STABLE 2-WAY FUZZY ADAPTIVE CONTROL OF A FLEXIBLE-JOINT ROBOT ARM

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Abstract: We analyze the stability of our novel 2-way fuzzy adaptive controller using describing function technique. We use additivity property of fuzzy systems to develop a systematic analytical approach for the design of a multi-input single-output fuzzy controller that we use for the control of a flexible-joint robot arm.

Keywords: 2-Way fuzzy control, describing function, flexible arm.

1. INTRODUCTION

Flexible robotic systems have been developed due to the need of having faster, lighter and more precise robots handling heavy payloads (Tang *et al.*, 2001), (Carrera and Serna, 1996), (Yang *et al.*, 1997). In the modeling and control of such systems, it is important to incorporate the flexibility. There are different controllers designed in the literature for such systems ((Tang *et al.*, 2001), (Nicosia and Tomei, 1995), (Ge, 1996), (Spong, 1995), (Ailon, 1996), (Wang, 1995)). We have modeled the inconsistencies and uncertainties in the system using our 2-way fuzzy adaptive system (Gürkan *et al.*, 2002).

In this paper, we use our 2-way fuzzy adaptive system to control the flexible-joint robot arm system, so it becomes important to analyze the stability of this controller. We use describing function technique to develop a systematic design procedure. Kim et al. (Kim *et al.*, 2000) derive analytical expressions for the describing functions of a fuzzy system with single input, and a fuzzy system with two inputs, where the second input is the derivative of the first. The existence of the limit-cycle of the fuzzy control system is predicted using the describing function analysis. In (Aracil and Gordillo, 2003), the describing function method is used to analyze the behavior of PD and PI fuzzy logic controllers. The existence of stable and unstable limit cycles are predicted. Describing function analysis of a T-S fuzzy system is done in (Cuesta *et al.*, 1999), where the describing function is evaluated experimentally. The existence of multiple equilibria and limit cycles are examined.

The describing function analysis is generally experimental in the discussed papers apart from the analysis in (Kim *et al.*, 2000). However, this analysis is only for two input fuzzy systems. We extend the analytical calculation of describing function to multi-input fuzzy systems by using the additivity property introduced in (Cuesta *et al.*, 1999). In Section 2, we briefly introduce our 2-way fuzzy adaptive system structure, and discuss the additivity property together with the describing function of our system. Section 3 gives the stability analysis of the controller. We apply the designed controller to a flexible-joint robot arm system in Section 4. Section 5 concludes the paper.

2. MATHEMATICAL OVERVIEW

2.1 2-Way Fuzzy Adaptive System

Our proposed 2-way fuzzy adaptive system models uncertainty and inconsistency using intuitionistic fuzzy sets and generate the novel architecture 2-way fuzzy adaptive system. The 2-way fuzzy adaptive system uses intuitionistic fuzzy sets that model intuitive uncertainty in place of classical fuzzy sets. An intuitionistic fuzzy set A (Atanassov, 1986), of a given underlying set E is represented by a pair of functions $\{\mu_A, \upsilon_A\}$ mapping $E \rightarrow [0,1]$, for all $x \in E$ where $\mu_A(x)$ gives the degree of membership to A, $\upsilon_A(x)$ gives the degree of nonmembership subject to the restriction: $\mu_A(x) + \upsilon_A(x) \leq 1$ stressing consistency in intuition.

The rule structure for our system is as follows:

$$R^{(ijk\ldots l)} : \text{IF } x_1 \text{ is } F_{1i} \text{ and } x_2 \text{ is } F_{2j} \text{ and } \ldots \text{ and } x_n \text{ is } F_{nl}, \text{ THEN } y \text{ is } y_{ijk\ldots l} \tag{1}$$

where F are the antecedent fuzzy sets, $x = (x_1, ..., x_n)^T \in U$ and $y \in V$ are respectively input and output linguistic variables.

The closed form of our fuzzy logic system with center average defuzzifier, product-inference rule and singleton fuzzifier is:

$$f(x) = \sum_i \sum_j \sum_k \cdots \sum_l \Omega_{ijk\ldots l} y_{ijk\ldots l} \tag{2}$$

where,

$$\Omega_{ijk\ldots l} = \frac{\mu_{1i}(x_1)\mu_{2j}(x_2)\ldots\mu_{nl}(x_n)}{\sum_p \sum_r \sum_s \cdots \sum_t \mu_{1p}(x_1)\ldots\mu_{nt}(x_n)} \tag{3}$$

We have used our system in modeling uncertainty and inconsistency (Gürkan *et al.*, 2002) and we have applied the developed technique to the modeling of a flexible-joint robot arm. The modeled inconsistencies and uncertainties have been reduced by a two-phase training procedure. In this paper, we analyze the stability of this system when used as a controller.

2.2 Additively Decomposable Fuzzy Systems

In order to compute the describing function of a fuzzy system with more than two inputs, we make use of the additivity property of fuzzy systems to reduce the multi-input single-output fuzzy system into one of single-input single-output. Towards this end, we bring an extension to the theory of additivity of fuzzy systems in Cuesta et al. (Cuesta *et al.*, 1999). In this paper, for simplicity, we develop the theory for $n = 4$ since in our application example (flexible-joint robot arm) the system is of degree 4, so there are four inputs to the fuzzy controller. Extension of the theory to higher degrees is straightforward.

For the fuzzy system to be additively decomposable, it should satisfy the following property (Cuesta *et al.*, 1999):

$$f(x) = f(x_1, x_2, \ldots, x_n) = f(x_1, 0, \ldots, 0) + f(0, x_2, \ldots, 0) + \ldots + f(0, 0, \ldots, x_n) \tag{4}$$

In order to simplify the notation, we consider $f_i(x_i) = f(0, \ldots, x_i, \ldots, 0)$, for $i = 1, 2, \ldots, n$.

The assumptions on the membership functions for the system to be decomposable are given in (Cuesta *et al.*, 1999) as follows:

1. $\mu_{qp}(x_q = 0) = 1$, $\mu_{qi}(x_q = 0) = 0$, $i \neq p$, $i = 1, \ldots,$ rule number and $q = 1, \ldots, n$.

2. $\sum_{i=1}^{ruleno} \mu_{qi}(x_q) = 1$, $\forall_{x_q}$, $q = 1, \ldots, n$.

Our fuzzy system uses triangular membership functions that satisfy the above assumptions:

$$\mu_{qi}(x_q) = \begin{cases} \dfrac{x_q - \phi_{qi-1}}{\phi_{qi} - \phi_{qi-1}}, & \phi_{qi-1} \leq x_q < \phi_{qi} \\ \dfrac{x_q - \phi_{qi+1}}{\phi_{qi} - \phi_{qi+1}}, & \phi_{qi} \leq x_q < \phi_{qi+1} \\ 0 & otherwise \end{cases} \tag{5}$$

where $\phi_{-qi} = -\phi_{qi}$. The reason for choosing these type of membership functions is that they also satisfy the assumptions in the calculation of the describing function, which is introduced in subsection 2.3.

The fuzzy controller is represented by Equations 2 and 3 with n replaced by the value 4. For all the inputs x_1, x_2, x_3 and x_4 in Equation 3, we assign the triangular memberships of Equation 5. These memberships satisfy the assumptions (1) and (2), and the denominator of Ω_{ijkl} in Equation 3 is:

$$\sum_p \sum_r \sum_s \sum_t \mu_{1p}(x_1)\mu_{2r}(x_2)\mu_{3s}(x_3)\mu_{4t}(x_4) = 1$$

so,

$$\Omega_{ijkl} = \mu_{1i}(x_1)\mu_{2j}(x_2)\mu_{3k}(x_3)\mu_{4l}(x_4) \tag{6}$$

When $\phi_{1a} \leq x_1 < \phi_{1a+1}$, two consequent rules are fired for x_1 with memberships $\mu_{1a}(x_1)$ and $\mu_{1a+1}(x_1)$. The same applies for the other inputs: for $\phi_{2b} \leq x_2 < \phi_{2b+1}$ with $\mu_{2b}(x_2)$ and $\mu_{2b+1}(x_2)$, for $\phi_{3c} \leq x_3 < \phi_{3c+1}$ with $\mu_{3c}(x_3)$ and $\mu_{3c+1}(x_3)$ and for $\phi_{4d} \leq x_4 < \phi_{4d+1}$ with $\mu_{4d}(x_4)$ and $\mu_{4d+1}(x_4)$. As a total, there are $2^4 = 16$ rules fired, an example of which is:

$$R^{(abcd)} : \text{IF } x_1 \text{ is } \mu_{1a} \text{ and } x_2 \text{ is } \mu_{2b} \text{ and } x_3 \text{ is } \mu_{3c} \text{ and } x_4 \text{ is } \mu_{4d}, \text{ THEN } y \text{ is } y_{abcd}.$$

We write $\mu_{i.}$ instead of $\mu_{i.}(x_i)$ in what follows in order to simplify the notation. The corresponding fuzzy system output is:

$$\begin{aligned} f(x) &= \mu_{1a}\mu_{2b}\mu_{3c}\mu_{4d}y_{abcd}+ \\ &+\ldots+\mu_{1a+1}\mu_{2b+1}\mu_{3c+1}\mu_{4d}y_{a+1b+1c+1d} \\ &+\mu_{1a+1}\mu_{2b+1}\mu_{3c+1}\mu_{4d+1}y_{a+1b+1c+1d+1} \end{aligned} \tag{7}$$

The decomposed system should have four single-input single-output systems of the form:

$$\begin{aligned} f_1(x_1) &= \mu_{1a}y_{afgh}+\mu_{1a+1}y_{a+1fgh} \\ f_2(x_2) &= \mu_{2b}y_{ebgh}+\mu_{2b+1}y_{eb+1gh} \\ f_3(x_3) &= \mu_{3c}y_{efch}+\mu_{3c+1}y_{efc+1h} \\ f_4(x_4) &= \mu_{4d}y_{efgd}+\mu_{4d+1}y_{efgd+1} \end{aligned} \tag{8}$$

We derive the condition under which $f_1(x_1)+f_2(x_2)+f_3(x_3)+f_4(x_4)=f(x)$ is satisfied. First we multiply the above equations by
$(\mu_{2b}+\mu_{2b+1})(\mu_{3c}+\mu_{3c+1})(\mu_{4d}+\mu_{4d+1})$,
$(\mu_{1a}+\mu_{1a+1})(\mu_{3c}+\mu_{3c+1})(\mu_{4d}+\mu_{4d+1})$,
$(\mu_{1a}+\mu_{1a+1})(\mu_{2b}+\mu_{2b+1})(\mu_{4d}+\mu_{4d+1})$,
and $(\mu_{1a}+\mu_{1a+1})(\mu_{2b}+\mu_{2b+1})(\mu_{3c}+\mu_{3c+1})$ respectively. All these four terms are equal to 1 for the membership assignments of Equation 5. The equations in 8 become:

$$\begin{aligned} f_1(x_1) &= \mu_{1a}\mu_{2b}\mu_{3c}\mu_{4d}y_{afgh}+ \\ &+\ldots+\mu_{1a+1}\mu_{2b}\mu_{3c}\mu_{4d}y_{a+1fgh}+\ldots+ \\ &\mu_{1a+1}\mu_{2b+1}\mu_{3c+1}\mu_{4d+1}y_{a+1fgh} \\[1em] f_2(x_2) &= \mu_{1a}\mu_{2b}\mu_{3c}\mu_{4d}y_{ebgh}+ \\ &+\ldots+\mu_{1a}\mu_{2b+1}\mu_{3c}\mu_{4d}y_{eb+1gh}+\ldots+ \\ &\mu_{1a+1}\mu_{2b+1}\mu_{3c+1}\mu_{4d+1}y_{eb+1gh} \\[1em] f_3(x_3) &= \mu_{1a}\mu_{2b}\mu_{3c}\mu_{4d}y_{efch}+ \\ &+\ldots+\mu_{1a}\mu_{2b}\mu_{3c+1}\mu_{4d}y_{efc+1h}+\ldots+ \\ &\mu_{1a+1}\mu_{2b+1}\mu_{3c+1}\mu_{4d+1}y_{efc+1h} \\[1em] f_4(x_4) &= \mu_{1a}\mu_{2b}\mu_{3c}\mu_{4d}y_{efgd}+ \\ &+\ldots+\mu_{1a+1}\mu_{2b}\mu_{3c}\mu_{4d}y_{efgd}+\ldots+ \\ &\mu_{1a+1}\mu_{2b+1}\mu_{3c+1}\mu_{4d+1}y_{efgd+1} \end{aligned} \tag{9}$$

Then, we add the above equations in Equation 9 and compare the terms with Equation 7. From the comparison of the first terms, we derive that if we choose $y_{afgh}+y_{ebgh}+y_{efch}+y_{efgd}=y_{abcd}$, the first terms become equal. If we do this comparison for the rest of the terms, we derive all the constraints under which the system is additively decomposable.

2.3 Describing Function of a 2-Way Fuzzy Adaptive System

We extend the analytical calculation of describing function of fuzzy systems introduced in (Kim *et al.*, 2000) so that it applies to our 2-way fuzzy adaptive system. We consider a single-input single-output case, since using the additivity property, fuzzy systems can be decomposed into single-input single-output fuzzy systems. The reason for the need of such a decomposition is that, for more than two inputs the calculation of describing function is experimental (Kim *et al.*, 2000). Our aim is to find an analytical expression to be used in the design of a 2-way fuzzy adaptive controller.

First, we review the describing function for a 1-way fuzzy system without giving the proofs (the basic assumptions and proofs can be found in (Kim *et al.*, 2000)). The system has the rule structure:

$$R^i : \text{IF } x \text{ is } F_i, \text{ THEN } u \text{ is } u_i \tag{10}$$

where x and u are the input and output variables respectively, F_i's are the fuzzy sets corresponding to the i^{th} rule, and u_i is the output fuzzy set, which is a singleton in this case.

The membership functions are in the form of Equation 5, and the closed form of the system is given by Equation 2 for $n=1$. The describing function of a single-input single-output 1-way fuzzy system is then given as:

$$\begin{aligned} N(A,w) = N(A) = \frac{b_1}{A} = \frac{4}{\pi A}\sum_{i=0}^{d}\{\frac{\Delta u_i A}{2\Delta\phi_i} \\ (\delta_{i+1}-\sin\delta_{i+1}\cos\delta_{i+1}-\delta_i+\sin\delta_i\cos\delta_i) \\ +\frac{1}{\Delta\phi_i}(\phi_i u_{i+1}-\phi_{i+1}u_i)(\cos\delta_{i+1}-\cos\delta_i)\} \end{aligned} \tag{11}$$

where d satisfies $\phi_d \le A < \phi_{d+1}$, $d>0$, and varies with A; $\{\delta_i\}$ are defined to be the angles where the input sinusoid $x = A\sin\delta$ intersects the centers $\{\phi_i\}$'s of membership functions. For $\{\delta_i\}$'s, we have:

$$\begin{aligned} &\delta_0 \equiv 0 \\ &\delta_i \equiv \sin^{-1}(\frac{\phi_i}{A}), (i=1,\ldots,d, 0<\delta_i<\frac{\pi}{2}) \\ &\delta_{d+1} \equiv \frac{\pi}{2} \end{aligned} \tag{12}$$

The describing function of a 2-way fuzzy adaptive system has two components: the describing function of the system with membership functions, and the describing function of the system with 1-nonmemberships. The one for the system with membership functions is the same as that of a 1-way fuzzy system. We derive the expression for the system with 1-nonmemberships, together with adaptation of the assumptions in the 1-way fuzzy system case.

The closed form of the fuzzy system is the same as 1-way fuzzy system apart from the definition of membership functions. In this case, we have $1-v(x)$, and the closed form of the system becomes:

$$u = f(x) = \sum_l \Omega_l(x)u_l \tag{13}$$

where

$$\Omega_l(x) = \frac{1 - \upsilon_l(x)}{\sum_{k=1}^{ruleno} 1 - \upsilon_k(x)} \quad (14)$$

The assumptions, lemmas and the proofs of the lemmas for the case with 1-nonmemberships can be found in (Gürkan, 2003). The describing function of the fuzzy system with nonmembership functions satisfying the assumptions, and lemmas given in (Gürkan, 2003) is given in Theorem 1, the proof of which is also in (Gürkan, 2003).

Theorem 1: The describing function of the fuzzy system given by Equation 13 that satisfies the four assumptions is a real number, which depends only on the amplitude A of the input sinusoid, and is in the following form:

$$\begin{aligned} &\bar{N}(A,w) = \bar{N}(A) = \frac{b_1}{A} \\ &= \frac{4}{\pi A}\sum_{i=0}^{d}((u_i + u_{i+1})(\cos\delta_i - \cos\delta_{i+1})) - \\ &\{\frac{\Delta u_i A}{2\Delta\alpha_i}((\delta_{i+1} - \sin\delta_{i+1}\cos\delta_{i+1}) - \\ &(\delta_i - \sin\delta_i\cos\delta_i)) \\ &+\frac{1}{\Delta\alpha_i}(\alpha_i u_{i+1} - \alpha_{i+1}u_i)(\cos\delta_{i+1} - \cos\delta_i)\} \end{aligned} \quad (15)$$

where α_i's are the centers of the triangular nonmembership functions. The rest of the definitions are the same as for $N(A)$ with ϕ_i's replaced with α_i's.

The describing function of the 2-way fuzzy adaptive system is given by $\{N(A), \bar{N}(A)\}$.

3. STABILITY ANALYSIS

We use describing function method for the stability analysis of our 2-way fuzzy adaptive system. In this method, the describing function of the fuzzy system is considered in cascade with a linear plant with transfer function $G(s) = \frac{n(s)}{d(s)}$, which has a low-pass property.

The characteristic equation of the feedback system with the fuzzy controller replaced by the describing function $\{N(A), \bar{N}(A)\}$ that is in cascade with the linear plant $G(s)$ is: $C(s) = 1 + \{N(A), \bar{N}(A)\}G(s)$, so we have:

$$\begin{aligned} C_1(s) &= 1 + N(A)G(s) = d(s) + N(A)n(s) \\ C_2(s) &= 1 + \bar{N}(A)G(s) = d(s) + \bar{N}(A)n(s) \end{aligned} \quad (16)$$

The C_1 and C_2 in the above equation are both interval polynomials, since $N(A)$ and $\bar{N}(A)$ are real and interval-valued that depend on A. For the stability analysis of these interval polynomials, we use Kharitonov's theorem for real polynomials (Bhattacharyya *et al.*, 1995).

Theorem 2: Let $I(s)$ be the set of real polynomials of degree n of the form $\delta(s) = \delta_0 + \delta_1 s + \delta_2 s^2 + \delta_3 s^3 + \ldots + \delta_n s^n$, where the coefficients lie within given ranges, $\delta_0 \in [x_0, y_0]$, $\delta_1 \in [x_1, y_1]$,..., $\delta_n \in [x_n, y_n]$.

Every polynomial in the family $I(s)$ is Hurwitz if and only if the following four extreme polynomials are Hurwitz:

$$\begin{aligned} K_1(s) &= x_0 + x_1 s + y_2 s^2 + y_3 s^3 + x_4 s^4 + \ldots \\ K_2(s) &= x_0 + y_1 s + y_2 s^2 + x_3 s^3 + x_4 s^4 + \ldots \\ K_3(s) &= y_0 + x_1 s + x_2 s^2 + y_3 s^3 + y_4 s^4 + \ldots \\ K_4(s) &= y_0 + y_1 s + x_2 s^2 + x_3 s^3 + y_4 s^4 + \ldots \end{aligned}$$

The proof of the theorem can be found in (Bhattacharyya *et al.*, 1995).

For our system in Equation 16, we need to check the Kharitonov polynomials for each characteristic equation C_1 and C_2, with $N(A) \in [N_{min}, N_{max}]$ and $\bar{N}(A) \in [\bar{N}_{min}, \bar{N}_{max}]$. If both polynomials are found to be Hurwitz, then we conclude that our system is stable.

4. APPLICATION EXAMPLE

In this section, we design a fuzzy controller given by Equation 2 for a flexible-joint robot arm system defined by the state equations:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\tfrac{mgl}{I_1}\sin(x_1) + \tfrac{k}{I_1}(x_3 - x_1) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \tfrac{k}{I_2}(x_1 - x_3) + \tfrac{u}{I_2} \end{aligned} \quad (17)$$

where $x_1 = \theta_1$ (joint1 angular position), $x_2 = \dot{\theta}_1$, $x_3 = \theta_2$ (joint2 angular position) and $x_4 = \dot{\theta}_2$, whereas u is the torque input, I_1 the link inertia, I_2 the motor inertia, m the mass, g the gravity constant, l the link length, k the stiffness.

We use the input-state linearization of the system introduced in (Slotine and Li, 1991) to be able to apply the stability analysis derived in this paper. The linear state equations are as follows:

$$\dot{z}_1 = z_2 \quad \dot{z}_2 = z_3 \quad \dot{z}_3 = z_4 \quad \dot{z}_4 = v \quad (18)$$

with the corresponding input transformation:

$$u = \frac{I_1 I_2}{k}(v - a(x)) \quad (19)$$

where $a(x) = \frac{mgl}{I_1}\sin x_1(x_2^2 + \frac{mgl}{I_1}\cos x_1 + \frac{k}{I_1}) + \frac{k}{I_1}(x_1 - x_3)(\frac{k}{I_1} + \frac{k}{I_2} + \frac{mgl}{I_1}\cos x_1)$.

The transfer function of this linearized system is $G(s) = \frac{1}{s^4}$. The degree of this system is $n = 4$, so there are four inputs to our fuzzy controller. In order to be able to find an analytical expression for the describing function of our fuzzy controller, we use additivity property reviewed in Section 2.2. We assume that the system parameters y_{ijkl} in Equation 2 are assigned such that the fuzzy system is additively decomposable, so the output of the fuzzy controller is:

$$\begin{aligned} v &= f(z) = f(z_1, z_2, z_3, z_4) = \\ &= f_1(z_1) + f_2(z_2) + f_3(z_3) + f_4(z_4) \end{aligned} \tag{20}$$

If the describing functions of $f(z)$ are $\{N(A), \bar{N}(A)\}$, then under the light of Equation 20, the describing functions become $N(A) = N_1(A) + N_2(A)s + N_3(A)s^2 + N_4(A)s^3$, and $\bar{N}(A) = \bar{N}_1(A) + \bar{N}_2(A)s + \bar{N}_3(A)s^2 + \bar{N}_4(A)s^3$, where $\{N_1(A), \bar{N}_1(A)\}$ are the describing functions of the system with input z_1, calculated using Equations 11 and 15, and so on. The reason for having terms like $N_2(A)s$ is that for example, the input to $N_2(A)$ is $z_2 = \dot{z}_1 = sz_1$, so the effect of $N_2(A)$ in $N(A)$ is $N_2(A)s$. It is straightforward to derive the rest of the terms in both $N(A)$ and $\bar{N}(A)$.

The characteristic equation of the closed loop system is calculated as:

$$C_1(s) = s^4 + N_4 s^3 + N_3 s^2 + N_2 s + N_1 \tag{21}$$

for the fuzzy system with membership functions and

$$C_2(s) = s^4 + \bar{N}_4 s^3 + \bar{N}_3 s^2 + \bar{N}_2 s + \bar{N}_1 \tag{22}$$

for the fuzzy system with nonmembership functions. N_1 stands for $N_1(A)$ and so on. We have dropped (A) for a simpler representation.

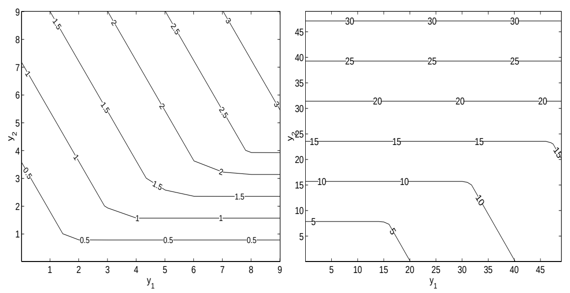
The ranges for N_i's are: $N_1(A) \in [a_1, b_1]$, $N_2(A) \in [a_2, b_2]$, $N_3(A) \in [a_3, b_3]$, and $N_4(A) \in [a_4, b_4]$, and for $\bar{N}_i$'s: $\bar{N}_1(A) \in [c_1, d_1]$, $\bar{N}_2(A) \in [c_2, d_2]$, $\bar{N}_3(A) \in [c_3, d_3]$, and $\bar{N}_4(A) \in [c_4, d_4]$. For stability, we check the Kharitonov polynomials, and since our system is of degree 4, we only need to check K_3 and K_4 of Theorem 2 (Bhattacharyya *et al.*, 1995). For $C_1(s)$:

$$\begin{aligned} K_3(s) &= b_1 + a_2 s + a_3 s^2 + b_4 s^3 + s^4 \\ K_4(s) &= b_1 + b_2 s + a_3 s^2 + a_4 s^3 + s^4 \end{aligned} \tag{23}$$

and for $C_2(s)$:

$$\begin{aligned} K_3(s) &= d_1 + c_2 s + c_3 s^2 + d_4 s^3 + s^4 \\ K_4(s) &= d_1 + d_2 s + c_3 s^2 + c_4 s^3 + s^4 \end{aligned} \tag{24}$$

Since there are too many parameters to be adjusted, we fix the ranges for the first three describing functions and we only solve for the range of $N_4(A)$ and $\bar{N}_4(A)$, i.e. we solve for a_4, b_4, c_4 and d_4.



(a) (b)

Fig. 1. Contour Plots for (a) Minimum N_4, (b) Maximum N_4

We take five rules for each system $f_1(z_1)$, $f_2(z_2)$, $f_3(z_3)$ and $f_4(z_4)$, and the parameters for the first three systems are given in Table 1.

Table 1. Fuzzy Controller Parameters

For f_1	$\phi_{-2} = -\pi$ $\phi_{-1} = -0.5\pi$ $\phi_0 = 0$ $\phi_1 = 0.5\pi$ $\phi_2 = \pi$	$y_{-2} = -5$ $y_{-1} = -1$ $y_0 = 0$ $y_1 = 1$ $y_2 = 5$
For f_2	$\phi_{-2} = -\pi$ $\phi_{-1} = -0.5\pi$ $\phi_0 = 0$ $\phi_1 = 0.5\pi$ $\phi_2 = \pi$	$y_{-2} = -10$ $y_{-1} = -8$ $y_0 = 0$ $y_1 = 8$ $y_2 = 10$
For f_3	$\phi_{-2} = -\pi$ $\phi_{-1} = -0.5\pi$ $\phi_0 = 0$ $\phi_1 = 0.5\pi$ $\phi_2 = \pi$	$y_{-2} = -20$ $y_{-1} = -25$ $y_0 = 0$ $y_1 = 25$ $y_2 = 20$

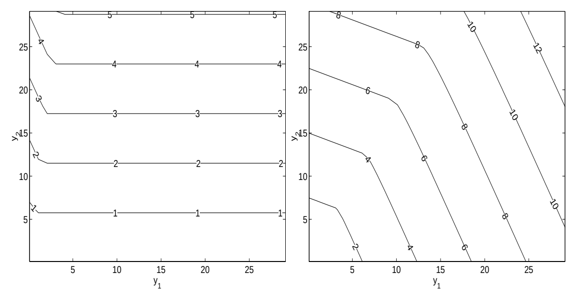
For the parameters in Table 1, the corresponding ranges for N_i's are: $N_1 \in [0.6366, 1.3831]$, $N_2 \in [3.6, 5.0930]$, and $N_3 \in [8.4506, 15.9155]$, and for $\bar{N}_i$'s: $\bar{N}_1 \in [0.1739, 1.7608]$, $\bar{N}_2 \in [1.3916, 4.3835]$ and $\bar{N}_3 \in [4.3487, 10.0923]$.

We use these ranges in Equations 23 and 24 to solve for the ranges of N_4 and $\bar{N}_4$, which are found to be $N_4 \in (0.6148, 21.5611)$ and $\bar{N}_4 \in (1.1249, 3.0798)$. In order to have a stable controller, we need to assign the parameters for $f_4(z_4)$ such that its describing functions $\{N_4, \bar{N}_4\}$ fall in the calculated ranges for stability.

We take the same ϕ_i's for f_4 as in the other f_i's. Then, we find the range of y_i's so that the controller is stable. The contour plots for minimum and maximum of N_4 are shown in Fig.1 (a) and (b) respectively.

The contour plots for minimum and maximum of $\bar{N}_4$ are given in Fig.2 (a) and (b) respectively.

From the plots, we see that if we choose $y_1 = y_2 = 1$, the system is unstable, since N_4 and $\bar{N}_4$ are out of the stability range. When we apply the controller with these settings for f_4 and with the settings in Table 1, the result is as expected and it is represented in Fig.3. Since we have chosen the parameters of the system outside the



(a) (b)

Fig. 2. Contour Plots for (a) Minimum $\bar{N}_4$, (b) Maximum $\bar{N}_4$

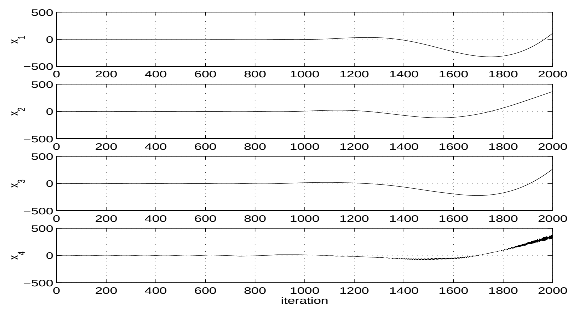


Fig. 3. Unstable System States

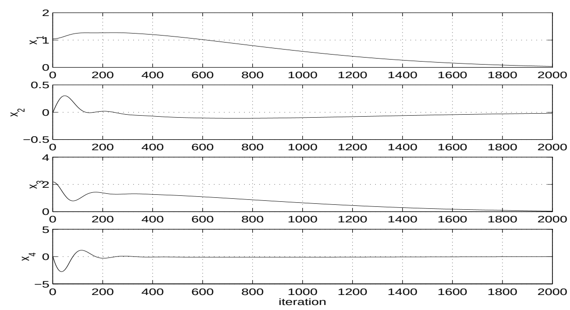


Fig. 4. Stable System States

stable region, the states of the closed loop system become unstable (Fig.3). If we choose $y_1 = 5$ and $y_2 = 8$ such that $N_4 \in (0.6148, 21.5611)$ and $\bar{N}_4 \in (1.1249, 3.0798)$ from Figures 1 and 2, wee see that N_4 and $\bar{N}_4$ are both in the stability range and the stable system states are shown in Fig.4. As can be seen from the figure, the closed loop system states are stable. This agrees with the theoretical results stating that if the parameters are chosen from the stability range for N_4 and $\bar{N}_4$, the closed loop system becomes stable.

5. CONCLUSION

We have extended the theory of describing function of a fuzzy system to multi-input single-output 2-way fuzzy adaptive systems using additivity property, and developed a systematic method for the design of a stable 2-way fuzzy adaptive controller. We have applied the theoretical results to the control of a flexible-joint robot arm system, where the simulation results are found to agree with the theory.

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