

HYBRID FAULT DETECTION AND ISOLATION METHOD FOR UAV INERTIAL SENSOR REDUNDANCY MANAGEMENT SYSTEM

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Abstract: Redundant system with three two-degree-of-freedom (2-DOF) inertial sensors is one of the possible candidates for the UAV inertial sensor redundancy management system. In the conventional FDI techniques using hardware redundancy, at least four 2-DOF inertial sensors are needed to detect and isolate the faulty sensor. Since two input axes of 2-DOF inertial sensors are mechanically correlated with each other, the fault of one axis sensor can affect the fault of the other axis sensor. When three sensors are used, singular direction problem can be occurred in detecting process, where false alarm may be declared such that two sensors have bias fault although only one sensor is out of order. Therefore, the study of multiple fault detection and isolation (FDI) technique is required to deal with these problems. In this study, a hybrid FDI technique is proposed for multiple FDI of three 2-DOF sensor system. The proposed FDI algorithm is based on hardware redundancy and is combined with an analytic redundancy by utilizing the unscented Kalman Filter. Numerical simulations are performed to verify the effectiveness of the proposed FDI technique. *Copyright © 2005 IFAC*

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1. INTRODUCTION

Failures or faults in the flight control system result in the mission incompleteness or catastrophic situation such as crash. The reliability of the system, therefore, is an important issue. Inertial navigation system (INS) carries out central functions in maintaining the stability of the aircraft system which has complicated dynamic characteristics. Because the failure of INS may result in fatal accident, recent researches have focused on the improvement of the reliability of INS components. To deal with the fault of the INS components, redundant sensors have been equipped and the hardware redundancy management method has been used (Gilmore and McKern, 1972; Hung and Doran, 1973; Wilcox, 1974).

In fault detection and isolation (FDI), the minimum number of the redundant sensors is dependent on the

degree of freedom (DOF) of the sensors. In the case that each sensor has single DOF and only one fault occurs, the system needs at least 4 sensors for the fault detection, but 5 sensors are required for the fault isolation. In the case in which 2-DOF sensors are used, the system needs at least two sensors for the fault detection and 4 sensors for the fault isolation.

In this paper, three 2-DOF inertial sensor system is considered. The optimal configuration is obtained by optimizing the navigation performance as well as FDI performance. Based on the optimum configuration, the hybrid FDI algorithm for fault detection and isolation is proposed. To detect and isolate the faults in the system with limited hardware redundancy, the hybrid FDI algorithm is proposed using hardware redundancy as well as analytic redundancy. Unscented Kalman filter is adopted for analytic redundancy. To show the effectiveness of

the proposed FDI algorithm, numerical simulation is performed.

2. GN&C AND FDI PERFORMANCE INDICES

In this study, to construct the optimal configuration of the sensor system, two performance indices are defined; guidance, navigation, and control (GN&C) performance and FDI performance (Gilmore and McKern, 1972).

2.1 GN&C Performance Index

The measurement equation for the sensor system is represented by

$$\mathbf{m} = H\mathbf{x} + \mathbf{e} \quad (1)$$

where \mathbf{m} is the measurement vector, \mathbf{x} is the state vector including the navigation information of each axis, \mathbf{e} denotes the measurement noise, and H is the measurement matrix which is composed of the direction vectors of sensors with respect to the body frame. When n single DOF sensors are used, vectors are defined as $\mathbf{m} = [m_1 m_2 \cdots m_n]^T$, and $\mathbf{x} = [x_1 x_2 x_3]^T$, $\mathbf{e} = [e_1 e_2 \cdots e_n]^T$. It is assumed that the measurement noise \mathbf{e} is zero-mean white noise with covariance σ^2 , i.e., $E[\mathbf{e}] = 0$, $E[\mathbf{e}\mathbf{e}^T] = \sigma^2 I_n$.

From the measurement equation, the state vector can be estimated using the least square method using the following equation.

$$\hat{\mathbf{x}} = (H^T H)^{-1} H^T \mathbf{m} \quad (2)$$

By regularizing σ^2 , generalized error covariance matrix C can be derived as

$$C = E[(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^T] = (H^T H)^{-1} \quad (3)$$

Assuming that \mathbf{e} is a zero-mean Gaussian process, the probability distribution function can be defined as

$$P_e(\boldsymbol{\eta}) = (2\pi)^{-n/2} |C|^{-1/2} \exp\left(-\frac{1}{2} \boldsymbol{\eta}^T C^{-1} \boldsymbol{\eta}\right) \quad (4)$$

where $\boldsymbol{\eta} = \mathbf{x} - \hat{\mathbf{x}}$. Let us define a nonnegative variable k as $k = \boldsymbol{\eta}^T C^{-1} \boldsymbol{\eta}$. Equation (4) can be described as an ellipsoid and it can be transformed to a sphere of which radius is $k^{1/2}$ by coordinate transform, where the volume of the sphere is

$$V = \frac{4}{3} \pi k^{3/2} \sqrt{|C|} \quad (6)$$

The volume V represents the magnitude of the measurement error. Because the volume is proportional to $\sqrt{|C|}$, the GN&C performance index can be defined as

$$J_{GNC} = \min \sqrt{|C|} = \min \sqrt{|H^T H|^{-1}} \quad (7)$$

2.2 FDI Performance Index

Parity equations are constructed so that each equation is used to detect the fault of corresponding sensor. Parity equation is defined as a linear combination of the measurement output as follows

$$\mathbf{p} = V_n^T \mathbf{m} \quad (8)$$

To make the parity vector \mathbf{p} be independent of the sensor input \mathbf{x} , the following equation should be satisfied.

$$H^T V_n = 0 \quad (9)$$

Consider the equation that detects the faults of the 1st sensor, $p_1 = \mathbf{v}_{1n}^T \mathbf{m}$ (where, $\mathbf{v}_{1n}^T = [v_{11} v_{12} \cdots v_{1n}]$). To make p_1 be sensitive only to m_1 , $\sum_{k=2}^n v_{1k}^2$ should be small compared to v_{11} . By taking $v_{11} = 1$, the following equation can be obtained from Eq.(9).

$$H^T \mathbf{v}_{1n} = 0 \quad (10)$$

$$H_{n-1}^T \mathbf{v}_{1(n-1)} + \mathbf{h}_1 = 0 \quad (11)$$

where $\mathbf{v}_{1(n-1)}^T = [v_{12} v_{13} \cdots v_{1n}]$, $H_{n-1}^T = [\mathbf{h}_2 \mathbf{h}_3 \cdots \mathbf{h}_n]$.

The solution of the Eq.(11) can be obtained as

$$\mathbf{v}_{1(n-1)} = -H_{n-1} (H_{n-1}^T H_{n-1})^{-1} \mathbf{h}_1 \quad (12)$$

Substituting Eq.(12) into the Eqs.(9) and (10) yields

$$p_1 = m_1 - \mathbf{h}_1^T (H_{n-1}^T H_{n-1})^{-1} H_{n-1}^T \mathbf{m}_{n-1} = m_1 - \hat{m}_1 \quad (13)$$

where $\mathbf{m}_{n-1} = [m_2 m_3 \cdots m_n]^T$.

Similarly, n parity equations can be defined for each sensor. Let us assume that the statistical properties of the parity equation are

-when there is no fault

$$E[p_i] = 0, \quad \sigma_i^2 = \mathbf{v}_{in}^T \mathbf{v}_{in} \sigma^2$$

-when a fault has occurred

$$E[p_i] = v_{ij}, \quad \sigma_i^2 = \mathbf{v}_{in}^T \mathbf{v}_{in} \sigma^2$$

From this statistical property, the following distance measure can be defined,

$$J_{ij} = (\mathbf{v}_{in}^T \mathbf{v}_{in})^{-1} v_{ij}^2 \quad (14)$$

where J_{ij} means the distance measure of the i -th parity equation between the normal state and the faulty state of j -th sensor. Now, the figure of merit of FDI can be defined as

$$F_i = J_{ii} / \max J_{ij}, \quad i \neq j \quad (15)$$

Note that large F_i' represents that the i -th parity equation p_i is much sensitive to the i -th sensor fault. Therefore, the FDI performance index can be defined as follows

$$J_{FDI} = \min[F_i'] \text{ for all } i \quad (16)$$

3. OPTIMAL CONFIGURATION OF THE SENSOR SYSTEM

In this study, a system with 2-DOF sensors is considered. To obtain the optimal configuration of the sensor system, the GN&C performance should be optimized first. The input axis of the sensor is orthogonal to the spin axis in the case of 2-DOF inertial sensor. From this property the following equation can be obtained (Harrison and Gai, 1977).

$$\begin{aligned} H^T H &= \mathbf{h}_{11} \mathbf{h}_{11}^T + \mathbf{h}_{12} \mathbf{h}_{12}^T + \mathbf{h}_{21} \mathbf{h}_{21}^T \cdots + \mathbf{h}_{n2} \mathbf{h}_{n2}^T \\ &= (I - \mathbf{h}_{s1} \mathbf{h}_{s1}^T) + \cdots + (I - \mathbf{h}_{sn} \mathbf{h}_{sn}^T) \\ &= nI - H_s^T H_s \end{aligned} \quad (17)$$

where \mathbf{h}_{ij} denotes the j -th input axis direction vector of i -th 2-DOF sensor, \mathbf{h}_{si} denotes the direction vector of i -th sensor spin axis, n represents the number of 2-DOF sensors, and H_s is defined as

$$H_s = [\mathbf{h}_{s1}^T \cdots \mathbf{h}_{sn}^T]^T.$$

The determinant of $nI - H_s^T H_s$ can be calculated as

$$\det(H^T H) = \det(nI - H_s^T H_s) = \prod_{i=1}^n \lambda_i \quad (18)$$

where λ_i denotes the eigenvalues of $H^T H$.

To calculate the trace of the matrix $H^T H$, the matrix is expanded as follow

$$H^T H = \mathbf{h}_{11} \mathbf{h}_{11}^T + \mathbf{h}_{12} \mathbf{h}_{12}^T + \mathbf{h}_{21} \mathbf{h}_{21}^T \cdots + \mathbf{h}_{n1} \mathbf{h}_{n1}^T + \mathbf{h}_{n2} \mathbf{h}_{n2}^T \quad (19)$$

Because \mathbf{h}_{ij} is the direction cosine vector, its magnitude is 1. Therefore, the following equation is satisfied.

$$\text{trace}(\mathbf{h}_{ij} \mathbf{h}_{ij}^T) = \mathbf{h}_{ij}^T \mathbf{h}_{ij} = 1 \quad (20)$$

where $i=1, 2, 3, \dots, n$, and $j=1, 2$.

Hence,

$$\text{trace}(H^T H) = 2n = \sum_{i=1}^n \lambda_i \quad (21)$$

Since the arithmetic mean is always equal to or larger than the geometric mean, we have

$$\frac{1}{n} \sum_{i=1}^n \lambda_i \geq \left(\prod_{i=1}^n \lambda_i \right)^{1/n} \quad (22)$$

From Eqs. (7), (18) and (22), minimum GN&C performance can be obtained when $\lambda_1 = \lambda_2 = \cdots = \lambda_n$.

In this study, a triple 2-DOF sensor system is considered, i.e., $n=3$. Therefore, from Eq.(21) the eigenvalues of $H^T H$ for the optimum GN&C configuration are

$$\lambda_1 = \lambda_2 = \lambda_3 = \frac{2n}{3} = 2 \quad (23)$$

or

$$H^T H = \frac{2}{3} nI = 2I \quad (24)$$

From the Eqs.(17) and (24),

$$H_s^T H_s = \frac{n}{3} I = I \quad (25)$$

Therefore, the axes optimizing the GN&C performance of redundant 2-DOF sensors can be defined as

$$\mathbf{h}_{s1} = [0 \ 0 \ 1]^T, \quad \mathbf{h}_{s2} = [0 \ 1 \ 0]^T, \quad \mathbf{h}_{s3} = [1 \ 0 \ 0]^T \quad (26)$$

If a spin axis of 2-DOF inertial sensor is fixed, two input axes can be represented by one parameter. The measurement matrix H , therefore, is a function of three parameter, ϕ, θ, ψ . The matrix H can be represented as

$$H = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ \sin \theta & 0 & \cos \theta \\ -\cos \theta & 0 & \sin \theta \\ 0 & \cos \psi & \sin \psi \\ 0 & -\sin \psi & \cos \psi \end{bmatrix} \quad (27)$$

Using the gradient based optimization technique, FDI performance is optimized for the triple 2-DOF sensor system. One of the optimum configurations is obtained as

$$(\phi, \theta, \psi) = \left(\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{4} \right) \quad (28)$$

In this case, the value of the FDI performance index is $J_{FDI} = 4$, and the optimum configuration is shown in Fig.1.

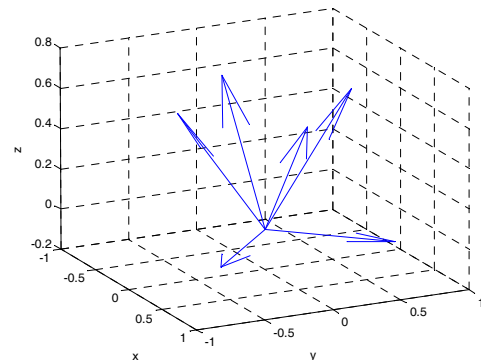


Fig1. The Optimal Configuration of Three 2-DOF Sensors

4. FDI ALGORITHM

For the 2-DOF inertial sensors system, there exists a strange property in the fault occurrence. Because two input axes of each sensor are correlated, simultaneous faults on both axes occur more frequently than the single axis fault. Therefore, if only the hardware redundancy method is used, then at least four sensors are required to carry out the FDI process. In this study, to overcome this problem, hardware redundancy method is combined with the analytic redundancy method. First, parity space approach (PSA) is modified to deal with the multiple axes faults (Potter and Deckert, 1972). And the unscented Kalman filter is used as an analytic redundancy method (Julier et al, 1995; Wan and Van Der Merwe, 2000).

4.1 Modified PSA and Singular Direction Problem

Consider a matrix V satisfying the following condition.

$$VH = 0, \quad VV^T = I \quad (29)$$

The parity vector p can be defined as follows

$$p = Vm = V(Hx + e + f) = Ve + Vf \quad (30)$$

The fault plane is defined as the plane on which the parity vector places, when a fault occurs. For example, when the 1st sensor has two-axis multiple fault, the parity vector lies on the plane which the 1st and the 2nd column vectors of matrix V , i.e., v_{c1} and v_{c2} , span. Now, three planes can be defined as

- 1st sensor fault plane : $span(v_{c1}, v_{c2})$
- 2nd sensor fault plane : $span(v_{c3}, v_{c4})$
- 3rd sensor fault plane : $span(v_{c5}, v_{c6})$

For the FDI process, a fault detection function and a fault isolation function should be defined. The fault detection function is defined as follows

$$FD = p^T p \quad (31)$$

If the function FD has a value over the threshold value T_D , fault detection process makes an alarm that a fault occurs in the sensor system. Note that the function has a χ^2 distribution when the sensor system is healthy, and therefore, the threshold value is determined to satisfy the following condition.

$$P(\chi^2 \geq T_D) = a \quad (32)$$

where a is a false alarm probability.

The fault isolation function FI_k ($k = 1, 2, 3$) is defined by the angle between each fault plane and the parity vector. The specific sensor will be declared the fault occurrence, when the corresponding

isolation function has a value less than the threshold value T_i , and the faulty sensor can be isolated.

When three 2-DOF inertial sensors are used, most of the fault can be detected and isolated by the modified parity space approach. However, if the parity vector lies near the intersection line of the fault planes, then the FDI algorithm may declare that two sensors are out of order, even though fault occurs in only one sensor. Therefore, to cope with this singular direction problem, additional sensors (hardware redundancy) or additional algorithms (software redundancy) are required. In this study, an analytic redundancy method is combined to deal with this problem.

4.2 Analytic method using Unscented Kalman Filter

When the modified parity space method declares that faults occur in two sensors, the additional analytic method is used to confirm the number of the faulty sensors. In this study, the unscented Kalman filter is used to generate an analytic residual for FDI. To avoid the risk of using contaminated data from the faulty sensor, the filter uses the data that are several steps ahead from the instance of fault alarm. The system model of the Kalman filter of a nonlinear aircraft model is represented by

$$\begin{aligned} x(k+1) &= f(x(k), u(k), v(k), k) \\ z(k) &= h(x(k), u(k), k) + w(k) \end{aligned} \quad (33)$$

where

$$\begin{aligned} E[v(i)v(i)^T] &= Q(i) \\ E[w(i)w(i)^T] &= R(i) \\ E[v(i)w(i)^T] &= 0, \quad \forall i, j \end{aligned} \quad (34)$$

UKF estimation algorithm is summarized as follows.

Step 1. Initialization

$$\hat{x}_0 = E[x_0] \quad (35)$$

$$P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] \quad (36)$$

Step 2. Sigma points generation

$$X_{k-1} = [\hat{x}_{k-1} \quad \hat{x}_{k-1} + \sqrt{(L+\lambda)P_{k-1}} \quad \hat{x}_{k-1} - \sqrt{(L+\lambda)P_{k-1}}] \quad (37)$$

Step 3. Time update

$$X_{k|k-1} = f(X_{k-1}, u_{k-1}) \quad (38)$$

$$\hat{x}_k^- = \sum_{i=0}^{2L} W_i^{(m)} X_{i,k|k-1} \quad (39)$$

$$P_k^- = \sum_{i=0}^{2L} W_i^{(c)} [X_{i,k|k-1} - \hat{x}_k^-][X_{i,k|k-1} - \hat{x}_k^-]^T + Q(k) \quad (40)$$

$$Y_{k|k-1} = h(X_{k|k-1}) \quad (41)$$

$$\hat{y}_k^- = \sum_{i=0}^{2L} W_i^{(m)} Y_{i,k|k-1} \quad (42)$$

Step 4. Measurement update

$$P_{\hat{y}_k \hat{y}_k} = \sum_{i=0}^{2L} W_i^{(c)} [Y_{i,k|k-1} - \hat{y}_k^-] [Y_{i,k|k-1} - \hat{y}_k^-]^T + R(k) \quad (43)$$

$$P_{x_k y_k} = \sum_{i=0}^{2L} W_i^{(c)} [X_{i,k|k-1} - \hat{x}_k^-] [Y_{i,k|k-1} - \hat{y}_k^-]^T \quad (44)$$

$$K_k = P_{x_k y_k} P_{\hat{y}_k \hat{y}_k}^{-1} \quad (45)$$

$$\hat{x}_k = \hat{x}_k^- + K_k (y_k - \hat{y}_k^-) \quad (46)$$

$$P_k = P_k^- - K_k P_{\hat{y}_k \hat{y}_k} K_k^T \quad (47)$$

The state vector estimated by UKF is used for generating the residuals as follows

$$r_i = m_i - H_i \hat{x}_k \quad (48)$$

$$r_j = m_j - H_j \hat{x}_k \quad (49)$$

Each residual is used to isolate the fault sensor among the i -th and j -th sensors that are declared as faulty sensors by modified PSA. Comparing each residual with the threshold value, actual faulty sensor can be isolated. Figure 2 shows the state update process using UKF. If multiple fault (2nd and 3rd sensors in this particular example) is declared, the states are initialized using the states that have much less possibility of being contaminated by the fault effects (states of three steps earlier before the fault alarm in this particular example). Estimated states by this process is used to compute the residual of Eqs. (48) and (49).

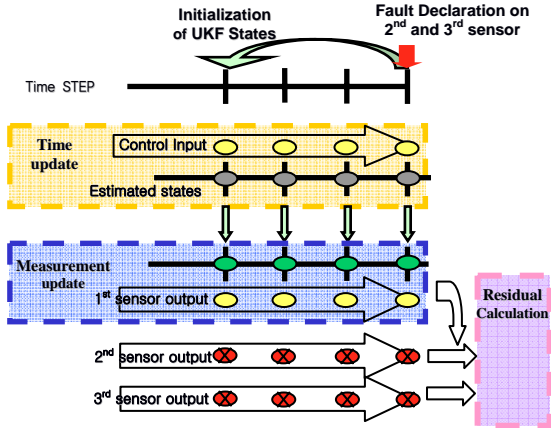


Fig.2 Updating States by UKF

5. NUMERICAL EXAMPLES

To verify the performance of the proposed algorithm, numerical simulation is performed using a nonlinear F-16 aircraft model which has a sensor system of three 2-DOF inertial sensors. The optimal configuration discussed in Section 3 is considered. It is assumed that each sensor signal includes a white noise with the standard deviation of 0.01rad/sec. Numerical simulation considers the flight mission that the aircraft climbs with the velocity of 120ft/sec for 5 seconds and performs a bank turn maneuver for 10 seconds. Figure 3 shows the flight trajectory of the considered aircraft mission.

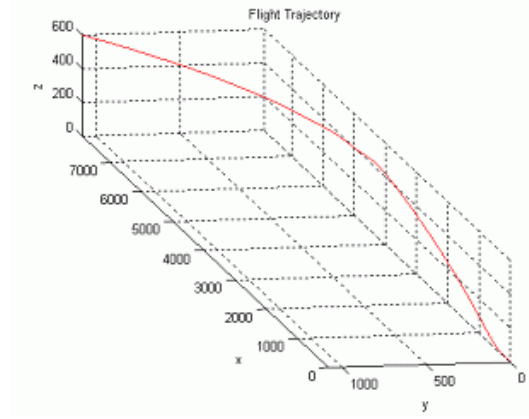


Fig.3 Flight Mission

Stability augmentation system is designed using LQR. The block diagram of the FDI process is shown in Fig.4. It is assumed that fault occurs in the first sensor at one second, i.e., $f^T = [0.1 \ 0.1 \ 0 \ 0 \ 0 \ 0]$ for $t \geq 1$ sec.

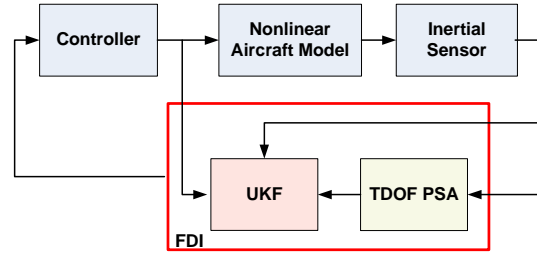


Fig.4 Block Diagram of FDI Process

Figures 5 and 6 show the result of the PSA. From Fig.5, it can be easily seen that the fault detection process works properly. However, although only the 1st sensor has fault, PSA reports that the 3rd sensor is also out of order as shown in Fig.6. That is, fault isolation cannot be performed properly without additional information.

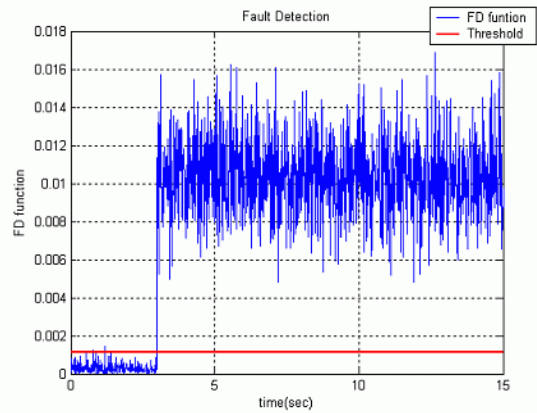


Fig.5 Fault Detection Function

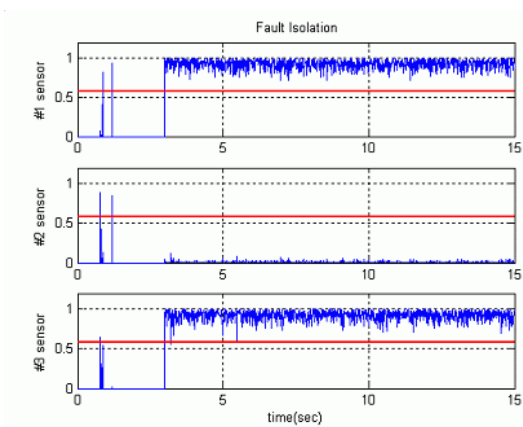


Fig.6 Fault Isolation Function

Figure 7 shows the residual between the output of the sensor system and the estimated output from the UKF. It shows that the fault occurs only in the 1st sensor. By using the additional analytical redundancy, correct isolation is properly performed.

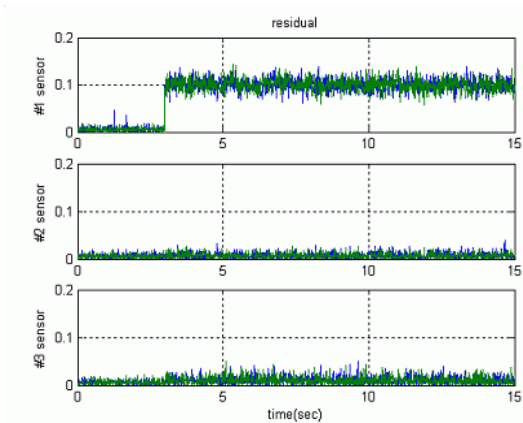


Fig.7 Residual Produced by UKF

The flight trajectory using the proposed algorithm is shown in Fig.8. The flight path shows that the proposed FDI algorithm works properly to accomplish the given mission.

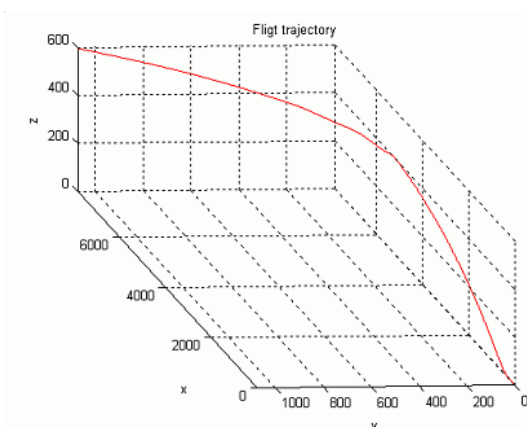


Fig.8 Flight Path with the Proposed Algorithm

6. CONCLUDING REMARKS

In this paper, the optimal configuration of three 2-DOF inertial sensors is obtained by optimizing the GN&C performance and FDI performance.

Unscented Kalman filter is used to compensate the restriction of the hardware redundancy method of PSA. UKF is used to generate the residuals only when the false alarm is given by the fault detection process. Numerical simulation is performed using nonlinear aircraft model to verify the performance of the proposed FDI.

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