

IDENTIFICATION OF DYNAMIC ERRORS-IN-VARIABLES SYSTEMS WITH PERIODIC DATA

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Abstract: Using instrumental variable methods to estimate the parameters of dynamic errors-in-variables systems with a periodic input signal is the focus in this paper. How to choose suitable instrumental variable vectors is the key point. Two variants are proposed; both of them can generate consistent estimates. An analysis shows that the best accuracy is achieved by using a specific overdetermined instrumental variable vector. Numerical illustrations demonstrate the effectiveness of the proposed Extended IV method for both white and colored measurement noise. It is superior to alternative methods under low signal to noise ratios. *Copyright ©2005 IFAC*

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1. INTRODUCTION

The problem of identification of dynamic errors-in-variables (EIV) systems appear in very wide scientific areas, such as time series modelling, array signal processing for direction-of-arrival estimation, blind channel equalization, multivariate calibration in analytical chemistry, image processing, astronomical data reduction, etc. A number of methods were elaborated in the past few decades. For example, attempts using instrumental variable estimates (Söderström and Stocica, 1983) or the Frisch scheme (Beghelli *et al.*, 1990) and other bias-compensating of the least-squares estimates were developed. Other methods are based on the frequency domain (Pintelon and Schoukens, 2001), or use higher-order statistics (Tugnait and Ye, 1995). It is also possible to apply the maximum likelihood and the prediction error approaches (Söderström, 1981). Some comparison between different approaches are given in (Söderström *et al.*, 2002).

For a system, when data are measured from a number of independently repeated experiments, the identification process can be designed under the periodic excitation condition. Periodic input signals offer interesting advantages over non-periodic excitations not only in frequency domain but also in time domain identification, see (Schoukens *et al.*, 1997), (Forsell *et al.*, 1999). For example, in (Forsell *et al.*, 1999), the periodic excitation was utilized to separate the driving signal and noise. Estimated noise models can be further exploited as pre-whitening filters implemented before the estimation. In this paper, instrumental variable (IV) methods will be considered because IV estimators are computationally inexpensive and are applicable under fairly general noise conditions. Our main motivation is to analyze what type of instrumental variables should be chosen to maximally utilize the information of periodic measurement data to achieve the optimal estimation accuracy. The derived IV estimator shows considerably improved accuracy compared to traditional ones.

The outline of this paper is as follows: Section 2 gives some assumptions and preliminary issues regarding errors-in-variables models. Two instrumental variable variants are also proposed in this section. Consistency and accuracy of these IV estimators are analyzed in Sections 3 and 4. Simulations are given in Section 5. Finally, some conclusions are drawn in Section 6.

2. PRELIMINARIES

Consider the system depicted in Figure 1.

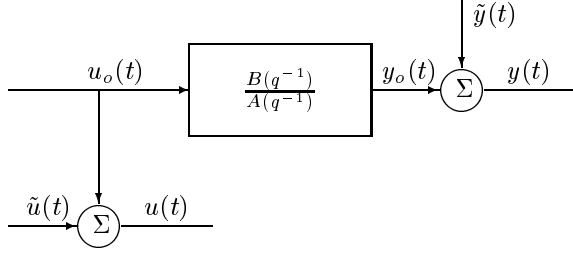


Fig. 1. The basic setup for a dynamic errors-in-variables problem.

The system is given by the difference equation

$$A(q^{-1})y_o(t) = B(q^{-1})u_o(t) \quad (1)$$

where

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + \dots + a_n q^{-n} \\ B(q^{-1}) &= b_1 q^{-1} + \dots + b_n q^{-n} \end{aligned} \quad (2)$$

are polynomial in the backward shift operator q^{-1} , i.e. $q^{-1}x(t) = x(t-1)$, and $y_o(t)$, $u_o(t)$ denote the noise free output and input, respectively.

It is generally assumed that the system is asymptotically stable and that the model order n is known. The model can easily be extended by allowing different degrees of A and B and by introducing a further delay. To keep the treatment reasonably simple these extensions are not dealt with here.

The input and the output are not directly available, but can be measured with some noise: $\tilde{u}(t)$ and $\tilde{y}(t)$, respectively. It is the noise corrupted measurements $u(t)$ and $y(t)$ that are available.

We next introduce some general assumptions.

A1. The noise free signal $u_o(t)$ is a periodic function. The length of the period is denoted N . It is assumed that m periods of the data $u(t)$, $y(t)$ are available. In each period $u_o(t)$ is a stationary process.

A2. The measurement noise signals $\tilde{u}(t)$ and $\tilde{y}(t)$ are uncorrelated with the noise free input $u_o(s)$ for all t and s . Further, the measurement noise signals within different periods are uncorrelated.

For simplicity we will generally assume that all signals have zero mean values.

Next introduce the regressor vector $\varphi(t)$ as

$$\varphi(t) = [-y(t-1) \dots -y(t-n) \ u(t-1) \dots u(t-n)]^T \quad (3)$$

We can then form a linear regressive model as

$$y(t) = \varphi^T(t)\theta + \varepsilon(t) \quad (4)$$

where the parameter vector θ is

$$\theta = [a_1 \dots a_n \ b_1 \dots b_n]^T \quad (5)$$

Due to the presence of noise in both y and u , a linear least squares estimate of θ will be biased and not consistent. Instead we will consider instrumental variable estimates, which are obtained by correlating the model with a vector $z(t)$ of instruments:

$$\sum_t z(t)y(t) \cong \left[\sum_t z(t)\varphi^T(t) \right] \theta \quad (6)$$

The vector $z(t)$ must have at least as many elements as θ . When $z(t)$ is larger, we solve (6) in a weighted least square sense to get

$$\hat{\theta} = \arg \min_{\theta} \left\| \left[\sum_t z(t)\varphi^T(t) \right] \theta - \left[\sum_t z(t)y(t) \right] \right\|_Q^2 \quad (7)$$

leading to

$$\hat{\theta} = (\hat{R}^T Q \hat{R})^{-1} \hat{R}^T Q \hat{r} \quad (8)$$

with

$$\hat{R} = \frac{1}{Nm} \sum_{t=1}^{Nm} z(t)\varphi^T(t) \quad \hat{r} = \frac{1}{Nm} \sum_{t=1}^{Nm} z(t)y(t). \quad (9)$$

In (7), (8) Q is a positive definite weighting matrix. The summations in (6)-(9) are over all time points (in the m periods), that is, t goes from 1 to Nm .

Next we introduce some more detailed notation, where we exploit the periodicity of the noise free data $u_o(t)$, $y_o(t)$. In period j (where $1 \leq j \leq m$), we write the regressor vector as

$$\varphi_j(t) = \varphi^0(t) + \tilde{\varphi}_j(t) \quad t = 1, \dots, N \quad (10)$$

where $\varphi^0(t)$ contains the noise free data and $\tilde{\varphi}_j(t)$ denotes the noise contributions. Similarly, $z_j(t)$ (for $1 \leq j \leq m$) will denote the instrumental vector for period j .

We proceed to give two variants of instrumental variable vectors.

IV. It is natural to let the vector $z_j(t)$ be formed by regressors other than $\varphi_j(t)$. A simple choice is to take

$$z_j(t) = \begin{cases} \varphi_{j+1}(t) & j = 1, \dots, m-1 \\ \varphi_1(t) & j = m \end{cases} \quad (11)$$

■

In the above example z_j and φ_j will have the same dimensions. Then \hat{R} in (8), (9) will be a square matrix and the weighting Q becomes superfluous.

Extended IV. Consider an overdetermined instrumental variable vector as

$$\begin{aligned} z_1(t) &= [\varphi_2^T(t) \dots \varphi_m^T(t)]^T \\ z_2(t) &= [\varphi_3^T(t) \dots \varphi_m^T(t) \varphi_1^T(t)]^T \\ &\vdots \\ z_j(t) &= [\varphi_{j+1}^T(t) \dots \varphi_m^T(t) \varphi_1^T(t) \\ &\quad \dots \varphi_{j-1}^T(t)]^T \end{aligned} \quad (12)$$

■

For a general case we introduce the following assumption:

A3. The instrumental vector $z_j(t)$ is uncorrelated to the measurement noise in period j . Further, the covariance matrix of $z_j(t)$ has rank at least equal to $\dim(\theta)$.

It will be convenient to further introduce the following matrices

$$\begin{aligned} \phi(t) &= [\varphi_1(t) \dots \varphi_m(t)] \\ Z(t) &= [z_1(t) \dots z_m(t)] \end{aligned} \quad t = 1, \dots, N \quad (13)$$

and the vector

$$Y(t) = [y_1(t) \dots y_m(t)]^T \quad t = 1, \dots, N \quad (14)$$

where $y_j(t)$ is the output at time t within period j :

$$y_j(t) = y(t + (j - 1)N). \quad (15)$$

The basic equation (6) can be compactly written as

$$\left[\sum_{t=1}^N Z(t)\phi^T(t) \right] \theta \cong \left[\sum_{t=1}^N Z(t)Y(t) \right] \quad (16)$$

We write the estimate as, see (8)

$$\hat{\theta} = (\hat{R}^T Q \hat{R})^{-1} \hat{R}^T Q \hat{r} \quad (17)$$

where now

$$\hat{R} = \frac{1}{Nm} \sum_{t=1}^N Z(t)\phi^T(t) = \frac{1}{Nm} \sum_{j=1}^m \sum_{t=1}^N z_j(t)\varphi_j^T(t) \quad (18)$$

$$\hat{r} = \frac{1}{Nm} \sum_{t=1}^N Z(t)Y(t) = \frac{1}{Nm} \sum_{j=1}^m \sum_{t=1}^N z_j(t)y_j(t) \quad (19)$$

When analyzing the general estimate (17), it is worth noticing that the underlying model equation (16) take the form of a multivariable instrumental variance estimate (Söderström and Stoica, 1989, page 262). This observation will be useful when examining the statistical properties of the estimate (17).

In the analysis, we generally assume that the model structure captures the true dynamic. More precisely, we assume that there is a true parameter vector θ_0 such that

$$y(t) = \varphi^T(t)\theta_0 + v(t) \quad (20)$$

where

$$v(t) = A_0(q^{-1})\tilde{y}(t) - B_0(q^{-1})\tilde{u}(t). \quad (21)$$

Introduce the vector $V(t)$ similarly to $Y(t)$, (14). The relation (20) now simplifies to

$$Y(t) = \phi^T(t)\theta_0 + V(t) \quad (22)$$

Inserting (22) into (19), and combining this with (17) gives

$$\hat{\theta} - \theta_0 = (\hat{R}^T Q \hat{R})^{-1} \hat{R}^T Q \hat{r} \quad (23)$$

where

$$\hat{r} = \frac{1}{Nm} \sum_{t=1}^N Z(t)V(t) \quad (24)$$

3. CONSISTENCY ANALYSIS

To analyze consistency (*i.e.* $\lim_{N \rightarrow \infty} \hat{\theta} = \theta_0$), it follows from (23) and standard conditions, see (Söderström and Stoica, 1983), (Söderström and Stoica, 1989) that consistency is guaranteed if

$$\lim_{N \rightarrow \infty} \hat{R} \triangleq R \quad (25)$$

has full rank and

$$\lim_{N \rightarrow \infty} \hat{r} = 0 \quad (26)$$

To examine these conditions further, note that the left hand side of (26) can be written

$$\begin{aligned} \lim_{N \rightarrow \infty} \hat{r} &= \lim_{N \rightarrow \infty} \frac{1}{Nm} \sum_{j=1}^m \sum_{t=1}^N z_j(t)v_j(t) \\ &= \frac{1}{m} \sum_{j=1}^m E z_j(t)v_j(t) \end{aligned} \quad (27)$$

Hence, (26) follows as the instrumental variables are constructed such that $z_j(t)$ is uncorrelated with $v_j(t)$, see Assumption A3. Under Assumption A2 this holds for all the variants of choosing $Z(t)$ which were considered above.

To examine the condition (25) introduce the covariance matrix R_0 of the noise-free regressor vector $\varphi^0(t)$:

$$R_0 = \frac{1}{N} \sum_{t=1}^N \varphi^0(t)\varphi^0(t)^T \quad (28)$$

where R_0 is assumed to be positive definite. Note that this assumption is basically a condition on persistent

excitation of the noise-free input signal $u_0(t)$, see (Söderström and Stoica, 1989).

We find from (18) that

$$\begin{aligned} R &= \lim_{N \rightarrow \infty} \frac{1}{Nm} \sum_{j=1}^m \sum_{t=1}^N z_j(t) [\varphi^{0T}(t) + \tilde{\varphi}_j^T(t)] \\ &= \frac{1}{m} \sum_{j=1}^m \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=1}^N z_j(t) \varphi^{0T}(t) \end{aligned} \quad (29)$$

as $z_j(t)$ is uncorrelated with the noise during period j . (see Assumption A3).

We next evaluate the limit matrix R in (29), for the instrumental vectors introduced in (11) and (12). For IV we get

$$\begin{aligned} R &= \frac{1}{m} \sum_{j=1}^m \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=1}^N [\varphi^0(t) + \tilde{\varphi}_{j+1}(t)] \varphi^{0T}(t) \\ &= R_0 \end{aligned} \quad (30)$$

which is nonsingular and of full rank since R_0 is positive definite. For Extended IV we get

$$\begin{aligned} R &= \frac{1}{m} \sum_{j=1}^m \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=1}^N \left\{ \begin{bmatrix} \varphi^0(t) \\ \vdots \\ \varphi^0(t) \end{bmatrix} + \begin{bmatrix} \tilde{\varphi}_{j+1}(t) \\ \vdots \\ \tilde{\varphi}_1(t) \\ \vdots \\ \tilde{\varphi}_{j-1}(t) \end{bmatrix} \right\} \\ &\quad \times \varphi^{0T}(t) = \begin{bmatrix} R_0 \\ \vdots \\ R_0 \end{bmatrix} = e_{m-1} \otimes R_0 \end{aligned} \quad (31)$$

where $e_{m-1} = (1 \dots 1)^T$ has dimension $(m-1) \times 1$, and \otimes denotes Kronecker product. Apparently, the matrix R is of full rank in (31).

4. ACCURACY ANALYSIS

It follows from the general theory of instrumental variable estimation for multi-variable system (Söderström and Stoica, 1983), (Söderström and Stoica, 1989), that the estimation error is asymptotically Gaussian distributed as

$$\sqrt{mN}(\hat{\theta} - \theta_0) \xrightarrow{dist} \mathcal{N}(0, P) \quad (32)$$

where

$$P = P(Q) = (R^T Q R)^{-1} R^T Q S Q R (R^T Q R)^{-1} \quad (33)$$

and

$$S = E \left[\sum_{j=0}^{\infty} Z(t+j) H_j \right] \Lambda \left[\sum_{k=0}^{\infty} H_k^T Z^T(t+k) \right] \quad (34)$$

In (34) $\{H_j\}_{j=0}^{\infty}$ and Λ are defined by a spectral factorization:

$$\Phi_V(w) = \mathbb{H}(e^{iw}) \Lambda \mathbb{H}^*(e^{iw}) \quad (35)$$

along with the condition that $H_0 = I$, $\mathbb{H}(q^{-1}) = \sum_{j=0}^{\infty} H_j q^{-j}$ and $\mathbb{H}^{-1}(q^{-1})$ being asymptotically stable. In (35), $\Phi_V(w)$ denotes the spectral density matrix of the vector $V(t)$, see (22). Note that all quantities in (35) as well as $\{H_j\}$ are $m \times m$ matrices.

Due to Assumption A2, the measurement noise sequences in different periods are uncorrelated. Hence the components $v_j(t)$ of $V(t)$ are uncorrelated. Therefore the spectral density matrix $\Phi_V(w)$ is diagonal, and in fact, its diagonal elements are equal. We write this as

$$\Phi_V(w) = \phi_v(w) I \quad (36)$$

It follows that the spectral factorization on (35) can be substituted by a scalar spectral factorization:

$$\phi_v(w) = H(e^{iw}) \lambda^2 H(e^{-iw}) \quad (37)$$

$$H(q^{-1}) = \sum_{k=0}^{\infty} h_k q^{-k}, \quad h_0 = 1 \quad (38)$$

It then follows that

$$\begin{aligned} H_k &= h_k I \\ \Lambda &= \lambda^2 I \end{aligned} \quad (39)$$

Therefore, the matrix S in (34) can be simplified:

$$\begin{aligned} S &= E \left[\sum_{j=0}^{\infty} h_j Z(t+j) \right] \lambda^2 I \left[\sum_{k=0}^{\infty} h_k Z^T(t+k) \right] \\ &= E \lambda^2 \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} [h_k Z(t'-k) h_j Z^T(t'-j)] \\ &= \lambda^2 E [H(q^{-1}) Z(t)] [H(q^{-1}) Z(t)]^T \end{aligned} \quad (40)$$

4.1 Optimal weighting

The covariance matrix P in (33) apparently depends on the weighting matrix Q . There is in fact an optimal choice of the weighting matrix. It is shown in (Söderström and Stoica, 1989) that

$$P \geq P^{opt} \quad (41)$$

meaning that the difference $P - P^{opt}$ is nonnegative definite where

$$P^{opt} = (R^T S^{-1} R)^{-1} \quad (42)$$

Further, equality holds in (41) if

$$Q = S^{-1} \quad (43)$$

The weighting matrix Q is irrelevant in IV, but does appear explicitly for Extended IV. We next examine how to compute the matrix S when the instruments

are chosen as in Extended IV. First the noise $V(t)$, see (21), has to be considered. Its spectral density $\phi_v(w)$ is factorized to give $H(q^{-1})$ and λ^2 , see (39). This is a standard procedure, see for example (Söderström, 2002). Next we find from (13) and (40)

$$S_{EIV} = \lambda^2 E [H(q^{-1}) [z_1(t) \dots z_m(t)] [H(q^{-1}) [z_1(t) \dots z_m(t)]]^T] \quad (44)$$

Split the instrumental vector $z_j(t)$ into a noise-free part and a noise contribution as, see (12),

$$z_j(t) = z_j^0(t) + \tilde{z}_j(t) = e_{m-1} \otimes \varphi^0(t) + \begin{pmatrix} \tilde{\varphi}_{j+1}(t) \\ \vdots \\ \tilde{\varphi}_m(t) \\ \tilde{\varphi}_1(t) \\ \vdots \\ \tilde{\varphi}_{j-1}(t) \end{pmatrix} \quad (45)$$

We then have

Lemma Consider the instrumental vectors chosen in Extended IV. It holds that

$$S_{EIV} = \lambda^2 e_{m-1} e_{m-1}^T \otimes C_1 + \lambda^2 I_{m-1} \otimes C_2 \quad (46)$$

where

$$\begin{aligned} C_1 &= E [H(q^{-1})\varphi^0(t)] [H(q^{-1})\varphi^0(t)]^T \\ C_2 &= E [H(q^{-1})\tilde{\varphi}_j(t)] [H(q^{-1})\tilde{\varphi}_j(t)]^T \end{aligned} \quad (47)$$

Furthermore, the optimal covariance matrix becomes in this case

$$P_{EIV}^{opt} = \lambda^2 R_0^{-1} \left(C_1 + \frac{C_2}{m-1} \right) R_0^{-1} \quad (48)$$

Proof. See (Söderström and Hong, 2004).

The expression (48) gives the asymptotic covariance matrix of the parameters estimates when the optimal weighting $Q = S^{-1}$ is applied. It hence provides a lower bound on the achievable accuracy for a large class of estimators.

Next we examine whether the lower bound can be achieved for other Q . Let us evaluate the degradation of using $Q = I$. Due to the general form of the covariance matrix P in (33), it follows that

$$P(I) = (R^T R)^{-1} R^T S R (R^T R)^{-1}. \quad (49)$$

By inserting (31) and (46) in the (49), we find

$$P_{EIV}(I) = \lambda^2 R_0^{-1} \left(C_1 + \frac{C_2}{m-1} \right) R_0^{-1}. \quad (50)$$

Proof. See (Söderström and Hong, 2004).

Comparing (50) with (48), we find that the optimal performance is in fact achieved also with $Q = I$. The choice $Q = I$ is to be preferred, as it leads to significantly simpler computations than the choice (43).

4.2 Comparing the asymptotic covariance matrix of the parameters for different estimates

For IV, the covariance matrix of $\hat{\theta}$ is:

$$P_{IV} = \lambda^2 R_0^{-1} (C_1 + C_2) R_0^{-1} \quad (51)$$

Proof. See (Söderström and Hong, 2004).

From the explicit expressions (48), (50), (51) we can easily reestablish (41):

$$P_{IV} \geq P_{EIV} = P^{opt} \quad (52)$$

Further, P_{IV} does not depend on m , the number of periods, while P_{EIV} decreases with m . The noise contribution in P_{EIV} is reduced a factor $1/(m-1)$.

One of the important advantages of using periodic excitation is that signal-to-noise ratio can be improved by averaging over m periods. In Extended IV, this averaging benefit was obtained by choosing the over-determined instrumental variable vector. For m periods data, as long as the Assumption A3 is met, there are totally $(m-1)^m$ different combinations of $\varphi_k(t)$ that can be exploited for forming the instruments $z_j(t)$. We may choose instrumental vector $Z(t)$ as gathering all these $(m-1)^m$ corporations of data. Using this kind of instrumental vector was proved to have the same estimation accuracy as that of using Extended IV (see (Söderström and Hong, 2004) for details). As Extended IV is computationally the simplest alternative, it is the appropriate choice (with $Q = I$) for achieving optimal accuracy.

5. NUMERICAL ILLUSTRATIONS

5.1 Simulation results of proposed IV methods

To illustrate numerically the identification methods introduced in the previous section, we consider a second order system with

$$\begin{aligned} A(q^{-1}) &= 1 - 1.5q^{-1} + 0.7q^{-2} \\ B(q^{-1}) &= 1.0q^{-1} + 0.5q^{-2} \end{aligned} \quad (53)$$

The true input $u_0(t)$ is an ARMA process given by

$$u_0(t) = \frac{1 + 2q^{-1} + q^{-2}}{1 - 1.8q^{-1} + 0.9q^{-2}} e(t) \quad (54)$$

where $e(t)$ is a zero mean white noise sequence with variance 0.25. The number of data points N in each period is 1024, and the number of periods m is 6. For each simulation 500 realizations are done.

As long as Assumption A2 is met, there is no specified restriction on the correlation of $\tilde{u}(t)$ and $\tilde{y}(t)$ within a period. We first let the noise signals $\tilde{u}(t)$ and $\tilde{y}(t)$ be mutually uncorrelated white noise signals. The variances of the measurement noise sequences $\tilde{u}(t)$ and $\tilde{y}(t)$ are both equal to 10.

Simulation results of the IV estimates using the instrumental variable vectors as IV and Extended IV are numerically summarized in Table 1. It is clear that the theoretical analysis in the above section is well supported by the Monte-Carlo simulation. We also compared these two IV estimators for both small and large amounts of measurement noise. The benefit of using the overdetermined instrumental variable vector in Extended IV is more obvious in small SNR conditions.

Table 1. Simulation results with the white measurement noise, $\text{SNR}_{input} = 13\text{dB}$.
(s)= simulation, (t)= theory

Estimate		a_1	a_2	b_1	b_2
$\hat{\theta}_{IV}$	Mean	-1.500	0.700	1.001	0.499
	std(s)	± 0.0099	± 0.0033	± 0.137	± 0.197
	std(t)	± 0.0091	± 0.0032	± 0.129	± 0.185
$\hat{\theta}_{EIV}$	Mean	-1.499	0.700	0.996	0.507
	std(s)	± 0.0058	± 0.0030	± 0.064	± 0.093
	std(t)	± 0.0054	± 0.0028	± 0.063	± 0.090

Next, we let $\tilde{u}(t)$ and $\tilde{y}(t)$ be mutually uncorrelated, but colored measurement noise. It was found that both the IV estimators also work well in the colored noise case.

5.2 Comparison with other EIV methods with periodic data

Compensated least squares (CLS) and Compensated total least squares (CTLS) are two algorithms for dynamic EIV system identification using periodic excitation signals presented in (Forsell *et al.*, 1999). To remove the bias in the least squares estimation, these two methods first subtract away the disturbances by a non-parametric noise model, estimated directly from the measured data (based on averaging or FFT idea), and then use the least squares or total least squares estimation. For comparison, we run the Extended IV method of this paper under the same conditions as in (Forsell *et al.*, 1999). The results of a Monte-Carlo simulations are shown in the Table 2. We note that the Extended IV method works better than CLS and CTLS.

Table 2. Comparing simulation results with other algorithms.

Estimate		a_1	a_2	b_1	b_2
$\hat{\theta}_{CLS}$	Mean	-1.4991	0.6992	0.9970	0.5058
	std(s)	± 0.0246	± 0.0238	± 0.0586	± 0.0647
$\hat{\theta}_{CTLS}$	Mean	-1.5102	0.7131	1.0170	0.4698
	std(s)	± 0.0255	± 0.0235	± 0.0636	± 0.0634
$\hat{\theta}_{EIV}$	Mean	-1.5005	0.7006	1.0015	0.4983
	std(s)	± 0.0145	± 0.0141	± 0.0318	± 0.0367

6. CONCLUSIONS

The motivation of this paper was to show how the information of periodic measurement data could be

used in identifying the system with errors-in-variables models by using the instrumental variable estimates. Two different instrumental variable vectors are considered. Consistency and accuracy analysis shows that both of these IV methods would give consistent estimates.

The best accuracy is achieved when using an overdetermined instrumental variable vector. The optimal weighting matrix and the lower bound of the accuracy were discussed. The theoretical results are further supported by Monte-Carlo simulation for both white and colored measurement noise conditions. For low SNR, the advantages of using the extended instrumental variable vector is more manifest. The Extended IV estimator proposed in this paper achieves better estimation accuracy in examples than CLS and CTLS methods previously proposed in the literature.

REFERENCES

- Beghelli, S., R.P. Guidorzi and U. Soverini (1990). The Frisch scheme in dynamic system identification. *Automatica* **26**, 171–176.
- Forsell, U., F. Gustafsson and T. McKelvey (1999). Time-domain identification of dynamic errors-in-variables systems using periodic excitation signals. In: *Proc. IFAC 14th World Congress*. Beijing, P.R.China.
- Pintelon, R. and J. Schoukens (2001). *System Identification. A Frequency Domain Approach*. IEEE Press. New York, NY, USA.
- Schoukens, J., R. Pintelon, G. Vandersteen and P. Guillaume (1997). Frequency domain system identification using non-parametric noise models estimated from a small number of data sets. *Automatica* **33**, 1073–1086.
- Söderström, T. (1981). Identification of stochastic linear systems in presence of input noise. *Automatica* **17**, 713–725.
- Söderström, T. (2002). *Discrete-Time Stochastic Systems - Estimation and Control, 2nd edition*. Springer-Verlag. London, UK.
- Söderström, T. and M. Hong (2004). Identification of dynamic errors-in-variables systems with periodic data (full version). Technical Report No:2004-037.
- Söderström, T. and P. Stoica (1983). *Instrumental Variable Methods for System Identification*. Springer-Verlag, Berlin.
- Söderström, T. and P. Stoica (1989). *System Identification*. Prentice Hall International. Hemel Hempstead, UK.
- Söderström, T., U. Soverini and K. Mahata (2002). Perspectives on errors-in-variables estimation for dynamic systems. *Signal Processing* **82**(8), 1139–1154.
- Tugnait, J. K. and Y. Ye (1995). Stochastic system identification with noisy input-output measurement using polyspectra. *IEEE Transactions on Automatic Control* **AC-40**, 670–683.