UNSCENTED TRANSFORM AND ITS APPLICATION IN ATC TRACKING

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Abstract: Nonlinear filtering is certainly very important in estimation since most real-world problems are nonlinear. In this paper, we devote the effort to use the unscented transform (UT) to improve Air Traffic Control (ATC) tracking. The simulation results show that the UT improved the tracking performance compared to the traditional methods for the track-while-scan (TWS) ATC application.

Keywords: Unscented Transform, Unscented Kalman Filter, Unscented Particle Filter, Air Traffic Control

1. INTRODUCTION

Recently, many efforts have been devoted in extending the unscented transform (UT) to the Bayesian filtering problems. The UT is a method for calculating the statistics of a random variable which undergoes a nonlinear transformation and builds on the principle that it is easy to approximate a Gaussian distribution than it is to approximate an arbitrary nonlinear function or transformation (Julier *et al.*, 1997).

The straightforward extension of the UT to the recursive estimation is the unscented Kalman filter (UKF) (Julier *et al.*, 1997). The UKF is a powerful nonlinear estimation technique and has been shown to give well performance in a variety of applications over extended Kalman filter (EKF).

Another extension of the UT to the recursive estimation is the unscented particle filter (UPF). Merwe et al. uses a UKF for proposal distribution within the particle filter framework (Merwe *et al.*, 2000). The UPF takes advantage of the good features of both UKF and particle filters, and avoids their limitations and has been shown to perform better than other sequential estimation algorithms.

Although the UKF and UPF have been applied to a wide range of Bayesian estimation problems, to the best of our knowledge there has been no attempt to use it to improve ATC tracking. Therefor, in this paper, we explore their potential benefits in this area. In a simulation study we compare these filters to the classical filter such as Extended Kalman filter and particle filter for an Air Traffic Control (ATC) track-while-scan (TWS) application, respectively. The problem under consideration incorporates nonlinear effects both in the dynamic and measurement model and some constraints on the system states.

The remaining part of this paper is organized as follows. In Section 2, we will begin with Bayesian description for maneuvering target tracking. The unscented transform and its application in nonlinear/non-Gaussian Bayesian tracking will be proposed in section 3. The mathematical model and system dynamics for ATC tracking and the parameter selection for the algorithms will be presented in section 4. In section 5, the simulation results will be presented. Conclusion will be drawn in Section 6.

2. BAYESIAN DESCRIPTION FOR MANEUVERING TARGET TRACKING

Many recursive estimation problems can be formulated as

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{w}_k) \quad (1)$$

$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{v}_k) \quad (2)$$

where $\mathbf{f}$ is a possibly non-linear function of the state $\mathbf{x}_k \in \Re^n$ and the observation $\mathbf{y}_k \in \Re^m$ is often a non-linear mapping of the current state. Both the dynamic model and the measurement model are inaccurate, due to modeling and/or sensor errors. This is described by the stochastic processes $\mathbf{w}_k$ and $\mathbf{v}_k$. Then the objective of tracking is to recursively estimate and predict the state $\mathbf{x}_k$ using the observations $\mathbf{y}_{1:k} = \{\mathbf{y}_i\}_{i=1}^{k}$ up to and including time k.

From a Bayesian perspective, the tracking problem is required to construct the pdf $p(\mathbf{x}_k|\mathbf{y}_{1:k})$, given the data $\mathbf{y}_{1:k}$ up to time k. It is assumed the signals are independent with probability densities $p(\mathbf{w}_k)$, $p(\mathbf{v}_k)$, and the initial pdf, $p(\mathbf{x}_0|\mathbf{y}_0) \equiv p(\mathbf{x}_0)$, of the state vector, also known as the prior, is available ($\mathbf{y}_0$ being the set of no measurements) and independent. Then, in principle, the pdf $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ may be obtained recursively in two stages: *time update* in (3) and *measurement update* in (4).

$$p(\mathbf{x}_{k+1}|\mathbf{y}_{1:k}) = \int_{\Re^n} p(\mathbf{x}_{k+1}|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{y}_{1:k})d\mathbf{x}_k \quad (3)$$

$$p(\mathbf{x}_k|\mathbf{y}_{1:k}) = \frac{p(\mathbf{y}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{y}_{1:k-1})}{p(\mathbf{y}_k|\mathbf{y}_{1:k-1})} \quad (4)$$

These equations can easily be derived using the Markov property, Bayes's rule and some standard calculations from probability theory.

The recurrence relations (3) and (4) form the basis for the optimal Bayesian solution. This recursive propagation of the posterior density is only a conceptual solution in that in general, it cannot be determined analytically.

3. UNSCENTED KALMAN FILTER

3.1 The Unscented Transform (UT)

Consider propagating a n-dimension random variable $\mathbf{x}$ through a nonlinear transformation $\mathbf{y} = g(\mathbf{x})$, where $\mathbf{x}$ is assumed to be multivariate normally distributed with mean $\bar{\mathbf{x}}$ and covariance $\mathbf{P}$, and we wish to estimate the mean and variance of $\mathbf{y}$. The UT bares a superficial resemblance to Monte Carlo-type methods, but uses a small deterministically chosen set of sigma points. The sigma points are selected so that distribution of $\mathbf{y}$ can be estimated from these transformed sigma points as accurately as possible. The procedure is presented in Fig. 1. The nonlinear function is applied to each point in turn to yield a cloud of transformed points and $\bar{\mathbf{y}}$ and $\mathbf{P}_{yy}$ are the statistics of the transformed points. Since the problems of statistical convergence are not an issue, high order information about the distribution can be captured using only a very small number of points.

1. Compute the set of $2n+1$ points from the rows or columns of the matrix $\sqrt{(n+\lambda)\mathbf{P}_{xx}}$:

$$\mathbf{x}_0 = \mathbf{m}_x$$
$$\mathbf{x}_0 = \mathbf{m}_x + (\sqrt{(n+\lambda)\mathbf{P}_{xx}})_i, \ i = 1, \dots, n$$
$$\mathbf{x}_0 = \mathbf{m}_x - (\sqrt{(n+\lambda)\mathbf{P}_{xx}})_{n+i}, \ i = 1, \dots, n$$

and the associated weights:

$$W_0^m = \lambda/(n+\lambda)$$
$$W_0^c = \lambda/(n+\lambda) + (1-\alpha^2+\beta)$$
$$W_i^m = \lambda/\{2(n+\lambda)\} \quad i = n+1, \dots, 2n$$
$$W_i^c = \lambda/\{2(n+\lambda)\} \quad i = n+1, \dots, 2n$$

Parameter λ is a scaling parameter defined as

$$\lambda = \alpha^2(n+\kappa) - n$$

The positive constants α, β and κ are used as parameters of the method.
2. Transform each of the sigma points as

$$\mathcal{Y}_i = g(\mathbf{x}_i), \ i = 0, \dots, 2n$$

3. Mean and covariance estimates for y can be calculated as

$$\mathbf{m}_y \approx \sum_{i=0}^{2n} W_i^m \mathcal{Y}_i$$
$$\mathbf{P}_{yy} \approx \sum_{i=0}^{2n} W_i^c (\mathcal{Y}_i - \mathbf{m}_y)(\mathcal{Y}_i - \mathbf{m}_y)^T$$

Fig. 1. **Unscented Transform**

In the rest of the section, we will first introduce the standard UKF, and will have an overview of other various UKF algorithms. On this base, the complete UPF algorithm that uses the UKF to generate its proposal distribution will be presented.

3.2 The Unscented Kalman Filter

Unscented Kalman filter was first proposed by Julier and Uhlmann in (Julier *et al.*, 1997), which is a straightforward extension of the unscented transformation (UT) to the recursive estimation. The standard UKF implementation is given in (Merwe *et al.*, 2001) for state-estimation.

The deceptively simple approach taken with the UT results in approximations that are accurate to the third order for Gaussian inputs for all nonlinearities. For non-Gaussian inputs, approximations are accurate to at least the second-order, with the accuracy of third and higher order moments determined by the choice of α and β. It is interesting to note that no explicit calculation of Jacobians are necessary to implement this algorithm. The total number of computations is only $O(n^2)$ as compared to $O(n^3)$ for the EKF (Wan *et al.*, 2000a).

It builds on the principle that it is easier to approximate a Gaussian distribution than it is to approximate an arbitrary nonlinear function or transformation (J.K.Uhlmann, 1994).

The superior performance of the UKF over that of the EKF have been reported in numerous publications including (Wan *et al.*, 2000a) (Wan *et al.*, 2000b) (Julier *et al.*, 1997).

3.3 A Overview of Unscented Kalman filter

The UKF has superior performance and superior implementation properties to the EKF, however, all of these sigma point solutions share the property that as the dimension of the state space increases, the radius of the sphere that bounds all the sigma points increases as well. Even though the specified information is still captured correctly (i.e., the mean and covariance of the sigma points matches the apprior distribution for all dimensions), it does so at the cost of sampling non-local effects. For many kinds of nonlinearities (such as exponents or trigonometric functions) this can lead to significant difficulties. In (Julier *et al.*, 2000), a method was proposed for overcoming these difficulties through the use of negative weights and a "modified" form of the algorithm to guarantee positive semi-definiteness. Unfortunately, the approach was developed from studying the higher order properties of the system and no physical intuition was used. Second, it was only developed to study the problem of point scaling for the specific set introduced in (Julier *et al.*, 1995) and its applicability to other sigma point sets was not examined.

In (Merwe *et al.*, 2001), it introduce the square-root unscented Kalman filter (SR-UKF). It has better numerical properties and guaranteed positive semi-definiteness of the underlying state covariance.

In (James, 2001), a more robust unscented transform was proposed. The unscented transformation is extended to use extra test points beyond the minimum necessary to determine the second moments of a multivariate normal distribution. A convenient way to add test points is to introduce "hidden variables" and a suitable orthogonal transformation. The additional test points can improve the estimated mean and variance of the transformation distribution when the transforming function or its derivatives have discontinuities. But the UT is still accurate only to third order even with the added test points proposed here.

In (Julier, 2002), it re-examines the problem of sigma point scaling and introduces a new, general framework. Called the *scaled unscented transformation*, the method allows any set of sigma points to be scaled by an arbitrary scaling factor in such a manner that the first two moments of the set are preserved. It is equivalent to applying the conventional unscented transformation followed by a simple post-processing step. The storage and computational costs are exactly the same as a non-scaled version of the same transformation. The method can also be used to partially incorporate contributions higher order information into the estimates.

3.4 Unscented Particle Filter (UPF)

It has been shown that the UKF is able to more accurately propagate the mean and covariance of the Gaussian approximation to the state distribution, than the EKF. And the UKF also has the ability to scale the approximation errors in the higher tailed distributions. This makes the UKF very attractive for the generation of proposal distributions within the particle filter framework. Recently, Merwe et al. have proposed a new particle filter named Unscented particle filter (UPF) (Merwe *et al.*, 2000), which takes advantage of the good features of both UKF and particle filters, and avoids their limitations. Specifically, the proposal distribution for each particle is as follows:

$$q(\mathbf{x}_k^i|\mathbf{x}_{0:k-1}^i, \mathbf{y}_{1:k}) = \mathcal{N}(\bar{\mathbf{x}}_k^i, \hat{\mathbf{P}}_k^i), \quad i = 1, \ldots, N \quad (5)$$

where $\bar{\mathbf{x}}_k$ and $\hat{\mathbf{P}}_k$ are the mean and covariance of $\mathbf{x}$, computed using UKF. The UPF algorithm is easily obtained by plugging the UKF step and Equation (5) into the generic particle filter algorithm.

Since the UKF can theoretically have heavier tails than EKF, while still incorporating the latest information before the evaluation of the importance weights, the theory predicts that UPF can perform very well in situations where the likelihood is peaked or when one finds outliers in the data.

4. SIMULATION SETUP

4.1 Dynamic and Measurement Models For ATC Tracking

For civil aircraft a common model is to use the nearly coordinated turn model (Li, X. R. *et al.*, 2000). The model is a discretized continuous time nonlinear stochastic differential equation model where the turn rate state ω is gives a strong nonlinear behavior. The discrete two dimensional system is given by

$$\mathbf{X}_{t+1} = A(\omega_{t+1})\mathbf{X}_t + [B_{\mathbf{v}}\ B_\omega]\mathbf{w}_t \tag{6}$$

$$\mathbf{X}_t = (\mathbf{x}_t\ \omega_{t+1})', \mathbf{x}_t = (\xi\ \dot{\xi}\ \eta\ \dot{\eta})' \tag{7}$$

where ξ and η are the Cartesian position coordinates and $\dot{\xi}$, $\dot{\eta}$ are the velocity components, w_t is the process noise with zero mean and covariance Q_t. System constraints are incorporated in the model, so that non-feasible maneuvers are avoided using the UPF technique.

$$A(\omega) = \begin{bmatrix} 1 & \frac{\sin\omega T}{\omega} & 0 & -\frac{1-\cos\omega T}{\omega} & 0 \\ 0 & \cos\omega T & 0 & -\sin\omega T & 0 \\ 0 & \frac{1-\cos\omega T}{\omega} & 1 & \frac{\sin\omega T}{\omega} & 0 \\ 0 & \sin\omega T & 0 & \cos\omega T & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B_v = \begin{bmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \\ 0 & 0 \end{bmatrix}, \ B_\omega = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

The range, azimuth and elevation radar measurements are modeled as

$$\mathbf{y}_t = h(\mathbf{x}_t) + \mathbf{v}_t = \begin{bmatrix} \sqrt{\xi^2 + \eta^2} \\ \arctan\left(\frac{\eta}{\xi}\right) \end{bmatrix} + \mathbf{v}_t \tag{8}$$

where v_t is zero mean noise with covariance R_t. Note that for the general case, the measurement equation must be modified so the angle equations are continuous. Independence in time and between the measurement and process noise is assumed. For this model we have neglected the relative height value.

4.2 Parameter Selection

Selection of noise levels for dynamic models is an important part of the estimator design. The process noise w_t is used to model the air turbulence, wind change, and so forth. The right choice of the

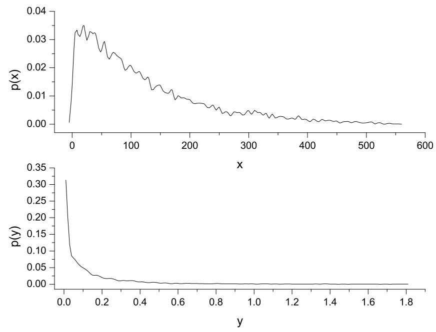

Fig. 2. Position and Velocity Posteriori Probabilities

noise level of the nearly coordinated turn model depends on what turn rate range is expected. In the experiment study, the Q_t is chosen as

$$Q_t = \begin{bmatrix} 0.001 & 0 & \\ 0 & 0.001 & 0 \\ 0 & 0 & 0.001 \end{bmatrix}$$

In the algorithm UKF, the constant α determines the spread of the sigma points around $\bar{\mathbf{x}}$. κ is a secondary scaling parameter, and β is used to incorporate prior knowledge of the distribution of $\mathbf{x}$. In this experiment, $\alpha = 1$, $\beta = 2$ and $\kappa = 0$.

5. SIMULATION RESULTS

A simulation study using the nearly coordinated turn model from section 4.1 is performed where the sampling period is chosen to $T = 4s$ to emulate a track-while-scan (TWS) behavior. The target was making a turn in a plane at nearly constant turn rate of $3°/sec$ and the distribution of the measurement noise is chosen to be Gaussian, with angular and distance standard deviations of $0.5°$ and 20 m respectively. In this simulation, UKF and UPF are implemented. For the purpose of comparison, other filters such as EKF and generic PF are also implemented.

In Figure 2, a posteriori probabilities for each coordinate is presented for the predicted particles for one realization.

In Table 1 the position Root Mean Square Error (RMSE) for the UKF and UPF are compared to the other methods, using $N_{mc} = 100$ Monte Carlo simulation and the particle filters used $N = 500$ particles. The RMSE using meausrements only is also presented in Table 1. The RMSE is defined as follows:

$$RMSE = \sqrt{\frac{1}{L}\sum_{k=1}^{L}\frac{1}{N_{mc}}\sum_{i=1}^{N_{mc}}((\hat{\xi}_k^i - \xi_k^i)^2 + (\hat{\eta}_k^i - \eta_k^i)^2)}$$

where $L = 60$ is the simulation path length and $\hat{\xi}_k^i$, $\hat{\eta}_k^i$ are the filter position estimates at time k in Monte Carlo run i.

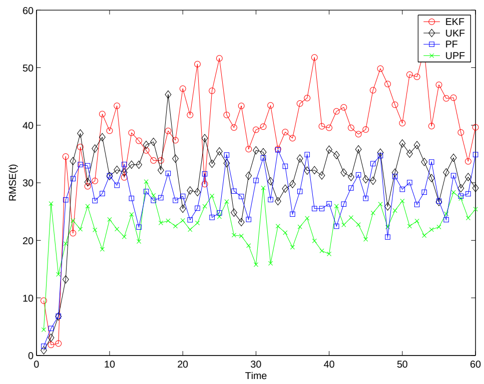

Fig. 3. RMSE(t) for different tracking methods

Table 1. RMSE for 100 Monte Carlo simulations with different mehtods

Algorithm	RMSE		Execution time
	mean	var	(s)
EKF	39.6427	20.9801	0.03685
UKF	30.6539	11.2789	0.03353
PF	27.4149	12.5382	0.08123
UPF	22.3407	11.9321	2.8613
Measurements	41.3064	21.3309	-

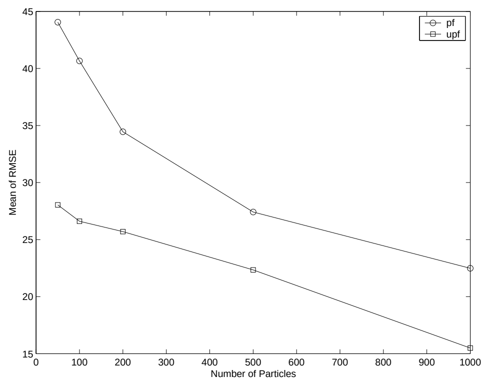

Fig. 4. particle filters with different number of particles

In Figure 3 the RMSE for different tracking methods is presented for each time, i.e., according to the following equation for the different methods

$$RMSE(t) = \sqrt{\frac{1}{N_{mc}} \sum_{i=1}^{N_{mc}} ((\hat{\xi}_k^i - \xi_k^i)^2 + (\hat{\eta}_k^i - \eta_k^i)^2)}.$$

Figure 4 shows performance of UPF using different number of particles: $50, 100, 200, 500, 1000$. The RMSEs using particles from 50 to 500 decrease rapidly , and 1000 and beyond have little room for improvement.

6. CONCLUSION

In the simulation study in section 5 the UT improved the tracking performance compared to the traditional methods for the track-while-scan (TWS) Air Traffic Control (ATC) application. The UKF showed superior performance over the EKF. And the UPF is flexible than traditional methods since it can also incorporate system constraints and non-Gaussian noise assumptions. However, UPF can be time consuming if many particles are used. To improve the real time execution performance the particle filter update could be run in parallel.

The UPF is shown to be superior to UKF at the cost of more computation but it can provide flexible accuracy by using more or less particles.

REFERENCES

E. A. Wan, R. van derMerwe, and A. T. Nelson, Dual Estimation and the Unscented Transformation, In: *Neural Information Processing Systems* 12.2000, pp. 666-672, MIT Press.

E. A. Wan and R. van der Merwe, The Unscented Kalman Filter for Nonlinear Estimation, In: *Proc. of IEEE Symposium 2000 (AS-SPCC)*, Lake Louise, Alberta, Canada, Oct. 2000.

E. A. Wan and R. van der Merwe. The Unscented Kalman Filter, In: *Kalman Filtering and Neural Networks, Chapter 7*, Edited by Simon Haykin. Wiley Publishing, 2001.

James R. Van Zandt, A more robust unscented transform Technical report, MITRE Corporation, http://www.mitre.org, July 2001.

J. K. Uhlmann. Simultaneous map building and localization for real time applications. Technical report, University of Oxford, 1994. Transfer thesis.

Li, X. R., and Jilkov, V.P. A survey of maneuvering target tracking—Part I: Dynamic models. In *Proceedings of the 2000 SPIE Conference on Signal and Data Processing of Small Targets*, Vol. 4048, Orlando, FL, Apr. 2000, pp. 212C236.

R. van der Merwe, A. Doucet, N. de Freitas and E. Wan, The Unscented Particle Filter, In: *Advances in Neural Information Processing Systems (NIPS13)*, MIT Press, Eds. T. K. Leen, T. G. Dietterich and V. Tresp, Dec, 2000

R. van der Merwe and E. A. Wan, The Square-Root Unscented Kalman Filter for State and Parameter-Estimation, In: *International Conference on Acoustics, Speech, and Signal Processing*, Salt Lake City, Utah, May, 2001

S. J. Julier and J. K. Uhlmann, A New Extension of the Kalman Filter to Nonlinear Systems, In: *Proceedings of AeroSense: The 11th International Symposium on Aerospace/Defense Sensing, Simulation and Controls*, 1997.

S. J. Julier, The Scaled Unscented Transformation, In: *Proceedings of the American Control Conference*, Vol. 6, pp. 4555-4559, 2002.

S. J. Julier, J. K. Uhlmann and H. F. Durrant-Whyte, A New Approach for Filtering Nonlinear Systems, In: *The Proceedings of the*

American Control Conference, Seattle, Washington, 1995, pp. 1628-1632.

S. J. Julier, J. K. Uhlmann and H. F. Durrant-Whyte, A New Approach for the Nonlinear Transformation of Means and Covariances in Linear Fil- ters, *IEEE Transactions on Automatic Control*, Vol. 5, No. 3, pp. 477-482, March 2000.