ON THE USE OF DYNAMIC INVERSION FOR THE IMPROVEMENT OF PID CONTROL

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Abstract: In this paper we discuss the use of a noncausal approach for the improvement of PID control, according to a method presented in (Piazzi and Visioli, 2004). In particular we verify that, despite the proposed methodology is based on an estimated first-order plus dead time (FOPDT) model of the process, different identification procedure can be employed for this purpose yielding in any case to a good result. Further, the role of the unique design parameter is analysed.

Keywords: Process control, PID control, feedforward control, inversion, process parameters estimation.

1. INTRODUCTION

It is well-known that Proportional-Integral-Derivative (PID) controllers are the most adopted controllers in industry due to the fact that, despite their relative simplicity, they are capable to solve in many cases the control task satisfactorily. However, the tuning of the parameters and the correct employment of different additional functionalities such as anti-windup, derivative filtering and so on, is often a crucial issue to obtain a high performance. To help the operator to select the controller gains to address given control specifications, many tuning formulas have been devised in the past (O'Dwyer, 2003) and autotuning functionalities are almost always available in commercial products (Leva *et al.*, 2001; Aström and Hägglund, 1995). The employment of these tuning formula generally depends on a simple (first or second order) model of the (possibly high-order) process to be controlled. In order to get a simple model of the process in an easy and cost effective way from an industrial point of view, many methods have been proposed in the literature. Obviously, the performance obtained by the control systems depends on the selected identification method.

From another point of view, it is also well-known that good performances both in the set-point following and in the load disturbances rejection task are often difficult to achieve at the same time. This problem is of concern in many applications and the typical approach in these cases is to adopt a two degrees-of-freedom controller, namely, to adopt a feedforward (linear) compensator (Kuo, 1995) (or simply a set-point weighting strategy (Araki, 1988)). The main disadvantage of this method is that the reduction of the overshoot is paid by a higher rise time in the set-point response. To overcome this drawback, the use of a variable set-point weight (Hang and Cao, 1996; Visioli, 1999) or of a feedforward action (Aström and Hägglund, 1995; Wallen, 2000; Wallen and

ström, 2002; Visioli, 2004) has been proposed.
In (Piazzi and Visioli, 2004) we propose to use a noncausal approach (namely, based on an input-output inversion procedure) in order to recover the set-point following performance when the PID parameters values are not appropriate for this task. There, it has been shown that the main merit of the devised methodology is to provide almost the same performance despite possible different values of the PID parameters. However, being based on an estimated FOPDT process transfer function, it is necessary to analyse how the result depends on the estimation method. To this purpose in this paper we address many identification methods that can be applied in the noncausal approach. Further, the role played by the unique design parameter is highlighted.
The paper is organised as follows. In Section 2 the overall noncausal approach is briefly reviewed. Then, in Section 3 the different considered identification methods taken from the literature are described. Results are presented and discussed in Sections 4 and 5 and finally conclusions are drawn in Section 6.

2. THE NONCAUSAL APPROACH

The method presented in (Piazzi and Visioli, 2004) is based on the control scheme shown in Figure 1. Basically, the technique consists of choosing a desired function to achieve a process output transition from y_0 to y_1 and then determining the command function r that causes the desired transition by inverting the closed-loop dynamics by means of a stable inversion procedure. In the following, without loss of generality we will assume $y_0 = 0$. This command function actually substitutes the classical step signal.
The process to be controlled is modelled as a FOPDT transfer function

$$P(s; K, T, L) = \frac{K}{Ts + 1} e^{-Ls}, \tag{1}$$

where the delay term is then approximated (in order to have a rational transfer function) by means of a second order Padé approximation, i.e. we have

$$\tilde{P}(s; K, T, L) \cong \frac{K}{Ts + 1} \frac{1 - Ls/6 + L^2 s^2/12}{1 + Ls/6 + L^2 s^2/12}. \tag{2}$$

Then, the PID controller transfer function is expressed as follows:

$$C(s; K_p, T_i, T_d, T_f) = K_p \left(1 + \frac{1}{T_i s} + T_d s\right) \frac{1}{T_f s + 1} \tag{3}$$

where K_p is the proportional gain, T_i is the integral time constant, T_d is the derivative time constant and T_f is the time constant of a first order filter that makes the transfer function proper.
It has to be noted that the PID controller can be tuned according to any of the many methods proposed in the literature (O'Dwyer, 2003) or even by a trial and error procedure. However, since the purpose of the dynamic inversion procedure is the attainment of high performances in the setpoint following task, disregarding of the controller gains, it is sensible to select the PID parameters aiming only at obtaining good load rejection performances.
The desired output function is chosen as a third-order polynomial parameterized by the transition time τ (see (Piazzi and Visioli, 2001; Piazzi and Visioli, 2004) for further details), i.e.

$$y_d(t; \tau) = y_1 \left(-\frac{2}{\tau^3} t^3 + \frac{3}{\tau^2} t^2\right). \tag{4}$$

Then, the (rational) closed-loop transfer function is determined as

$$H(s) := \frac{C(s)\tilde{P}(s)}{1 + C(s)\tilde{P}(s)} \tag{5}$$

and the stable inversion procedure described in (Piazzi and Visioli, 2004) is applied in order to determine the closed-form expression of the command input $r(t; K, T, L, K_p, T_i, T_d, T_f, \tau)$ that provides the desired output function (4). It has to be noted that $r(\cdot)$ is defined and bounded over the interval $(-\infty, +\infty)$ and from a practical point of view it is necessary to truncate it (with arbitrary precision), resulting therefore in an approximate generation of the desired output $y_d(t; \tau)$. This yields to a pre-action and a post-action time (Perez and Devasia, 2003).
In (Piazzi and Visioli, 2004) it has been highlighted than in addition to provide good set-point following performances, the great merit of the methodology is its capability of providing basically the same step response for very different tuning of the PID parameters. It has to be noted in any case that, although the stable inversion procedure can be applied to any stable rational transfer function, in order to determine a closed-form expression of $r(t; K, T, L, K_p, T_i, T_d, T_f, \tau)$ (so that the actual command signal can be simply calculated by substituting the system parameters in the symbolic expression, making the method suitable to implement in single station controller in addition to Distributed Control Systems), the process dynamics has to be modelled as a FOPDT transfer function (1) and then approximated by (2). Thus, as the devised technique is model based, it is interesting to evaluate the role played by the adopted identification procedure in the achieved performances. Further, it is necessary to evaluate better the role of the design parameters τ as well.
Remark 1. It is worth stressing that the devised methodology is indeed different from filtering (causally) the set-point (or, equivalently, from adopting a set-point weight). Indeed, by filtering

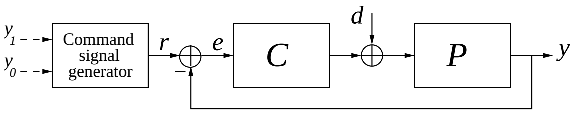

Fig. 1. The dynamic inversion based control scheme.

the set-point, performances are recovered when a too aggressive tuning has been selected and excessive overshoot is avoided by increasing the rise time. Conversely, by using a noncausal approach, set-point following performances are recovered for differently "wrong" settings of the parameters (for example, even if they determine a sluggish response).

3. IDENTIFICATION METHODS

Many methods for the estimation of a FOPDT transfer function have been proposed and proven to be effective in the literature. Here we consider a few methods that are suitable for industrial (PID) control, as they are based on simple and cost-effective experiments that are typically employed in industrial practice, namely, the application of a step signal to the process input (open-loop experiment) or of a relay feedback (closed-loop experiment). As the purpose of this paper is to evaluate the application of the different methods in the noncausal approach rather than evaluating and comparing the effectiveness of each method in estimate an accurate FOPDT model, measurement noise is not taken into account in the following.

3.1 Methods based on step response

The following methods based on the evaluation of an open-loop step response, where different approaches are exploited, have been considered:

a) the well-known area method (Åström and Hägglund, 1995, page 24);
b) the least squares method proposed in (Sung *et al.*, 1998), which is based on the integrated process input and output signals;
c) the least squares method based on the use of Laguerre functions described in (Wang and Cluett, 2000, chapter 2);
d) the least squares method proposed in (Wang *et al.*, 2001) applied to FOPDT transfer function.

It has to be noted that whereas methods a) and d) yield directly to a FOPDT transfer function, methods b) and c) actually yield to a rational transfer function of arbitrarily chosen order. Thus, in these cases, a fourth order transfer function with relative order equal to one has been estimated first. Then, it has been reduced to a FOPDT model by adopting the model reduction method proposed in (Sung *et al.*, 1998).

3.2 Methods based on relay feedback

Relay feedback based methods (Hang *et al.*, 2002) are the most adopted methods where a closed-loop experiment is used for automatic tuning of PID controllers. Actually, the original idea is to employ the relay feedback to estimate the ultimate gain and the ultimate period of the system. However, recently, various techniques have been devised in order to estimate a FOPDT transfer function. The following ones have been considered in this paper:

e) the standard method with a symmetric relay without hysteresis. The FOPDT transfer function is then determined by straightforward calculations (see for example (Yu, 1999, chapter 2));
f) the technique based on the use of a biased relay (with hysteresis) (Hang *et al.*, 2002);
g) the method described in (Wei *et al.*, 2002) where two points on the Nyquist curve (the one where the phase is -90 deg in addition to that where the phase is -180 deg) are estimated. The FOPDT transfer function parameters are then estimated by assuming the knowledge of the process gain (note that this can be simply estimated by considering steady-state values of the input and the output);
h) the method in which an asymmetrical relay is adopted (Srinivasan and Chidambaram, 2003);
i) the method based on the so-called curvature factor of the process response described in (Luyben, 2001).

It turns out that the addressed techniques represent rather different approaches.

4. RESULTS

The influence of the estimation method in the approach based on the input-output inversion has been evaluated on different benchmark systems (Åström and Hägglund, 2000):

$$P_1(s) = \frac{1}{Ts+1}e^{-s} \qquad T = 0.5, 1, 2, 5, 10 \tag{6}$$

$$P_2(s) = \frac{1}{(Ts+1)^2}e^{-s} \qquad T = 0.5, 1, 2 \tag{7}$$

$$P_3(s) = \frac{1}{(s+1)^n} \qquad n = 3, 4, 8 \tag{8}$$

$$P_4(s) = \frac{1}{(s+1)(0.5s+1)(0.5^2s+1)(0.5^3s+1)} \tag{9}$$

For each of these systems, a FOPDT model has been estimated with the techniques considered in Section 3. Then, the PID controller has been tuned, by means of a genetic algorithm (Houck *et al.*, 1995) in which the nominal process transfer function has been considered, in order to minimize the integrated absolute error defined as

$$IAE = \int |e(t)|dt \tag{10}$$

where $e(t)$ is the difference between the set-point value and the process output, when a step load disturbance occurs on the control system. Thus, in order to analyse the effect of the identification method just on the inversion approach and not on the PID controller tuning, the best PID parameters (in the sense of those that provide the best load disturbance response) have been adopted for each case. Hence, the tuning of the PID controller depends only on the considered process and it is independent from the adopted identification method (i.e. we have the same tuning for each process disregarding the adopted estimation method).

Then, for each estimated process transfer function the command signal $r(t; K, T, L, K_p, T_i, T_d, T_f, \tau)$ has been calculated with different values of τ and two values have been calculated with respect to the closed-loop response, namely, the integrated absolute error with respect to a step set-point signal and the integrated absolute error with respect to the desired output response (4), i.e.:

$$IAE_a = \int_{t_0}^{+\infty} |y_1 - y(t)|dt \tag{11}$$

and

$$IAE_b = \int_{t_0}^{+\infty} |y_d(t) - y(t)|dt \tag{12}$$

where t_0 is the time instant in which the process output y leaves the zero value, so that the dead time of the process and the preaction time do not bias the result.

For the sake of brevity and clarity, hereafter we will focus on few significant results which are representative of the behavior of the overall methodology. For example, consider processes $P_3(s)$ with $n = 3$ and $n = 8$ (see (8)). The values of the PID parameters for these systems are reported in Table 1 ($T_f = 0.01$ in both cases), whilst the parameters of the FOPDT transfer functions estimated with the different techniques are shown in Tables 2 and 3. The values of IAE_a and IAE_b for different values of the specified output transition time τ that have been obtained by employing the nine considered identification methods are plotted in Figures 2-5 respectively. For clarity, the nine plots in each figure have not been always labelled, because they are actually very difficult to distinguish in some cases. Indeed, it is worth stressing the general result more than providing detailed quantitative results. Note however that the value of IAE_a obtained by applying a step set-point signal instead of the inversion based command signal is 0.94 for the third-order process and 4.57 for the eighth-order one.

For a better analysis of the technique, Figure 6 shows the different outputs obtained for increasing values of τ (i.e. $\tau = 0.1, 1, 2, \dots, 10$) when the process $P_3(s)$ with $n = 3$ has been estimated with method d) (see subsection 3.1).

Table 1. PID parameters for the considered processes.

Process	K_p	T_i	T_d
$P_3(s)$, $n = 3$	20.48	0.75	0.73
$P_3(s)$, $n = 8$	1.05	4.40	2.62

Table 2. Estimated FOPDT transfer function parameters for $P_3(s)$, $n = 3$.

Method	K	T	L
a	1	1.64	1.41
b	1	1.83	1.28
c	1	1.87	1.28
d	1	2.18	0.87
e	1	1.54	0.99
f	0.99	4.41	0.97
g	1	2.35	1.14
h	1	4.35	0.99
i	0.65	2.94	1.04

Table 3. Estimated FOPDT transfer function parameters for $P_3(s)$, $n = 8$.

Method	K	T	L
a	1	2.88	5.25
b	1	3.15	5.25
c	1	3.15	5.25
d	1	4.24	3.88
e	1	3.81	5.13
f	1	4.66	5.09
g	1	3.67	4.79
h	1	3.85	5.14
i	2.40	10.51	4.32

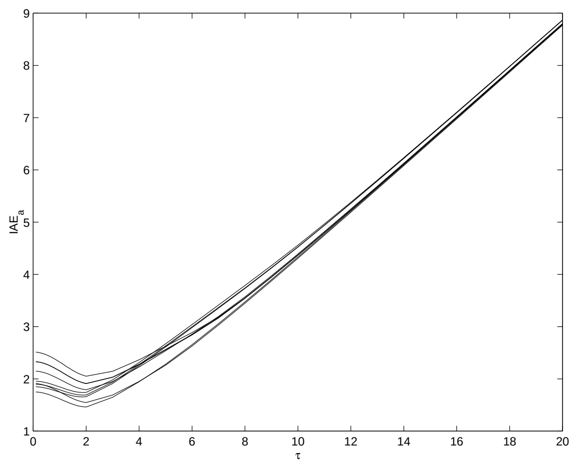

Fig. 2. IAE_a for different values of τ obtained by considering the nine different identification methods for $P_3(s)$, $n = 3$.

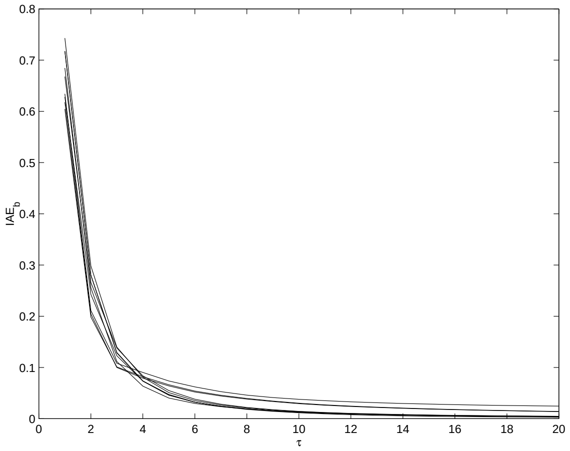

Fig. 3. IAE_b for different values of τ obtained by considering the nine different identification methods for $P_3(s)$, $n = 3$.

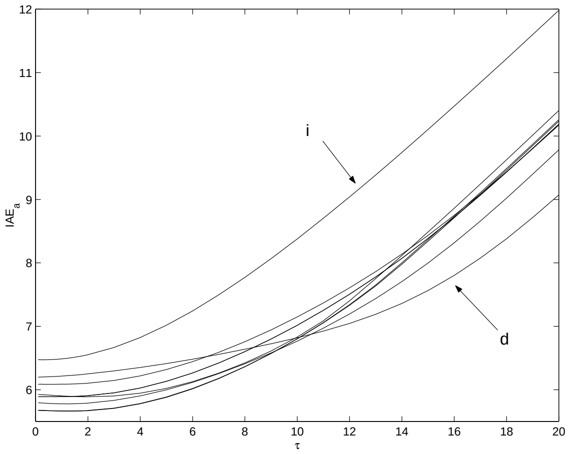

Fig. 4. IAE_a for different values of τ obtained by considering the nine different identification methods for $P_3(s)$, $n = 8$.

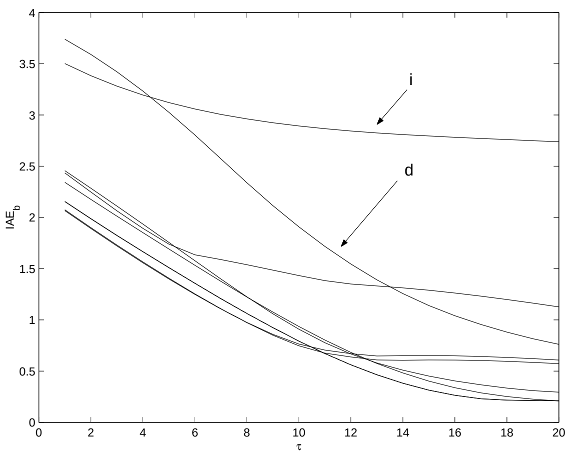

Fig. 5. IAE_b for different values of τ obtained by considering the nine different identification methods for $P_3(s)$, $n = 8$.

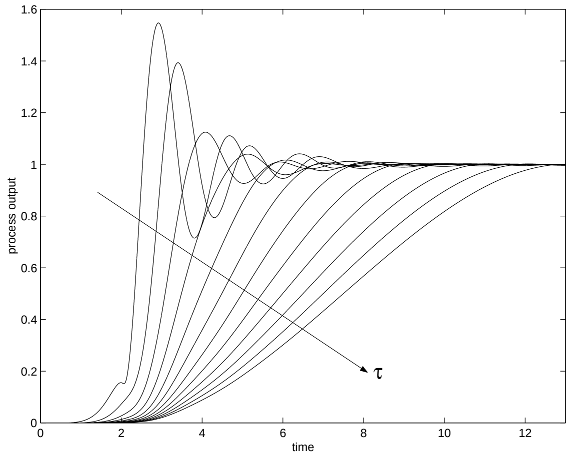

Fig. 6. Process $P_3(s)$ $(n = 3)$ outputs with different values of τ.

5. DISCUSSION

From all the results obtained, the following considerations can be done:

- as expected, the desired transition time τ handles the trade-off between robustness and aggressiveness (and control activity), independently from the adopted estimation method. Indeed, by increasing the value of τ the desired response is obtained more accurately (IAE_a decreases), although this implies that IAE_a increases as the rise time increases. In any case good performances are obtained for low values of τ (thus, excessively high values of τ are not worth to being employed);

- the considered estimation methods are practically equivalent, because very often they provide basically the same performances and in case a method is worse than another for a given process and a given value of τ, it can be better for another process and another value of τ (see for example method d) in Figures 4 and 5). The only exception is the method i) (Luyben, 2001) when the process dynamics is rather different from being FOPDT, mainly because it does not provide a very accurate estimation of the process gain;

- if the PID controller is well tuned from the point of view of minimising the value of IAE_a for a step set-point signal, then the inversion based approach might give a higher value of IAE_a than using a step set-point signal. However, it has to be taken into account that a low value of the integrated absolute error is often achieved at the expense of high overshoots, high control activity and poor robustness and these latter aspects are almost always of major concern in the industrial context. Indeed, they can be easily handled by the devised noncausal approach.

6. CONCLUSIONS

In this paper, the use of a noncausal approach for the improvement of PID control proposed in (Piazzi and Visioli, 2004) has been further investigated. The methodology is based on the derivation of a parameterized closed-form expression of the command signal to be applied to the closed-loop system in order to achieve a desired output transition. Predefined performances can be obtained almost independently from the tuning of the PID parameters and from the employed model estimation method. Further, the presence of the desired output transition time τ as a design parameter allows the user to easily handle the trade-off between aggressiveness and robustness (in the sense that the desired output is accurately achieved). It appears therefore that the devise technique appears to be suitable to implement in the industrial context.

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