COEFFICIENT DIAGRAM METHOD IN MIMO APPLICATION: AN AEROSPACE CASE STUDY

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Abstract: The longitudinal control of the fighter with dual control surfaces is a typical MIMO control problem, where various modern control design techniques are employed. Although Coefficient Diagram Method (CDM) is proven effective in SISO or SIMO control design, the concrete procedure for MIMO design is not established yet. A trial design by CDM is made for this MIMO problem and the result is compared with the standard H-inf design.

Keywords: Control system design, Control theory, Aircraft control, MIMO, Polynomials.

1. INTRODUCTION

The classical control and modern control are mainly used in control design. However, there is a third approach generally called as algebraic design approach. The Coefficient Diagram Method (CDM) (Manabe, 1998, 2002b) is one of the algebraic design approaches, where the coefficient diagram is used instead of Bode diagram, and the sufficient condition for stability by Lipatov (Lipatov and Sokolov, 1978) constitutes its theoretical basis.

The purpose of this paper is to present one example of MIMO design by CDM and to make comparison with H-inf design. For this purpose, the problem is taken from the well-known example of the longitudinal control of a modern fighter in Robust Control Toolbox of MATLAB (Chiang and Safonov, 1994). The procedures for CDM MIMO design have not been established yet, and this paper is the continuation of the previous effort (Manabe, 2002a, 2004). In the effort, a new concept, called determinant transfer function, is found to be very effective. This concept is a natural result of the effort by Kwakernaak (2002a, b) on pole-zero analysis of H-inf control. Also Polynomial Toolbox by Poly-x is fully utilized (Kwakernaak, 2000)(Henrion, 2000).

This paper is organized as follows: In Section 2, the basics of CDM are briefly explained. In Section 3, the mathematical model and the problem statement are presented. In Section 4, analysis of plant is made. In Section 5, H-inf design results are analysed by determinant transfer function concept. In Section 6, a controller is designed by CDM. In section 7, frequency responses and singular value plots are shown.

2. BASICS OF CDM

Some notations used in CDM is briefly explained. The characteristic polynomial $P(s)$ is given in the following form.

$$P(s) = a_n s^n + \cdots + a_1 s + a_0 = \sum_{i=0}^{n} a_i s^i \qquad (1)$$

The stability index γ_i, the equivalent time constant τ, and the stability limit γ_i^* are defined as follows.

$$\gamma_i = a_i^2 / (a_{i+1}\, a_{i-1}), \quad i = 1 \sim n-1, \qquad (2)$$

$$\tau = a_1 / a_0, \qquad (3)$$

$$\gamma_i^* = 1/\gamma_{i+1} + 1/\gamma_{i-1}, \qquad (4)$$

γ_n and γ_0 are defined as ∞.

The equivalent time constant of the i-th order τ_i is defined as follows;

$$\tau_i = a_{i+1} / a_i, \qquad i = 1 \sim n-1. \tag{5}$$

Then the following relations are derived.

$$\tau_i = \tau_{i-1} / \gamma_i = \tau / (\gamma_i \cdots \gamma_2 \gamma_1), \tag{6}$$

$$a_i = \tau_{i-1} \ldots \tau_2 \tau_1 \tau a_0 = a_0 \tau^i / (\gamma_{i-1} \gamma_{i-2}^2 \cdots \gamma_2^{i-2} \gamma_1^{i-1}). \tag{7}$$

The sufficient condition for stability (Lipatov and Sokolov, 1978) (Manabe, 1999) is given as

$$\gamma_i > 1.12 \gamma_i^* \quad \text{for all} \quad i = 2 \sim n-2. \tag{8}$$

In CDM, the following stability indices are recommended. These values are improvement of Kessler (1960) standard form.

$$\gamma_{n-1} = \cdots = \gamma_3 = \gamma_2 = 2, \quad \gamma_1 = 2.5. \tag{9}$$

For more relaxed form, with very small sacrifice of stability,

$$\gamma_i > 1.5 \gamma_i^*, \quad i = n-2 \sim 3,$$

$$\gamma_{n-1} = \gamma_2 = 2, \quad \gamma_1 = 2.5. \tag{10}$$

In these cases, the step response has no overshoot, and the settling time is about $2.5 \sim 3\tau$.

3. MATHEMATICAL MODEL AND PROBLEM STATEMENT

The problem selected is the longitudinal control of a modern fighter, shown in Fig. 1 (Chiang and Safonov, 1994) (Safonov et al., 1981) (Safonov and Chiang, 1988). This aircraft is trimmed at 25000 ft and 0.9 Mach. The linear model in state space expression is given as follows, where the MATLAB type expression is adopted, such that vector $[2\ 4\ 5]^T$ is expressed as [2; 4; 5].

$$[\delta \dot{V}; \dot{\alpha}; \dot{q}; \dot{\theta}; \dot{\delta}_e; \dot{\delta}_c] = A_g\, [\delta V; \alpha; q; \theta; \delta_e; \delta_c] + B_g [u_e; u_c],$$

$$[\alpha; \theta] = C_g [\delta v; \alpha; q; \theta; \delta_e; \delta_c] + D_g [u_e; u_c],$$

$$A_g = \begin{bmatrix} -0.022567 & -36.617 & -18.897 & -32.090 & 3.2509 & -0.76257 \\ 9.2572e-5 & -1.8997 & 0.98312 & -7.2562e-4 & -0.17080 & -0.49652e-3 \\ 0.012338 & 11.720 & -2.6316 & 8.7582e-4 & -31.604 & 22.396 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -30 & 0 \\ 0 & 0 & 0 & 0 & 0 & -30 \end{bmatrix},$$

$$B_g = [0\ 0;\ 0\ 0;\ 0\ 0;\ 0\ 0;\ 30\ 0;\ 0\ 30],$$

$$C_g = [0\ 1\ 0\ 0\ 0\ 0;\ 0\ 0\ 0\ 1\ 0\ 0], \quad D_g = [\ 0\ 0;\ 0\ 0\ ]. \tag{11}$$

The state variables are velocity deviation (δV), angle of attack (α), attitude rate (q), attitude angle (θ), elevon angle (δ_e), and canard angle (δ_c). The output variables are α and θ. The control input variables are elevon actuator input (u_e) and canard actuator input (u_c).

By the use the of two control inputs, the non-conventional precision flight path control becomes possible. Vertical translation mode keeps θ while varying α. Pitch pointing mode keeps both α and θ. Direct lift mode keeps α while varying θ. The stated objective of the control is interpreted as making α and θ to follow the respective commands (α_r and θ_r). The more precise design specification is given in singular value specification as follows;

(1) Robustness Spec.: -40 dB/decade roll-off and at least –20 dB at 100 rad/sec.
(2) Performance Spec.: Minimize the sensitivity function as much as possible.

These specifications given in terms of singular value are interpreted as follows;

(1) Each control channel should be independent and no interaction is expected.
(2) Each channel should have the same characteristics.
(3) The auxiliary sensitivity function of each channel should show –40 dB/decade roll-off and at least –20dB at 100 rad/sec.

Usually the sensitivity function becomes larger when the interaction exists between two channels. Thus the minimization of sensitivity function makes the interaction the minimum. The singular value specification takes worse value between the two channels, and naturally each channel should show the same characteristics. In this situation, the two singular values take the same value and they are equal to the characteristics of each channel.

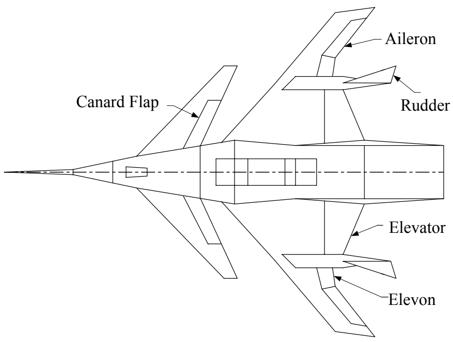


Fig. 1. Fighter model

4. ANALYSIS OF PLANT

In order to make CDM MIMO design, the plant has to be expressed in a right polynomial matrix fraction (RMF). Also the nature of the plant must be clarified in order to make the design to be systematic. First the plant is converted to a left polynomial fraction (LMF) by "ss2lmf" of Poly-x. Then a proper unimodular matrix is multiplied from the left. Then the LMF is given as follows;

$$A_u(s)[\alpha; \theta] = B_u(s)[u_e; u_c]. \tag{12}$$

Now new input variables are introduced such that

$$[u_e^*; u_c^*] = B_u(s)[u_e; u_c]. \tag{13}$$

$$B_u(s) = \begin{bmatrix} s + 0.020981 & 0 \\ -0.38337 & 1 \end{bmatrix}$$

Then the plant becomes RMF with numerator matrix $B_p(s) = I$.

$$A_p(s)[\alpha; \theta] = [u_c^*; u_e^*] \tag{14}$$

The subscript is changed to p from u to reflect conversion to RMF.

Now the denominator is further factorized.

$$A_p(s) = A_{p3} A_{p2} F_0 \,. \tag{15}$$

$$A_{p3}(s) = U_{p1} A_{p1} V_{p1} \tag{16}$$

Matrices U_{p1} and V_{p1} are unimodular matrices whose determinants are 1. F_0 is a scalar matrix representing the actuator dynamics.

$$F_0(s) = [s+30 \;\; 0; 0 \;\; s+30] \tag{17}$$

The matrix A_{p1} is monic and contains unstable poles, which cannot be cancelled. The matrix A_{p2} contains stable poles, which can be cancelled. By this arrangement, the determinants of these matrices are obtained as follows;

$$\begin{aligned} \det(A_{p1}) &= (s-0.6898 \pm j0.2488) \\ &= s^2 - 1.3796s + 0.53773 \end{aligned} \,, \tag{18}$$

$$\begin{aligned} \det(A_{p2}) &= k_{ap2}(s+5.6757)(s+0.25779) \\ &= k_{ap2}(s^2 + 5.9335s + 1.4631) \end{aligned} \,, \tag{19}$$

$$k_{ap2} = -0.00027902 \,.$$

The factorization procedures are not unique and depend on the design philosophy. The controller is assumed in the following form.

$$A_c(s)[u_e; u_c] = B_c(s)[\alpha_r - \alpha; \theta_r - \theta] \,, \tag{20}$$

$$A_c(s) = A_{c1} A_{c2} A_{c3} \,, \qquad A_{c3} = U_{p1}^{-1} B_u \,,$$

$$B_c(s) = B_{c1} B_{c2} \,, \qquad B_{c2} = V_{p1} A_{p2} \,.$$

A_{c1} is the main diagonal controller. A_{c2} is for decoupling. A_{c3} is an matrix to compensate the plant numerator and unimodular matrix U_{p1}. B_{c1} is the main diagonal controller. B_{c2} is used for pole-zero cancel. When controller equation, Eq. (20), is combined with plant equations, Eqs.(13)(14)(15)(16), the closed-loop input-output relation is obtained as follows;

$$[\alpha; \theta] = B_{c2}^{-1} A_1^{-1} B_{c1} B_{c2} [\alpha_r; \theta_r] \,, \tag{21}$$

$$A_1 = A_{c1} A_{c2} A_{p1} F_0 + B_{c1} \,.$$

The input-output transfer matrix, $T(s)$, is shown as follows;

$$T(s) = B_{c2}^{-1} T^* B_{c2} \,, \tag{22}$$

$$T^*(s) = A_1^{-1} B_{c1} \,.$$

If $T^*(s)$ is diagonal and each channel has the same characteristics, $T^*(s)$ is called as scalar type. Then $T(s)$ is equal to $T^*(s)$ irrespective to the choice of B_{c2}. If $T^*(s)$ is diagonal, but two channels show different characteristics, $T^*(s)$ is called as quasi-scalar type. If B_{c2} is diagonal, $T(s)$ is equal to $T^*(s)$. If B_{c2} has large cross terms, $T(s)$ is no longer equal to $T^*(s)$. $T(s)$ has large cross-coupling terms. Thus great care has to be taken in selection of B_{c2}. The design of $T^*(s)$ can be done by usual SISO CDM design procedures.

5. ANALYSIS OF H-INF DESIGN

The controller obtained by H-inf design can be reproduced from the following program (Chiang, 1994).

MATLABR11\toolbox\robust\hmatdemo.m

The controller is given as follows;

$$\begin{aligned} [\dot{x}_1; \dot{x}_2; \cdots; \dot{x}_8] &= A_{cp}[x_1; x_2; \cdots; x_8] + B_{cp}[\alpha_r - \alpha; \theta_r - \theta], \\ [u_e; u_c] &= C_{cp}[x_1; x_2; \cdots; x_8] + D_{cp}[\alpha_r - \alpha; \theta_r - \theta], \end{aligned} \tag{23}$$

$$A_{cp} = \begin{bmatrix} -317.05 & 1240.7 & -283.53 & 440.61 & -4866.8 & 1027.7 & -18310 & 1.6956e{+}005 \\ 399.27 & -1674.3 & 371.67 & -386.1 & 5938.6 & -503.6 & 15718 & -2.0946e{+}005 \\ -3.8377 & 16.223 & -21.941 & 0.13885 & -171.42 & -43.1 & 55.091 & 6242.8 \\ 0.34403 & -4.5148 & 2.9631 & -7.9561 & 19.138 & -31.586 & 315.17 & -572.65 \\ -0.4725 & 1.8575 & -0.87267 & 0.51799 & -10.188 & 0.10697 & -20.012 & 361.45 \\ 0.13614 & -0.086709 & -0.44814 & 0.24502 & -0.13245 & 0.70603 & -7.831 & 2.7619 \\ 0.36525 & 0.56536 & -0.80295 & 2.9163 & -4.4699 & 12.565 & -110.04 & 112.3 \\ -0.057762 & -0.1471 & -1.8057 & -0.055972 & 9.6264 & -1.1137 & 3.4438 & -305.65 \end{bmatrix}$$

B_{cp} = [1.1045e-005 -1.2335e-005;1.742e-005 -4.1339e-005;-0.0098439 -0.0088096;0.24628 -0.12981; 27.589 61.738; 63.391 -24.267; 115.47 -81.719; 28.836 62.778]

C_{cp} = [2.1013 -7.2505 1.5861 1.8654 16.741 13.204 -84.78 -640.42; -16.939 72.242 -16.476 23.297 -277.11 49.651 -964.27 9684.4]

D_{cp} = [0 0; 0 0]

It is very complicated and difficult to understand the meaning. In order to clarify the nature of the controller, it is converted to LMF and factorized.

$$A_c(s)[u_e; u_c] = B_c(s)[\alpha_r - \alpha; \theta_r - \theta] \,, \tag{24}$$

$$A_c(s) = A_{c0} A_{c1} A_{c2} F_1 \,,$$

$$B_c(s) = B_{c1} B_{c2} F_0 \,,$$

$$F_0(s) = [s+30 \;\; 0; 0 \;\; s+30] \,,$$

$$F_1(s) = [s+0.01 \;\; 0; 0 \;\; s+0.01] \,,$$

$$\begin{aligned} \det(A_{c0}) &= (s+2000.1)(s+263.59) \\ &= s^2 + 2263.7s + 527210 \end{aligned} \,,$$

$$\begin{aligned} \det(A_{c1}) &= (s+65.072 \pm j25.902)(s+52.606) \\ &= s^3 + 182.75s^2 + 11752s + 258040 \end{aligned} \,,$$

$$\det(A_{c2}) = s + 0.020981 \,,$$

$$\begin{aligned} \det(B_{c1}) &= k_{bc1}(s+0.60081 \pm j0.32062) \\ &= k_{bc1}(s^2 + 1.2016s + 0.46378) \end{aligned} \,,$$

$$k_{bc1} = -1.2699 \times 10^{10} \,,$$

$$\begin{aligned} \det(B_{c2}) &= (s+5.6757)(s+0.25779) \\ &= (s^2 + 5.9335s + 1.4631) \end{aligned} \,.$$

F_0 is for cancellation of the plant actuator dynamics poles. F_1 is a pseudo-integral matrix for integral control. Both F_0 and F_1 are scalars. A_{c0} represents high frequency filter. A_{c1} is the denominator of the main control. A_{c2} is for cancellation of the plant zero. B_{c1} is the numerator of main control. B_{c2} is for cancellation of the plant stable poles. From the poles and zeros of the auxiliary sensitivity function $T(s)$, it is confirmed that such cancellation really occurs. It has 14 poles and 7 zeros, of which 5 poles and zeros are cancelled out, and remaining 9 poles and 2 zeros are effective (Manabe, 2004)(Kwakernaak, 2002b).

Cancelled poles and zeros: -0.020981, -0.25779, -5.6757, -30, -30.

Poles: -0.68980 ± j0.24880, -22.705 ± j18.444, -23.834 ± j20.692, -95.031, -408.96, -2000.

Zeros: -0.60064 ± j0.32068.

In order to clarify the meaning of H-inf controller, the determinant of transfer function, abbreviated as detTF, is considered. The detTF is defined as the determinant of the transfer function matrix, and is equal to the ratio of the determinant of the numerator polynomial matrix and that of the denominator. If two channels are decoupled and have the same characteristics, detTF is simply the square of the transfer function of each channel. Thus the channel characteristic is estimated from the detTF for such decoupled system.

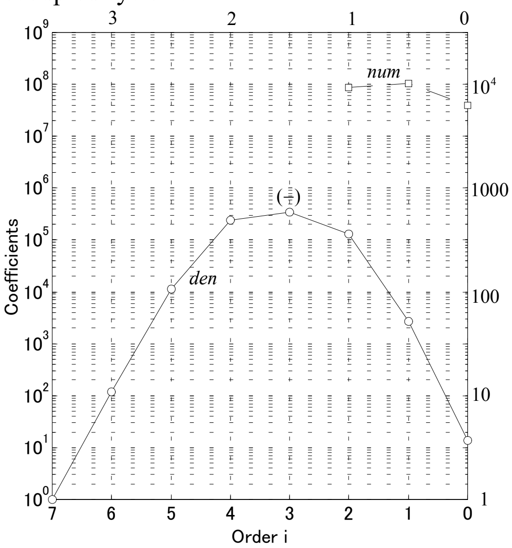
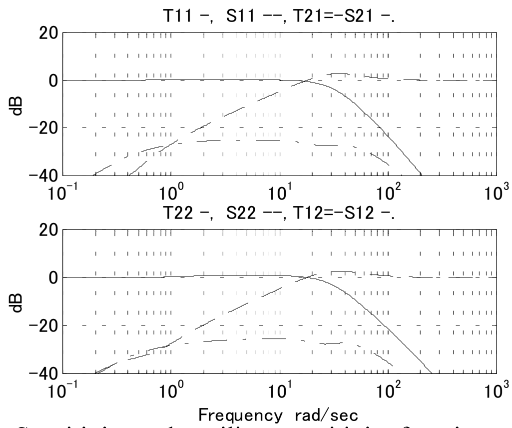
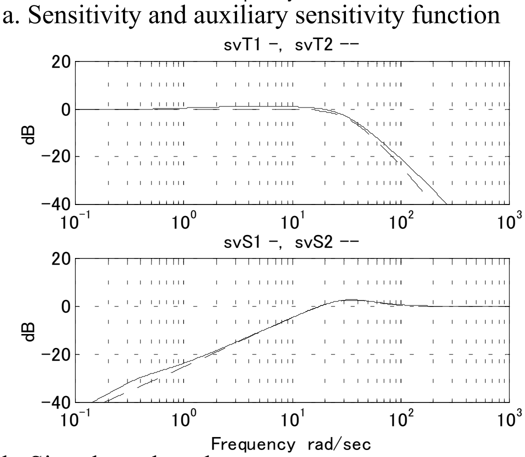


Fig. 2. Coefficient diagram of open-loop detTF



a. Sensitivity and auxiliary sensitivity function



b. Singular value plot

Fig. 3. H-inf 6/8 order controller

After pole-zero cancellation and high frequency filter replaced by constant, the detTF of the open-loop transfer matrix, $\det(G_{pc}(s))$, becomes as follows;

$$\det(G_{pc}(s)) = \frac{\det(B_{c1})\det(B_{c2})}{\det(A_{p1})\det(A_{p2})\det(A_{c0}(0))\det(A_{c1})\det(F_1)}$$
$$= \frac{8.6327\times10^7(s^2+1.2016s+0.46378)}{(s^2-1.3796s+0.53773)(s^3+182.75s^2+11752s+258040)(s+0.01)^2}$$
$$= \frac{8.6327\times10^7(s^2+1.2616s+0.46378)}{(s^7+181.39s^6+11504s^5+242160s^4-344840s^3+131790s^2+2740.2s+13.876)} \tag{25}$$

The coefficient diagram is shown in Fig. 2. The solid line with circle is for denominator and dotted line with square is for numerator. Left-lower scale is for usual definition. Right-upper scale is used to show roughly the characteristics of each channel. In *den* of Fig. 2, only one negative coefficient is found at the 3-rd order. This strongly suggests that the H-inf controller is a quasi-scalar type. If it is scalar type, 2 coefficients are negative.

The frequency characteristics and singular value plots are shown in Fig. 3. The system is almost decoupled, but small value of coupling, $T_{21}(s)$ and $T_{12}(s)$, is visible from the figure. This is another indication that the controller is quasi-scalar.

6. CDM DESIGN

When factorization is utilized, the design is made systematically as explained in Section 4. However the procedure depends on the design philosophy and various procedures are possible. The procedure used here is as follows. From A_p, extract F_0 by simple division. This is possible, because F_0 is a scalar. From $A_pF_0^{-1}$, extract a monic matrix A_{p3} with two instable poles by "fact" command. The rest is a matrix A_{p2} with two stable poles. The matrix A_{p3} is further decomposed to a unimodular matrix U_{p1}, a diagonal matrix A_{p1}, and a unimodular matrix V_{p1}.

$$U_{p1} = \begin{bmatrix} 1 & 0.94857s+0.01563 \\ 0 & 1 \end{bmatrix}, \tag{26}$$

$$A_{p1} = \begin{bmatrix} s^2-1.3796s+0.53773 & 0 \\ 0 & 1 \end{bmatrix},$$

$$V_{p1} = \begin{bmatrix} 0 & -0.94857 \\ 1.0542 & s-1.3961 \end{bmatrix},$$

$$A_{p2} = \begin{bmatrix} -0.18747s-0.35671 & -4.1562\times10^{-5}s^2-0.18419s+0.0013569 \\ 0.0049581 & 0.0014895s-0.0011256 \end{bmatrix}$$

Then two component matrices A_{c3} and B_{c2} of the controller are automatically defined.

$$A_{c3} = U_{p1}^{-1}B_u$$
$$= \begin{bmatrix} 1.3637s+0.026982 & -0.94857s-0.015653 \\ -0.38337 & 1 \end{bmatrix}, \tag{27}$$

$$B_{c2} = V_{p1}A_{p2} = \begin{bmatrix} -0.0047031 & -0.0014129s - 0.0010678 \\ -0.19268s - 0.38297 & 0.0014456s^2 + 0.19323s - 0.00014108 \end{bmatrix}.$$

Now assume the main diagonal controller as follows;

$$A_{c1} = [l_2 s^2 + l_1 s \;\; 0; 0 \;\; l_2 s^2 + l_1 s], \qquad l_1 = 1, \qquad (28)$$

$$A_{c2} = I,$$

$$B_{c1} = [30k_0(s^2 + 2.5s + 2.5) \;\; 0; 0 \;\; 30k_0].$$

Then the closed-loop system matrix after pole zero cancellation, A_1 defined in Eq. (19), is expressed as follows;

$$A_1 = A_{c1}A_{c2}A_{p1}F_0 + B_{c1} = [P_{11} \;\; 0; 0 \;\; P_{22}], \qquad (29)$$

$$P_{11} = (l_2 s^2 + s)(s^2 - 1.3796s + 0.53773)(s + 30) + 30k_0(s^2 + 2.5s + 2.5),$$

$$P_{22} = (l_2 s^2 + s)(s + 30) + 30k_0.$$

The value of k_0 roughly corresponds to the crossover frequency. The time constant l_2 is for the properness of controller and should be some small value. The following values are selected.

$$l_2 = 0.01, \quad k_0 = 25. \qquad (30)$$

The term $(s^2 + 2.5s + 2.5)$ in P_{11} is a replica of the plant unstable polynomial, $(s^2 - 1.3796s - 0.5377)$. The $2.5s$ is selected as about twice of 1.3796, and 2.5 is selected such that the stability index γ_1 of P_{11} is about 2.5. The coefficient diagram of each channel is shown in Fig. 4a, b.

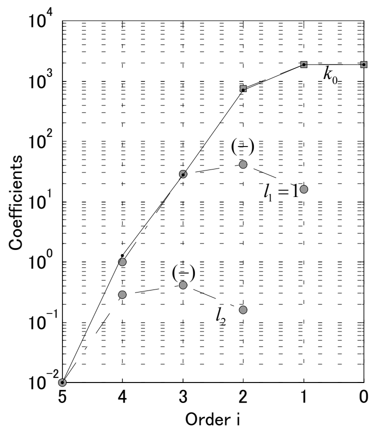


Fig. 4a. P_{11} channel coefficient diagram

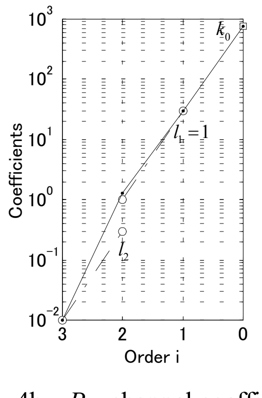


Fig. 4b. P_{22} channel coefficient diagram

The stability index γ_i and equivalent time constant τ for each channel are as follows;

$$P_{11}: \; \gamma_i = [5.8639 \;\; 0.8724 \;\; 9.4302 \;\; 2.6891], \qquad (31)$$

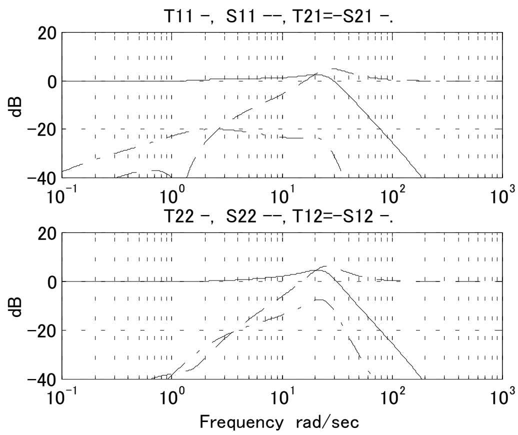
$$\tau = 1.0086,$$

$$P_{22}: \; \gamma_i = [5.6333 \;\; 0.92306], \quad \tau = 0.04.$$

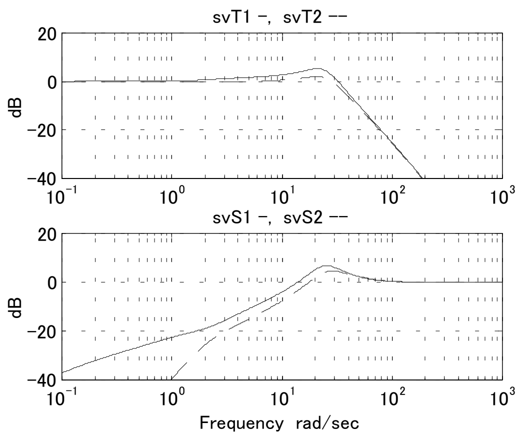
These γ_i values are not standard, because the selected $k_0 = 25$ is large. If it is chosen around 15, it becomes close to standard. The γ_2 of P_{11} is large. This comes from the requirement that the channel 11 should be similar to the channel 22 in order to minimize the cross-coupling. This controller is called 4/5-order quasi-scalar controller, because the orders of numerator/denominator are 4/5, and $T^*(s)$ is quasi-scalar.

7. FREQUENCY RRESPONSES

The frequency response of auxiliary sensitivity function $T(s)$ as in Eq. (22) and related sensitivity function $S(s)$ are shown in Fig. 5a. Singular value plots are shown in Fig. 5b. There are some cross-coupling due to the quasi-scalar nature. The attenuation at 100 rad/sec is –25 db, larger than H-inf controller, because the actuator dynamics of $(s + 30)$ is not compensated in this design.

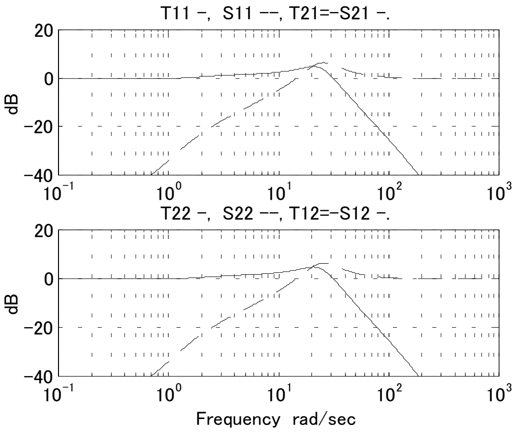


a. Sensitivity and auxiliary sensitivity function

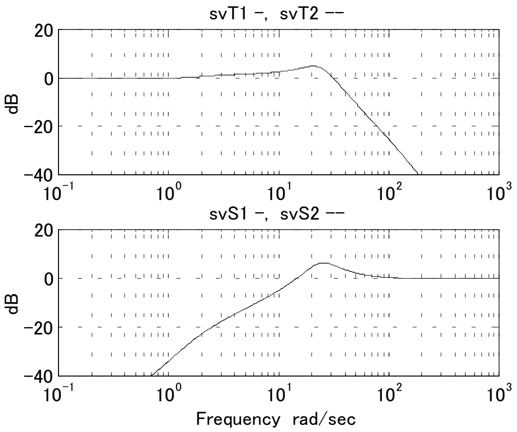


b. Singular value plot

Fig. 5. CDM 4/5 order Quasi-scalar controller



a. Sensitivity and auxiliary sensitivity function



b. Singular value plot

Fig. 6. CDM 6/7 order scalar controller

The 6/7 order scalar controller is designed by redesigning A_{c2} and B_{c1} as follows;

$$A_{c2} = [1\ \ 0; 0\ \ s^2 - 1.3796 + 0.53773]\,, \qquad (32)$$

$$B_{c1} = [30k_0(s^2 + 2.5s + 2.5)]I\,.$$

The frequency responses are shown in Fig. 6a, b. There is no cross-coupling.

8. CONCLUSION

The major results of this paper are as follows;

(1) Controllers are designed for the longitudinal control of a modern fighter with elevon and canard by CDM. The design procedures are based on factorization of polynomial matrices. Although the method is systematic, some caution is necessary, because such factorization is not unique due to unimodular matrices.

(2) The designed controllers are a 4/5 order quasi-scalar controller and a 6/7 order scalar controller.

(3) The 6/8 order controller designed by H-inf contains pole-zero cancellation of actuator dynamics, which is usually not recommended in practical design. It looks to be quasi-scalar type.

(4) MIMO design by CDM is still at the developing stage. Further efforts are keenly needed. The polynomial CAD should be improved. The role of unimodular matrix should be further clarified.

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