively. Considering 1→M+N+1 cloning process, it can be described by the transformation:

$$|s\rangle_a |M\rangle_x \to |s\rangle_a |s\rangle_b \cdots |s\rangle_{M+N+1} |\tilde{M}\rangle_x \qquad (2)$$

The cloner generates (M+N+1) copies and sends N copies into the state recognizer. Assume that the recognizer receives N unknown copies $U_R = \{|P_1\rangle, |P_2\rangle, \cdots, |P_N\rangle\}$, make some measurements on them and obtain some information about their states.

In quantum mechanics, states of quantum systems are represented by quantum states and quantum states can be represented by the vector $|\phi\rangle$ in Hilbert space:

$$|\phi\rangle = \sum_k a_k \, |\varphi_k\rangle \quad (3)$$

where a_k is complex number and satisfies $\sum_k |a_k|^2 = 1$. $\{|\varphi_1\rangle, |\varphi_2\rangle, \cdots, |\varphi_k\rangle\}$ is a set of bases of the vector $|\phi\rangle$ and is called k eigenstates of quantum states $|\phi\rangle$, namely all possible states when $|\phi\rangle$ is measured. $|a_k|^2$ represents the occurrence probability of eigenstate $|\varphi_k\rangle$ and the information about the phase is contained in the argument of complex number a_k.

After some measurement on the N unknown copies of the recognizer, one can get some information of unknown copies. Assume that $N(N \geq 1)$ quantum states $|P_s\rangle (1 \leq s \leq N)$ can be respectively represented as follows:

$$|P_s >= \sum_{j=1}^{n_s} \beta_j^{(s)} \, |p_j^{(s)}\rangle \quad (4)$$

where $s \in \{1, 2, \cdots, N\}$, $\beta_j^{(s)}$ are complex number coefficients and satisfy $\sum_{j=1}^{n_s} |\beta_j^{(s)}|^2 = 1$. $U_s = \{|p_1^{(s)}\rangle, |p_2^{(s)}\rangle, \cdots, |p_{n_s}^{(s)}\rangle\}$ is composed of n_s orthogonal eigenstates of $|P_s\rangle$ and these eigenstates can constitute a set of orthogonal bases in Hilbert space. We use $|U|$ to express the number of elements in the set U, then $|U_s| = n_s$. Let $\tilde{U}$ represent the set of the extended bases:

$$\tilde{U} = U_o \cup U_1 \cup U_2 \cup \cdots \cup U_N = \{|\tilde{p}_1\rangle, |\tilde{p}_2\rangle, \cdots, |\tilde{p}_l\rangle\} \quad (5)$$

that is to say, define the union of these eigenstate sets as the set of extended bases. The elements in the set of extended bases are not necessarily orthogonal. Now express the all quantum states with the extended bases $\{|\tilde{p}_1\rangle, |\tilde{p}_2\rangle, \cdots, |\tilde{p}_l\rangle\}$:

$$|\tilde{P}_s\rangle = \sum_{k=1}^{l} \beta_k^{(s)} \, |\tilde{p}_k\rangle \quad (6)$$

where if $|\tilde{p}_k\rangle$ is equal to $|p_j^{(s)}\rangle$, $\tilde{\beta}_k^{(s)}$ is equal to $\beta_j^{(s)}$; otherwise $\tilde{\beta}_k^{(s)}$ is zero. Then calculate their arithmetical mean value $|\overline{P}\rangle$:

$$|\overline{P}\rangle = \sum_{k=1}^{l} (\frac{1}{T} \sum_{s=1}^{T} \beta_k^{(s)}) \, |\tilde{p}_k\rangle = \sum_{k=1}^{l} \tilde{\alpha}_k \, |\tilde{p}_k\rangle \quad (7)$$

Regard $|\overline{P}\rangle$ as the "objective" state. Based on the above notations, we may define state-distance between the given quantum states $|P_s\rangle$ and the "objective" quantum state $|\overline{P}\rangle$:

Definition 1: State-distance between $N(N \geq 1)$ given quantum states $|P_s\rangle (1 \leq s \leq N)$ expressed by Eq.(4) and the "objective" state $|\overline{P}\rangle$ expressed by Eq.(7) is defined as:

$$d(|P_s\rangle, |\overline{P}\rangle) = (\sum_{k=1}^{l} \left|\tilde{\beta}_k^{(s)} - \tilde{\alpha}_k\right|^2)^{1/2} \quad (8)$$

where $\tilde{\beta}_k^{(s)}$ and $\tilde{\alpha}_k$ are corresponding complex numbers in Eq.(6) and Eq.(7), respectively.

According to the definition of state-distance, it is obvious that $d(|P_s\rangle, |\overline{P}\rangle) \geq 0$ and $d(|P_s\rangle, |\overline{P}\rangle) = 0$ if only if $|\overline{P}\rangle$ and $|P_s\rangle$ are the same, that is to say, state-distance will reach the minimum zero when all given quantum states are identical.

Calculate $d(|P_s\rangle, |\overline{P}\rangle)$ according to Eq.(8) respectively and compare them with the beforehand given state-distance threshold d_0. Here, the state-distance threshold should be selected according to the need of specific problem. If all state-distances satisfy $d(|P_s\rangle, |\overline{P}\rangle) < d_0$, recognize them as a class, the state recognizer generates an "on" signal, and the actuator receives another M copies from the cloner as feedback. Considering the feedback information and comparing the output with the control objective, the actuator generates new quantum signal to drive the object. Otherwise, the state recognizer will send an "off" signal to the actuator, and the actuator will not receive feedback copies from the cloner.

To demonstrate the above recognition process, consider the recognition of many states:

Example 1: Consider $U_R = \{|P_1\rangle, |P_2\rangle, |P_3\rangle, |P_4\rangle, |P_5\rangle\}$, and after some measurement we get:

$$|P_1\rangle = 0.700\,|11\rangle + 0.700\,|10\rangle + 0.100\,|01\rangle + 0.100\,|00\rangle \quad (9)$$

$$|P_2\rangle = 0.700|11\rangle + 0.690|10\rangle + 0.100|01\rangle + 0.155|00\rangle \tag{10}$$

$$|P_3\rangle = 0.707|11\rangle + 0.690|10\rangle + 0.118|01\rangle + 0.100|00\rangle \tag{11}$$

$$|P_4\rangle = 0.695|11\rangle + 0.703|10\rangle + 0.100|01\rangle + 0.113|00\rangle \tag{12}$$

$$|P_5\rangle = 0.707|11\rangle + 0.700|10\rangle + 0.100|01\rangle \tag{13}$$

Under the condition of given state-distance threshold $d = 0.1$, recognize these states.

Based on the above discussion, we calculate the set of the extended bases

$$\tilde{U} = \{|11\rangle, |10\rangle, |01\rangle, |00\rangle\} \tag{14}$$

and the "objective" state $|\bar{P}\rangle$ is:

$$|\bar{P}\rangle = 0.702|11\rangle + 0.697|10\rangle + 0.104|01\rangle + 0.094|00\rangle \tag{15}$$

According to the definition of state-distance, calculate the state-distance between every state and the "objective" state, respectively:

$$\begin{cases} d(|P_1\rangle, |\bar{P}\rangle) = 0.008 < d_0 \\ d(|P_2\rangle, |\bar{P}\rangle) = 0.062 < d_0 \\ d(|P_3\rangle, |\bar{P}\rangle) = 0.017 < d_0 \\ d(|P_4\rangle, |\bar{P}\rangle) = 0.022 < d_0 \\ d(|P_5\rangle, |\bar{P}\rangle) = 0.094 < d_0 \end{cases} \tag{16}$$

So we recognize them as a class, the state recognizer should generate an "on" signal, the actuator receives another M copies from the cloner as feedback and generates new quantum signal to drive the object.

Remark 1: In this feedback strategy, the problem of designing quantum feedback algorithms is separated into two steps: a state recognition step and a feedback control step. The aim of state recognition step is to obtain information and the process involves quantum measurement, so it is destructive. However, the feedback process doesn't necessarily acquire information and doesn't involve measurement, so it can preserve quantum coherence. In fact, we give a compromise between information acquisition and measurement disturbance in view of the characteristics of measurement in quantum control. The compromise is realized through approximately cloning and renouncing precise feedback. Since the coherence of feedback information can be preserved, this feedback strategy can perform coherent feedback control at the cost of feeding precisely back the output.

4. CONCLUSIONS

Quantum control theory and quantum information technology are two rapidly developing new subjects. This paper proposes a scheme of quantum feedback control with an optimal cloning machine in quantum information technology. In this method, the design of quantum feedback control algorithms is separated into a state recognition strategy, which gives "on-off" signal to the actuator through recognizing some copies from the cloning machine, and a feedback (control) strategy with the another copies of cloning machine. The recognition process involves quantum measurement and is destructive, however, the feedback process is preserving quantum coherence, so this scheme can perform some quantum control tasks with coherent feedback, and also points out a new path for quantum feedback control design using quantum information technology. Besides quantum control, this quantum feedback scheme also has important potential application to large-scale quantum computation, quantum communication network and quantum robot.

REFERENCES

Clark, J.W., D. G. Lucarelli and T. J. Tarn (2003). Control of quantum systems. *International Journal of Modern Physics B*, *17*, 5397—5411.

D`Alessandro, D., and M. Dahleh (2001). Optimal control of two-level quantum systems. *IEEE Transactions on Automatic Control*, *46*, 866—876.

Doherty, A.C., S. Habib, K. Jacobs, H. Mabuchi, and S. M. Tan (2000). Quantum feedback control and classical control theory. *Phys. Rev. A*, *62*, 012105.

Doherty, A.C., S. Habib, and G. Jungman (2001). Information, disturbance, and Hamiltonian quantum feedback control. *Phys. Rev. A*, *63*, 062306.

Doherty, A.C. and K. Jacobs (1999). Feedback control of quantum systems using continuous state estimation. *Phys. Rev. A*, *60*, 2700—2711.

Dong, D.Y. and Z. H. Chen (2003). Clustering recognition of quantum states based on quantum module distance. *Acta Sinica Quantum Optica*, *9*, 144—148.

Duan, L.M. and G. C. Guo (1998). Probabilistic cloning and identification of linearly independent quantum states. *Phys. Rev. Lett.*, *88*, 4999—5002.

Giovannetti, V., P. Tombesi and D. Vitali (1999). Non-Markovian quantum feedback from homodyne measurements: The effect of a nonzero feedback delay time. *Phys. Rev. A*, *60*, 1549—1561.

Gisin, N. and S. Massar (1997). Optimal quantum cloning machines. *Phys. Rev. Lett.*, *79*, 2153—2156.

Lloyd, S.(2000). Coherent quantum feedback. *Phys. Rev. A*, *62*, 022108.

Nielsen, M.A. and I. L. Chuang (2000). Quantum computation and quantum information. Cambridge: Cambridge University Press.

Rabitz, H., R. de Vivie-Riedle, M. Motzkus and K. Kompa (2000). Whither the future of controlling quantum phenomena?. *Science, 288*, 824－828.

Ruskov, R. and A. N. Korotkov (2002). Quantum feedback control of a solid-state qubit. *Phys. Rev. B*, *66*, 041401R.

Solomon, A.I. and S. G. Schrirmer (2002). Limitations in quantum control. *International Journal of Modern Physics B*, *16*, 2107－2112.

Ting, J.J.L.(2002). Alternative method for quantum feedback-control. *Superlattices and Microstructures*, *32*, 331－336.

Wang, J. and H. M. Wiseman (2001). Feedback-stabilization of an arbitrary pure state of a two-level atom. *Phys. Rev. A*, *64*, 063810.

Wiseman, H.M., S. Mancini and J. Wang (2002). Bayesian feedback versus Markovian feedback in a two-level atom. *Phys. Rev. A*, *66*, 013807.

Wiseman, H.M. and G. J. Milburn (1993). Quantum theory of optical feedback via homodyne detection. *Phys. Rev. Lett.*, *70(5)*, 548－551.

Wootters, W. K. and W. H. Zurek (1982). A single quantum cannot be cloned. *Nature*, *299*, 802－803.

Yamamoto, Y., N. Imoto and S. Machida (1986). Amplitude squeezing in a semiconductor laser using quantum nondemolition measurements and negative feedback. *Phys. Rev. A*, *33(5)*, 3243－3261.

Yanagisawa, M. and H. Kimra (2003a). Transfer function approach to quantum control—part I: Dynamics of quantum feedback systems. *IEEE Transactions on Automatic Control*,*48(12)*, 2107－2120.

Yanagisawa, M. and H. Kimra (2003b). Transfer function approach to quantum control—part II: Control concepts and applications. *IEEE Transactions on Automatic Control*,*48(12)*, 2121－2132.