ADAPTIVE CONTROL OF A COUPLED DRIVES APPARATUS USING DUAL YOULA-KUCERA PARAMETRIZATION

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Abstract: An adaptive algorithm based on the dual Youla-Kucera parametrization is introduced enabling simple closed-loop identification and adaptation of a class of symmetric MIMO systems. The methodology exploits the algebraic approach to control system design. Necessary conditions for usage of the developed method are discussed and results are presented for the case of coupled drives control.

Keywords: control system design, MIMO systems, adaptive control, closed-loop identification, model-based control, algebraic approaches, parametrization.

1. INTRODUCTION

Theoretical work on identification for control has shown that it is not always necessary to have a precise model of the system to design a good controller, even in the case of complex industrial processes. The important point is that this model should capture the essential dynamical characteristics that are important for control (e.g. Schrama, 1992; Åström, 1993). These ideas have led to the development of iterative schemes with separate stages of closed-loop identification and model based control design, and consequently to iterative feedback tuning methodology (Gevers, 2002; Hjalmarsson, 2002) which is now frequently used for optimization of parameters of any restricted complexity controller.

This contribution presents a modification of an iterative method for closed-loop identification and control design that arose from the above mentioned theoretical background. The method is based on a suitable way of plant-parametrization, also referred to as the *dual Youla-Kucera parametrization* (e.g. Anderson, 1998), and the modification (Gazdos, 2004) enables simple on-line identification and control of a class of symmetric MIMO systems; moreover, unlike the original algorithm, which uses separate stages of off-line closed-loop identification and subsequent model-based control design, the modification represents a true adaptive control scheme.

The aim of this work is to present the methodology together with conditions for its successful usage and show some results when controlling a two-input/two-output nonlinear system, namely a coupled drives apparatus. The first part of the contribution introduces both the original and the modified algorithm.

2. THEORETICAL BACKGROUND

Consider the closed-loop set-up presented by Figure 1 where $\boldsymbol{P}$ denotes a plant to be controlled and $\boldsymbol{C}$ is a corresponding controller. The input and output dimensions are given by $\boldsymbol{y}(t)$, $\boldsymbol{r_1}(t) \in \mathbb{R}^n$ and $\boldsymbol{u}(t)$, $\boldsymbol{r_2}(t) \in \mathbb{R}^m$ where $\boldsymbol{y}$ represents the controlled output and $\boldsymbol{u}$ the control input. External signals $\boldsymbol{r_1}$, $\boldsymbol{r_2}$ can be either reference signals or external disturbances uncorrelated to each other and to a noise signal $\boldsymbol{n}$.

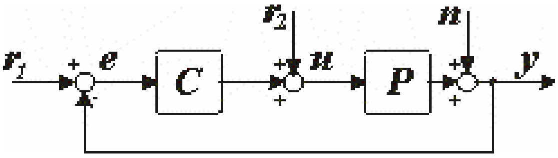

Fig. 1. Closed-loop set-up.

Let the plant give rise to a linear time-invariant transfer function matrix $\boldsymbol{P}$ and let $\boldsymbol{C}$ be also a linear time-invariant transfer function matrix of a controller. Further, let both $\boldsymbol{P}$ and $\boldsymbol{C}$ have right and left coprime factorizations as follows:

$$\boldsymbol{P} = \boldsymbol{N} \cdot \boldsymbol{D}^{-1} = \bar{\boldsymbol{D}}^{-1} \cdot \bar{\boldsymbol{N}} \qquad (1)$$

$$\boldsymbol{C} = \boldsymbol{N_C} \cdot \boldsymbol{D_C}^{-1} = \bar{\boldsymbol{D}}_{\boldsymbol{C}}^{-1} \cdot \bar{\boldsymbol{N}}_{\boldsymbol{C}} \qquad (2)$$

where $\boldsymbol{N}, \boldsymbol{D}, \boldsymbol{N_C}, \boldsymbol{D_C}$ denote matrices of the right matrix fractions whereas $\bar{\boldsymbol{N}}, \bar{\boldsymbol{D}}, \bar{\boldsymbol{N}}_{\boldsymbol{C}}, \bar{\boldsymbol{D}}_{\boldsymbol{C}}$ describe the left counterparts. All matrices are with elements in $\Re_{PS}$, the ring of proper and stable transfer functions, which helps to handle situations of unstable plants and/or controllers. The presented feedback system will be stable if and only if (Vidyasagar, 1985) the so-called double Bezout identity holds:

$$\begin{bmatrix} \bar{\boldsymbol{D}}_{\boldsymbol{C}} & \bar{\boldsymbol{N}}_{\boldsymbol{C}} \\ -\bar{\boldsymbol{N}} & \bar{\boldsymbol{D}} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{D} & -\boldsymbol{N_C} \\ \boldsymbol{N} & \boldsymbol{D_C} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \qquad (3)$$

The relation expresses both coprimeness of the matrices involved and internal stability of the feedback loop of Figure 1. This concept offers a tool for finding all plants that are stabilized by a given controller – in addition, expressed using a single parameter. The methodology, known as the *dual Youla-Kucera parametrization*, can be exploited for an iterative approach to closed-loop identification and control design as presented in the next part.

3. METHODOLOGY

First, the original ideas and algorithm are presented, followed by the developed modification.

3.1 Original idea and algorithm

Consider the closed-loop system of Figure 1 and suppose that a nominal plant-model (1) stabilized by the controller (2) is available and that the double Bezout identity (3) holds; then, the set of all plant models stabilized by the controller is given as:

$$\begin{aligned} \boldsymbol{P}(\boldsymbol{R}) &= (\boldsymbol{N} + \boldsymbol{D_C} \cdot \boldsymbol{R}) \cdot (\boldsymbol{D} - \boldsymbol{N_C} \cdot \boldsymbol{R})^{-1} = \\ &= (\bar{\boldsymbol{D}} - \boldsymbol{R} \cdot \bar{\boldsymbol{N}}_{\boldsymbol{C}})^{-1} \cdot (\bar{\boldsymbol{N}} + \boldsymbol{R} \cdot \bar{\boldsymbol{D}}_{\boldsymbol{C}}) \end{aligned} \qquad (4)$$

where $\boldsymbol{R}$ is a free parameter of $\Re_{PS}$ that satisfies non-singularity of $(\boldsymbol{D} - \boldsymbol{N_C} \cdot \boldsymbol{R})$ and $(\bar{\boldsymbol{D}} - \boldsymbol{R} \cdot \bar{\boldsymbol{N}}_{\boldsymbol{C}})$ (for proof, see e.g. Vidyasagar, 1985). This result can be used for closed-loop identification (e.g. Hansen, 1989; Schrama, 1991, Tay, *et al.*, 1989). When a plant-model is chosen in the form of (4), using e.g. the left coprime factorization, then a noise contaminated closed-loop can be alternatively formed according to Figure 2.

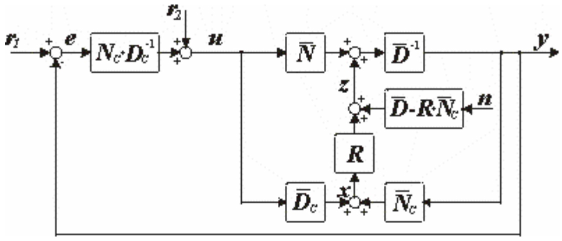

Fig. 2. Alternative closed-loop configuration.

In this set-up, the key idea is to identify the free parameter $\boldsymbol{R}$ using signals $\boldsymbol{x}$ and $\boldsymbol{z}$ rather than identifying all coefficients of the plant-model. This is an "open-loop-like" identification problem, i.e. identification of this parameter is not dependent on the noise contribution of the data. From the figure above, it is possible to derive following relations defining and relating the auxiliary signals $\boldsymbol{x}$ and $\boldsymbol{z}$:

$$\boldsymbol{x} = \bar{\boldsymbol{D}}_{\boldsymbol{C}} \cdot \boldsymbol{u} + \bar{\boldsymbol{N}}_{\boldsymbol{C}} \cdot \boldsymbol{y} \quad = \quad \bar{\boldsymbol{N}}_{\boldsymbol{C}} \cdot \boldsymbol{r_1} + \bar{\boldsymbol{D}}_{\boldsymbol{C}} \cdot \boldsymbol{r_2} \qquad (5)$$

$$\boldsymbol{z} = \bar{\boldsymbol{D}} \cdot \boldsymbol{y} - \bar{\boldsymbol{N}} \cdot \boldsymbol{u} \qquad (6)$$

$$\boldsymbol{z} = \boldsymbol{R} \cdot \boldsymbol{x} + (\bar{\boldsymbol{D}} - \boldsymbol{R} \cdot \bar{\boldsymbol{N}}_{\boldsymbol{C}}) \cdot \boldsymbol{n} \qquad (7)$$

The last equation reveals that the parameter $\boldsymbol{R}$ can be identified using the signals $\boldsymbol{x}$ and $\boldsymbol{z}$ which can be reconstructed from the data through (5)-(6). The estimated parameter, in fact, represents discrepancy between the actual plant and a nominal model. A nice property of this approach is that, as a result of the dual Youla-Kucera parametrization, the identified model is guaranteed to be stabilized by the current controller. A drawback represents the fact that the order of the resultant plant model is not simply tunable due to the required re-parametrization according to (4).

Practical implementation of the concept above leads to an iterative scheme consisting of a sequence of identification and model-based control design steps and it can be formed as follows: let there be a nominal model $\boldsymbol{P_M}$ of the plant stabilized by a controller $\boldsymbol{C}$. Denoting $\boldsymbol{C}^{i}$ the controller used to regulate the control input into the plant during i^{th} iteration step, the following algorithm can be prepared:

- *step 1*: perform an experiment on the closed-loop system of Figure 1 using the controller $\boldsymbol{C}^{i-1}$ and external signals $\boldsymbol{r_1}(t), \boldsymbol{r_2}(t)$ and collect the experimental data set $\{\boldsymbol{r_1}, \boldsymbol{r_2}, \boldsymbol{u}, \boldsymbol{y}\}$ of a length N;
- *step 2*: generate the auxiliary signals $\boldsymbol{x}(t)$, $\boldsymbol{z}(t)$ employing knowledge of the controller $\boldsymbol{C}^{i-1}$, of the nominal model $\boldsymbol{P}_{\boldsymbol{M}}^{i-1}$ and the input-output data record $\{\boldsymbol{u}, \boldsymbol{y}\}$ according to (5)-(6);

- *step 3*: identify the parameter $\boldsymbol{R}$ from the equation (7) using the constructed signals $\boldsymbol{x}(t), \boldsymbol{z}(t)$ and a least-squares algorithm with e.g. an output-error model structure;
- *step 4*: compute a new plant-model which replaces the nominal one using rel. (4) and if necessary, apply an order-reduction technique;
- *step 5*: design a new controller $\boldsymbol{C}^{i}$ using the Bezout equation (3) and continue by *step 1* with $\boldsymbol{C}^{i}$.

Note that both signals $\boldsymbol{x}$ and $\boldsymbol{z}$, due to the algebraic framework which employs $\Re_{PS}$-description, represent stably-filtered quantities $\boldsymbol{u}$ and $\boldsymbol{y}$. Hence, the signal $\boldsymbol{z}$ can be interpreted as a stably filtered equation error. As a result, in practice, if this signal is close to zero (which results in coefficients of the parameter $\boldsymbol{R}$ to be also close to zero), it is possible to state that the plant model represents a good approximation of the real plant. This is exploited for monitoring the iterations - if $\boldsymbol{R}$ is close to zero, the procedure is stopped.

3.2 *Modification for a class of symmetric systems*

The modification, originally considered in the work of Gazdos (2004), is focused on systems with the same number of input and output signals. It is based on the idea of choosing the identified matrix parameter $\boldsymbol{R}$ as simply as a diagonal matrix with constant elements, i.e. in the continuous-time form:

$$\boldsymbol{R}(s)=\begin{bmatrix} r_{011} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & r_{0nn} \end{bmatrix} \tag{8}$$

This choice offers significant simplification of the identification process and consequently, it enables recasting the original iterative algorithm into a true adaptive control scheme. Here, the idea is to perform the process of identification on-line, i.e. in real-time. Having the actual measurements of $\{\boldsymbol{u}(t), \boldsymbol{y}(t)\}$ with a nominal model and a controller at our disposal, it is possible to generate the signals $\boldsymbol{x}(t)$, $\boldsymbol{z}(t)$ continuously as stable filtration of $\{\boldsymbol{u}(t), \boldsymbol{y}(t)\}$ according to (5)-(6). As a result, the parameter $\boldsymbol{R}$ can directly be identified from the filtered signals. Having it in the simple form of (8), it is possible to *compute* $\boldsymbol{R}$-coefficients *continuously* from the equation (7) (provided that the noise contribution is neglected) without the need for using a least squares algorithm. The *computation* can be as simple as:

$$r_{011}=\frac{z_1(t)}{x_1(t)},\ r_{022}=\frac{z_2(t)}{x_2(t)},\dots,r_{0nn}=\frac{z_n(t)}{x_n(t)} \tag{9}$$

The division should be of course done cautiously and only if elements of the signal $\boldsymbol{x}$ differ from the zero value, which is monitored. Then $\boldsymbol{R}$-coefficients, computed in each discrete-time step according to (9), are considered *identified when they stabilize*, i.e. the period of updating plant-model's and controller's coefficients is *irregular* - after settling of the identification process. The procedure of settling is monitored by the standard deviation of the identified coefficients: if any of them has the standard deviation under a certain level within a given number of discrete-time steps, then adaptation of corresponding plant-model's and controller's coefficients is performed. The process of adaptation is controlled by the values of $\boldsymbol{R}$-coefficients – each channel of the controlled MIMO system is being adapted until the corresponding coefficient of the parameter $\boldsymbol{R}$ is close to zero; then, adaptation of relevant plant-model's and controller's coefficients is stopped and starts again only if there is a significant difference between the model and the real plant in the given channel, i.e. when the identified coefficient exceeds a given threshold. Details of the methodology can be found in the work of Gazdos (2004). One of the issues addressed in the thesis was whether such simple structure of the estimated parameter (8) is able to capture unknowness of the plant and under what conditions the process of identification stabilizes to enable adaptation. The obtained results can be summarized into the following points:

- *nominal-model's structure* must be comparable to the plant's structure;
- *initial estimates* of plant-model coefficients play an important role only for unstable and nonlinear plants. Then it is necessary to guess approximately at least gains and time-constants of the system. Apart from this, for unstable plants, it is advisable to pre-suppose the instability and this holds also for a negative gain or non-minimum-phase systems;
- *convergence* of the algorithm, particularly when controlling unstable or strongly-nonlinear plants, depends on an order-reduction technique applied after re-parametrization of a model. Therefore, it is advisable to employ an adequate order-reduction technique when one has a priori knowledge of possible instability or nonlinearity.

Taking into account these facts, the presented simplification makes sense and leads to a more transparent process of identification and control than would offer a classical adaptive scheme with recursive identification. Then, the introduced algorithm enables to identify a whole MIMO plant using only few coefficients in a simple way with independent identification and adaptation of each channel of the system. Nevertheless, one has to keep in mind the recommendations above.

4. CASE STUDY

The proposed method was tested by a carefully selected series of simulation and real-time experiments. One series of the experiments was performed using a product of TecQuipment Inc., the CE108 Coupled Drives Apparatus displayed in Figure 3.

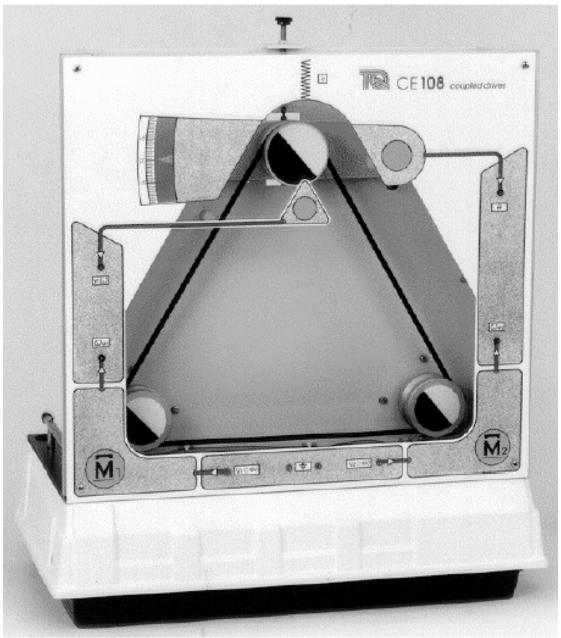

Fig. 3. The CE108 Coupled Drives Apparatus.

The system relates to industrial material transport problems as they occur in magnetic tape drives, textile machines, paper mills, strip metal production plants, etc. where the material is processed in continuous lengths, it is transported through work stations by drive systems and the material speed and tension have to be controlled within defined limits at all times.

4.1 Plant description

A scheme of the controlled system is sketched in Figure 4.

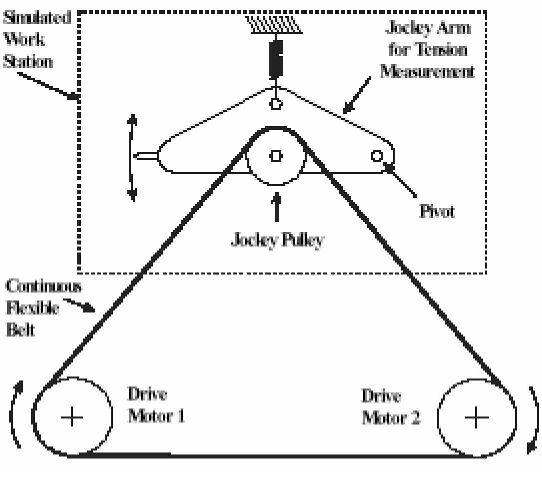

Fig. 4. Scheme of a coupled drives system.

It has two drive motors (Motor 1 and Motor 2). These drives operate together to control the speed of a continuous flexible belt that goes round pulleys on the drive motor shafts and so called *jockey pulley*. The jockey pulley is mounted on a swinging arm that is supported by a spring. The deflection of the arm is a measure of the tension in the drive belt. The pulley and arm assembly represents a *work station* where material that the belt represents can be processed. The control problem is to regulate the belt speed and tension by varying the motor torques.

The coupled drive apparatus is designed to have characteristics seen in industrial drives, but it is not any particular industrial application – it is a prototype for all industrial coupled drive applications. Detailed description of the system together with derivation of a mathematical model can be found e.g. in the work of Wellstead (1979).

4.2 Plant characteristics

For purposes of an experiment, the system is interpreted as a multivariable plant with interacting inputs and outputs where the coupling is given by the fact that both motors change both outputs (due to the drive belt). Static properties as they were measured are provided by next two graphs. Figure 5 shows a scaled final value of the first controlled variable – the pulley speed y_{ω} when applied various torques of the motors expressed by power supply in voltages (0-10V correspond to 0-3000 t/min.).

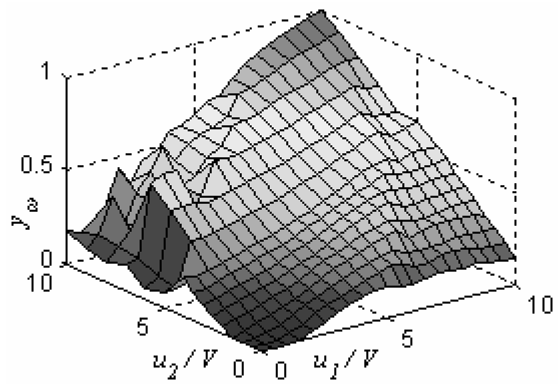

Fig. 5. Static characteristics – pulley speed y_{ω}.

As can be seen, the surface is quite smooth when speed of both motors does not differentiate much. When there is a significant difference between torques of the motors, the system starts to oscillate (the belt slips) and becomes nearly unstable. The second graph – Figure 6 demonstrates how the scaled tension in the belt y_x varies for different speed of the motors. Apparently, there is a smaller area where the system behaves well and conversely, there is a stronger possibility of oscillatory behaviour and instability.

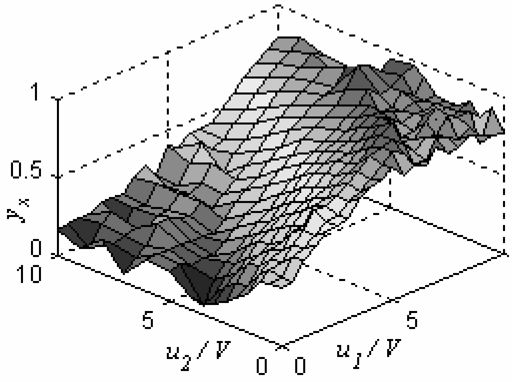

Fig. 6. Static characteristics – tension in the belt y_x.

4.3 Experimental conditions

Real-time measurements were performed using the Advantech MultiLab Analog and Digital I/O card PCL-812PG and MATLAB®/SIMULINK® environment with the help of Real Time Toolbox. The algorithm was discretized using the concept of δ-models (e.g. Middleton and Goodwin, 1990) enabling high-frequency sampling without risking possible instability of obtained models. Both controlled outputs y_ω, y_x were filtered using discrete-time filters of the form (10) and a sampling interval was set to $T_0 = 0.05\,\text{sec}$.

$$F_\omega = \frac{(1-0.83)}{1-0.83 \cdot z^{-1}}; \quad F_x = \frac{(1-0.95)}{1-0.95 \cdot z^{-1}} \quad (10)$$

As the plant to be controlled represents a two-input/two-output system, the estimated parameter $\boldsymbol{R}$ was chosen in the form of (11).

$$\boldsymbol{R}(s) = \begin{bmatrix} r_{011} & 0 \\ 0 & r_{022} \end{bmatrix} \quad (11)$$

A nominal plant-model was derived from the linearized model of the system and controller design methodology was based on the pole-assignment technique. The resultant closed-loop system had one double pole α_i for each channel and they were placed at: $\boldsymbol{\alpha} = [\alpha_1, \alpha_2] = [-3, -1]$. Variables controlling the adaptation process were tuned to have these values: $\varepsilon_1 = 0.005$, $n_R = 20$ and $\varepsilon_2 = 0.02$, where n_R is a number of sampled-data of $\boldsymbol{R}$-coefficients to compute the standard deviation, ε_1 stands for the threshold for deciding whether the coefficients are stable or not and ε_2 denotes the level for switching off/on the adaptation process (see section 3.2 for details).

4.4 Results and discussion

Control response for the setting described in the previous section is displayed in Figure 7 where the controlled variables y_ω, y_x were scaled to match the form presented by the graphs of static characteristics.

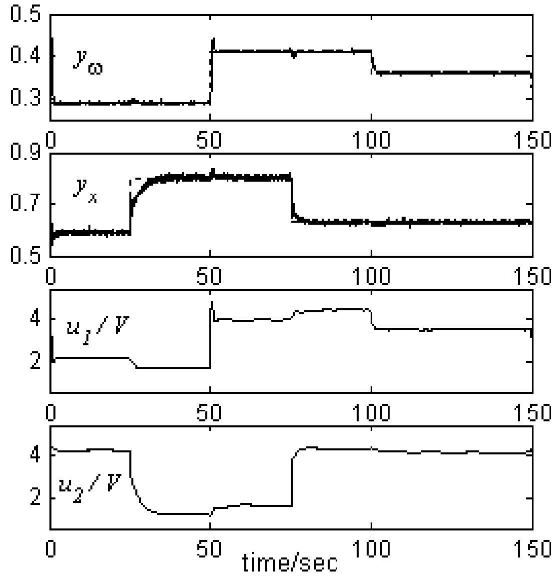

Fig. 7. Plant output and control input responses.

The figure shows stable and smooth control response with only minor over/under-shoots and minimal coupling. The identification record, as presented by Figure 8, where the adaptation points are indicated by circles, reveals that both identified coefficients soon settle near the zero value changing considerably only as a response to a change in an operating point of either one or the other controlled variable. This fact means that the model approximates soon the real plant well, adapting quickly to the new operating conditions.

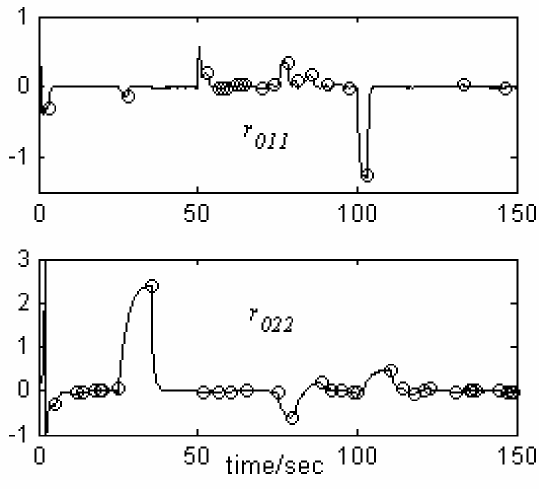

Fig. 8. Identified coefficients of the parameter $\boldsymbol{R}$.

A response to disturbances is demonstrated by Figure 9. During the experiment, transient disturbances acting on both control input u_1 and controlled output y_x (at approx. times $t = 60$ and $120\,\text{sec}$. respectively) were applied in order to test sensitivity of the method to disturbance signals. The figure shows that the method is able to cope with both disturbances. While disturbance acting on the controlled output does not affect the other controlled variable significantly, disturbance applied on the control input causes changes in all variables, which shows interlacing in the system.

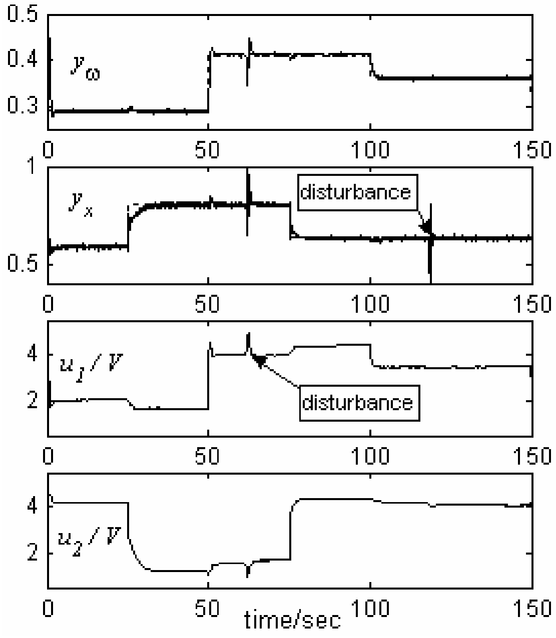

Fig. 9. Plant output and control input responses to disturbances.

Based on all performed experiments, it is possible to state that the effect of either transient or step-disturbances on both controlled output and control input can be entirely eliminated. This is attributable to the control design methodology employed where the controller compensates disturbances of the same form as the reference signal, i.e. if the reference signal is a step-function, then disturbances with up to the step-dynamics can be attenuated.

As far as the noise influence is concerned, the method is sensitive especially when there is a small signal-to-noise ratio (owing to the small sampling period and neglect of the noise contribution to the identification). Then, experiments indicated positive effects of filtering the measured data. In addition to this, it helps to increase values of the thresholds ε_1, ε_2 proportionally to the noise level. Then, despite the noise-effect, results are satisfactory.

Experimental results also revealed the need for a priori information about this controlled plant. When initial estimates of model coefficients were set without a priori information, the method did not give good results. It was necessary to guess approximately at least gains and time-constants of the system.

5. CONCLUSION

A modification of an iterative method for closed-loop identification and control design based on the dual Youla-Kucera parametrization has been presented, discussed and tested on a nonlinear two-input/two-output system. The resultant adaptive scheme enables simple on-line identification and control of a class of MIMO systems having the same input and output signal dimensions. For successful implementation of the method, it is advisable to follow presented recommendations, mainly the requirements concerning nominal-model's structure, adequate order-reduction techniques and initial estimates, especially when controlling unstable or strongly-nonlinear plants.

ACKNOWLEDGEMENTS

This work was supported in part by the Grant Agency of the Czech Republic under grant No. 102/03/0070 and by the Ministry of Education of the Czech Republic under grant MSM 2811 00001.

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