MODIFIED PQ METHOD FOR ROBUSTNESS TO MICROACTUATOR SATURATION AND FAILURE

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Abstract: Two concerns for the deployment of two stage actuators in disk drives are microactuator saturation and data recovery in the event of outright microactuator failure. Compensators designed for good performance with two intact actuators may be unstable when the microactuator saturates or stops working entirely. This paper develops a modification of the PQ Method to guarantee stable operation in the event of microactuator failure. Application of the Circle Criterion shows that the resulting design can be robust to actuator saturation. A controller design for a simplified but realistic model of a disk drive two stage actuator system illustrates the approach.

Keywords: Disk memory, fault tolerance, Nyquist diagram, robustness, saturation

1.INTRODUCTION

Two stage (or dual stage) actuators systems for head positioning in disk drives have been the subject of extensive research, and a number of techniques for the design of compensators have been proposed. Decoupled design [Li et al., 2001] and the PQ Method [Schroeck et al., 2001] (herein denoted the Standard PQ Method) are two techniques that exploit the structure of two stage systems. The H_∞ [Semba, 2000] and μ-synthesis [Young et al., 2003] techniques are two multi-input/multi-output methods that can explicitly account for the uncertainties in actuator dynamics, but are less intuitive to use.

Two concerns for the deployment of two stage systems are the susceptibility of the microactuator to saturation and data recovery in the event of outright microactuator failure. Neither the Decoupled Design nor the Standard PQ Method guarantees stability of the system when the microactuator saturates or fails. H_∞ and μ-synthesis can account for actuator failure by adjusting the weighting functions representing uncertainties in the plant. However, the resulting controllers may be too conservative to realize the full performance benefits of the two stage system.

This paper presents a modification of the PQ Method to guarantee stability of the closed loop system in the event of failure of the microactuator. The modified method requires the simultaneous stabilization of two plants by a single compensator. The paper also shows the resulting controller can be robust to actuator saturation.

This paper is organized as follows. Section II presents the derivation of the Modified PQ Method. Section III demonstrates the application of the new technique to achieve specifications for the intact and compromised system based on a simplified but realistic two stage actuator model. Section IV shows that the resulting controller is robust to actuator saturation, and shows how to relax the design requirements, if robustness to saturation, rather than robustness to total failure, is the goal of the design. Section V contains concluding remarks.

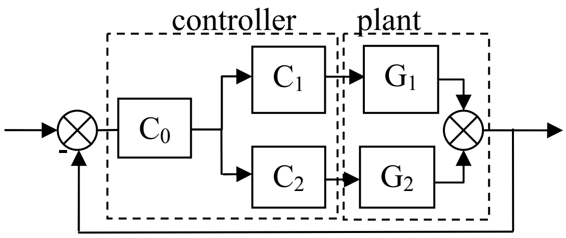

Figure 1 – DISO system block diagram.

2. DERIVATION OF THE MODIFIED PQ METHOD

All linear time-invariant feedback systems consisting of a dual-input single-output system (DISO) with a single-input dual-output (SIDO) compensator have Figure 1 as one block diagram representation. In this paper G_1 represents the microactuator subsystem, and G_2 represents the voice coil motor (VCM) subsystem.

The Standard PQ Method has two parts. The first part selects C_1 and C_2 to allocate the outputs from the G_1 and G_2 as a function of frequency such that (i) the zeros of the parallel subsystem of Figure 1 are stable and (ii) there is minimal destructive interference between the G_1 and G_2. The second part then treats the selection of C_0 as a standard SISO design problem. However, there is no guarantee that the closed loop will remain stable, if the microactuator saturates or fails.

Both the Standard and the Modified PQ Method begin by forming the ratios

$$P = \frac{G_2}{G_1} \text{ and } Q = \frac{C_2}{C_1}. \tag{1}$$

The zeros of the parallel system of Figure 1 are the roots of

$$1 + \frac{G_2 C_2}{G_1 C_1} = 1 + PQ = 0. \tag{2}$$

Equation 2 is the characteristic equation of the PQ feedback system depicted in Figure 2.

Choosing Q to stabilize the PQ feedback system ensures that the zeros of the parallel system of Figure 1 will be stable. The 0 dB crossover of PQ is the frequency at which the outputs of the G_1 and G_2 subsystems have equal magnitude. The phase margin

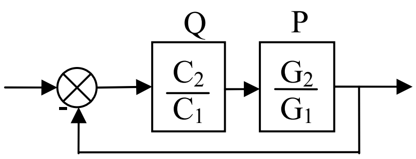

Figure 2 – PQ feedback system

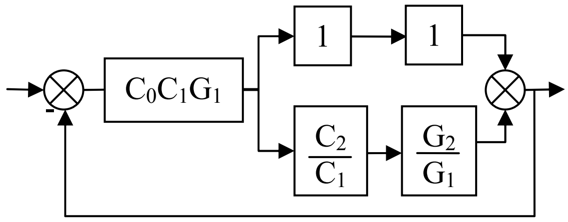

Figure 3 – Rearranged DISO feedback system.

of the PQ feedback system determines the relative phase of the outputs and thus how much the two subsystems destructively interfere.

The Modified PQ Method and the Standard PQ Method are identical through the selection of Q. The difference is that the second part of new method combines the selection of C_0 and C_1. The combined selection relies on the block diagram of Figure 3 which is a rearrangement of a Figure 1.

Substituting the definitions of P and Q and defining

$$K_1 = C_0 C_1 \tag{3}$$

leads to Figure 4, which depicts the system when the microactuator and the VCM are both working. Figure 5 shows the block diagram when the microactuator fails, and the upper branch of the parallel system disappears. Note that G_1 represents the dynamics of the *intact* microactuator, and therefore does not change when the microactuator fails.

The second part of the modified PQ Method is the selection of K_1 so that the feedback system of Figure 4 and the feedback system of Figure 5 are simultaneously stabilized. The selection of K_1 must also ensure that the feedback system of Figure 4 meets the performance specifications that required the microactuator in the first place. However, it is unclear a priori how much compromise in the two stage performance will be required to assure robustness to microactuator failure.

Since one may assume that $C_1 = 1$ without loss of generality, one also can assume $C_0 = K_1$. In that case, both K_1 and $C_2 = Q$ must be realizable.

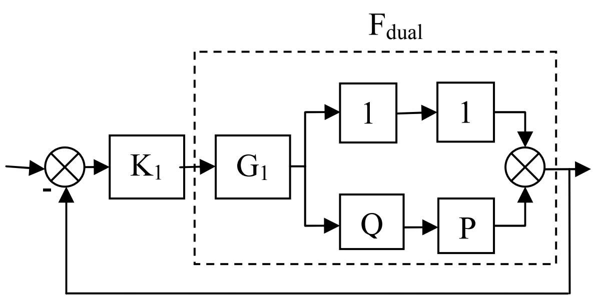

Figure 4 – DISO Feedback system with P, Q, and K_1 substitutions.

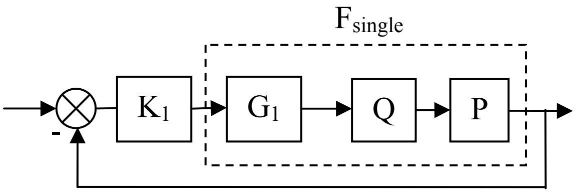

Figure 5. Block diagram of system when the microactuator fails.

3. ILLUSTRATION OF THE MODIFIED PQ METHOD

3.1. Plant Models and Servo Specifictions.

Figure 6 shows the frequency responses of two actuator subsystems models in discrete time with a zero order hold operating at a 30 kHz sampling rate. The specifications for the intact two stage system are

0 dB crossover	> 2,500 Hz
Phase margin	> 45 degrees
Gain margin	> 8 dB
Hand-off frequency	167 Hz
Hand-off phase margin	> 60 degrees
Zero steady-state error for a constant disturbance	

The performance objective in the event of micro actuator failure is to maintain at least 40 degrees of phase margin while maximizing the 0 dB crossover frequency.

3.2. Allocating the Actuator Effort: Selection of Q

The first step is to allocate the actuator effort by forming the ratio P and selecting Q. Figure 7 shows the frequency response of P. Since zero steady state error is desired, the final controller must contain an integrator. If Q contains an integrator, then C_2 will also contain an integrator (if it is not cancelled by K_1). A discrete-time PID compensator is a reasonable choice for Q. The following Q achieves the hand-off

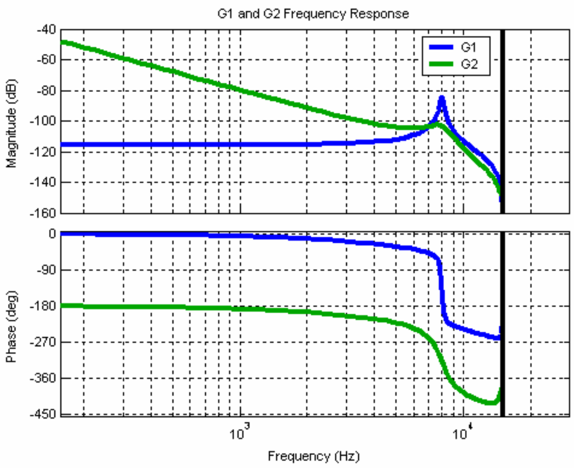

Figure 6. Frequency responses of the actuator subsystems.

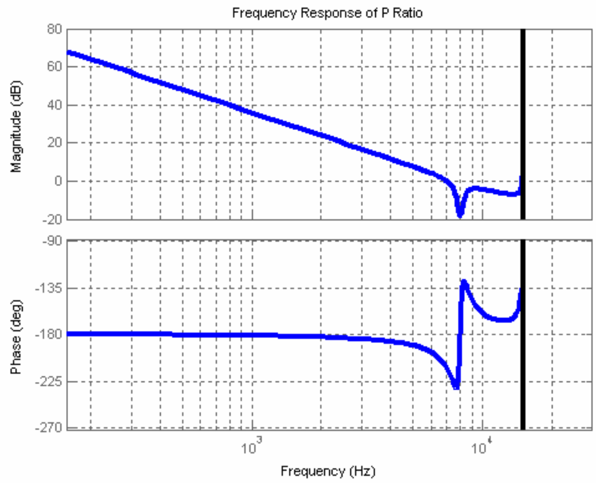

Figure 7 – Frequency response of $P = G_2 / G_1$.

specifications:

$$Q(z) = 0.025259\,\frac{(z^2 - 1.995z + 0.9953)}{(z+1)(z-1)} \quad . \qquad (4)$$

Figure 8 shows the frequency response of PQ. The hand-off phase margin is 83 degrees at 167 Hz.

3.3. Selection of K_1

The second step is to select K_1 to achieve the performance specifications of subsection 3.1. When the microactuator is intact, the parallel system inside the dashed box in Figure 4 represents the dynamics compensated by K_1. The expression for these intact dynamics is

$$F_{dual} = G_1(1 + PQ) \, . \qquad (5)$$

When the microactuator fails, the system inside the dashed box in Figure 5 represents the dynamics compensated by K_1. Equation 6 is the expression for these compromised dynamics.

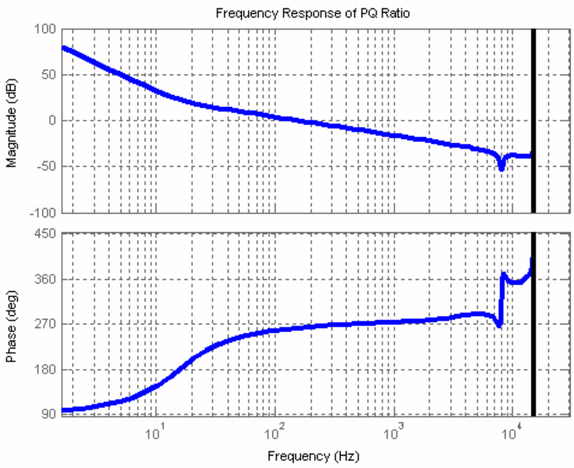

Figure 8. Frequency response of $PQ = G_2C_2 / G_1C_1$.

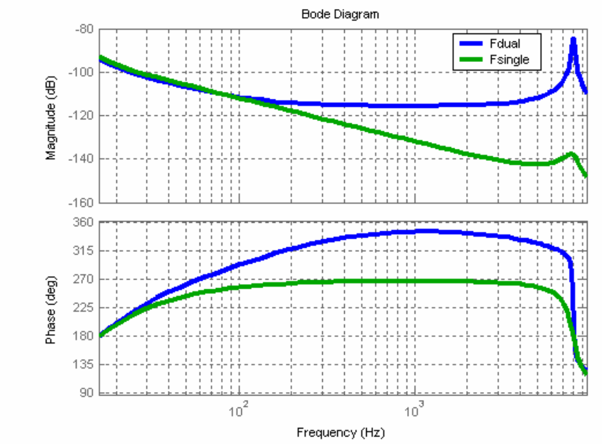

Figure 9 – Frequency response of F_{dual} and F_{single}.

$$F_{single} = G_1 PQ \cdot \quad (6)$$

Figure 9 shows the frequency responses of F_{dual} and F_{single}.

It is instructive to examine the implications of the shape of the magnitude plots of F_{dual} and F_{single} on the potential performance when the microactuator fails. At 2500 Hz, the intended crossover frequency when the microactuator is intact, the slope of F_{dual} is essentially zero, while the slope of F_{single} is approximately -20 dB/dec. The difference in magnitude between F_{dual} and F_{single} is about 23 dB. The slope of K_1 will add to the slope of both open loop responses.

Using these facts, an estimate of the 0 dB crossover frequency when the microactuator fails is

$$f_{0db\_single} \approx 2500 \times 10^{25/(-20+\mathrm{slope}\,K_1)}. \quad (7)$$

If K_1 has a slope of -20 dB/dec at 2,500 Hz, then the approximation is $f_{0dB\_single} \approx 665$ Hz. However, when the slope of K_1 is -20 dB/dec, its phase will be ~-90 degrees, resulting in an unacceptable phase margin for $F_{single}K_1$. Therefore f_{0dB_single} will be less than 665 Hz, because selecting K_1 with phase greater than -90 degrees results in a shallower magnitude slope.

The selection of K_1 begins with a 40 dB notch for the resonance. Deliberate mistuning by 200 Hz shows the robustness to small resonance shifts.

The slope at the 0 dB crossover of $F_{dual}K_1$ should be at least 20 dB/dec. There is no need for K_1 to include an integrator, since Q already contains one. A first-order low pass filter with corner frequency 167 Hz will make the slope of $F_{dual}K_1$ be -20 dB/dec, and the slope of $F_{single}K_1$ be -40 dB/dec above 167 Hz. Figure 10 shows the result of applying the notch and the low pass. The phase margin of $F_{dual}K_1$ is approximately 48 degrees at 2.5 kHz, but the phase margin of $F_{single}K_1$ is 0.5 degrees at 650 Hz.

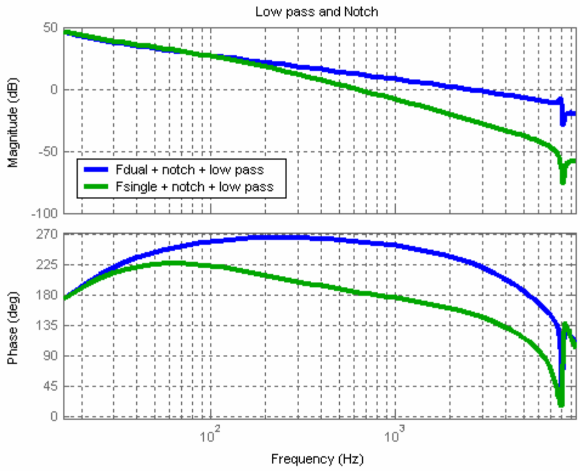

Figure 10 – F_{dual} and F_{single} with notch and low pass compensation.

Lead compensation is necessary, but it will flatten the magnitude responses and decrease the 0 dB frequency of $F_{single}K_1$. Adding 35 degrees of phase at 525 Hz by appying a complex lead compensator [Messner, 2000] with a 0.7 damping ratio results in the response shown in Figure 11. The phase margin of $F_{single}K_1$ is 40 degrees at 511 Hz. The phase margin for $F_{dual}K_1$ is 56 degrees at 2.5 kHz, and its gain margin is 8.1 dB at 5,500 Hz.

The final form of K_1 is

$$K_1 = 82285\left(\frac{z+1}{z-0.9672}\right)\left(\frac{z^2-1.876z+0.8836}{(z+0.0734)^2}\right)\left(\frac{z^2\,\_\,2893z+0.9804}{z^2-1.809z+0.8257}\right) \quad (8)$$

The three factors are the low pass filter, the notch filter, and the complex lead compensator, respectively.

The trade off between performance and robustness to microactuator failure is now apparent. If compensation of F_{single} were of no concern, then K_1

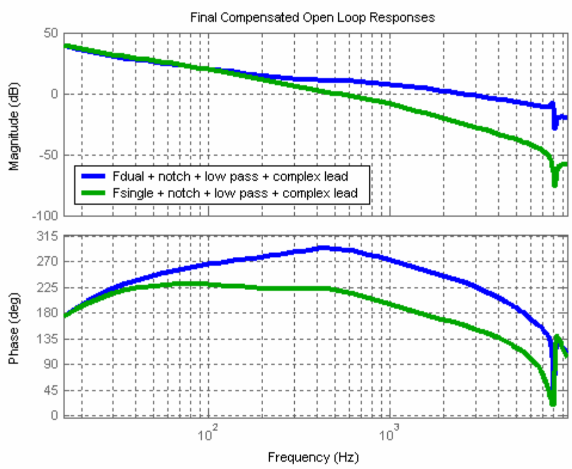

Figure 11. Bode plots of $F_{dual}K_1$ and $F_{single}K_1$.

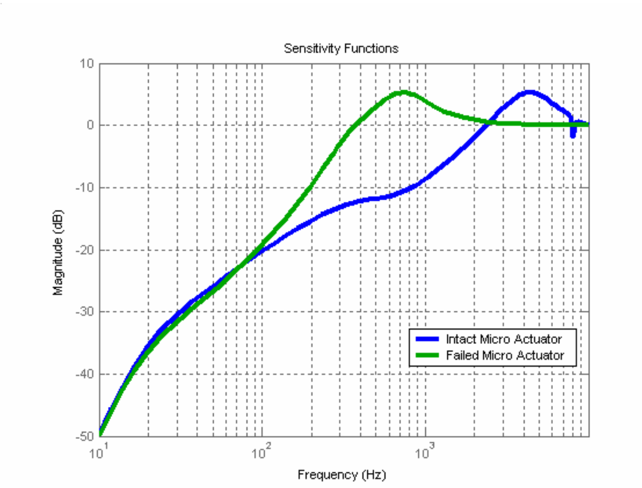

Figure 12. Sensitivity functions for intact and failed microactuator systems.

could have a steeper magnitude slope below 2.5 kHz. The magnitude of $F_{dual}K_1$, which could have been 20 dB at 250 Hz, is only 13 dB. Figure 12 shows the Bode magnitude plots of the sensitivity functions. The disturbance rejection below 100 Hz is about 8 dB less than would have been possible without the need to compensate for the F_{single}. Of course, if a lower $F_{single}K_1$ phase margin were acceptable, then a steeper K_1 slope would be possible also.

4. CONFIRMING STABILITY IN THE PRESENCE OF SATURATION

Microactuator failure is an extreme form of saturation where the saturation limit is zero. This fact suggests that the the controller designed for total actuator failure will be robust to actuator saturation. Checking for such robustness requires some block diagram algebra and the Circle Criterion [Vidyasagar, 1978].

Figure 13 is the result of rearranging Figure 4, and including the saturation nonlinearity ψ, where

$$\psi(y) = \begin{cases} y & |y| \le y_{\max} \\ \operatorname{sgn}(y) y_{\max} & |y| > y_{\max} \end{cases} \qquad (9)$$

Leaving out the reference input, judicious block diagram algebra leads to Figure 14 with ψ in the feedback path and the following definition of H:

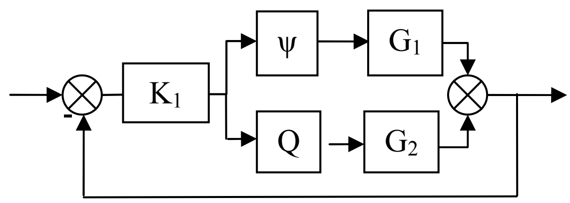

Figure 13. Two stage actuator system including saturation nonlinearity.

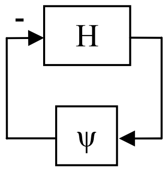

Figure 14. Block diagram with ψ the feedback path.

$$H = \frac{G_1 K_1}{1 + PQG_1 K_1}. \qquad (10)$$

One version of the Circle Criterion states that the feedback system of Figure 14 is stable if the (i) ψ belongs to the sector [0, 1], (ii) H has no unstable poles, and (iii) the Nyquist plot of H lies to the right of vertical line through -1. The nonlinearity ψ satisfies (i), because

$$\alpha y^2 \le y\psi(y) \le \beta y^2, \qquad (11)$$

where α=0, and β=1. The poles of H satisfy (ii), because they are the roots of

$$1 + PQG_1 K_1 = 0, \qquad (12)$$

which is the characteristic equation of the stable feedback system of Figure 5. Figure 15 clearly shows that Nyquist plot of H satisfies (iii). Thus, the dual stage system will be stable in the presence of microactuator saturation.

Note that the Circle Criterion only provides sufficient conditions for stability. Therefore, a controller that is robust to microactuator failure might result in an H, which does not satisfy (iii) but is still robust to microactuator saturation.

If robustness to saturation of the microactuator, rather robustness to its total failure, is the concern of the design, then there are two complementary approaches to improving the performance of the two stage system using the Modified PQ Method. First, the phase margin for $F_{single}K_1$ can be much lower. The system need only retain stability so that it can recover from saturation. It need not have good performance while in saturation, because the satur-

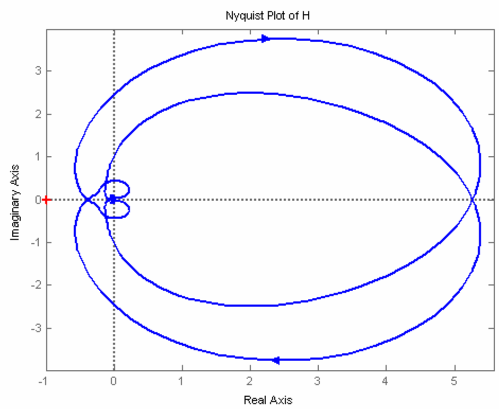

Figure 15. Nyquist plot of H.

ation is intermittent.

Figure 16 shows the improvement in the sensitivity function of the intact system when the phase margin of $F_{single}K_1$ is 15 degrees at 597 Hz. The only design change is that the complex lead compensator adds 13 degrees of phase at 609 Hz. The disturbance rejection increases by 4.8 dB below 100 Hz.

Second, rather than choosing K_1 to simultaneously stabilize F_{dual} and F_{single}, K_1 need only simultaneously stabilize of F_{dual} and F_{sat} where

$$F_{sat} = G_1(\alpha + PQ) \tag{13}$$

and

$$\alpha = \frac{y_{max}}{u_{max}} \tag{14}$$

with u_{max} being an upper bound on absolute value of the output of K_1. The "failure" mode of the microactuator is where the gain is reduced by the ratio of the maximum output to the maximum input, given by Equation 14. For this case, H becomes

$$H = \frac{G_1 K_1}{1 + (\alpha + PQ) G_1 K_1} \quad , \tag{10}$$

and the sufficient condition (iii) for stability in the presence of saturation relaxes to requiring that Nyquist plot of H remain to the right of $-1/(1-\alpha)$.

5. CONCLUSIONS

This paper derived and demonstrated the Modified PQ Method to achieve stable operation in the presence of actuator failure. The first part of the Modified PQ Method is the same as the Standard PQ Method, in which actuator effort is allocated as a function of frequency. The difference is in the second part, where the new method relies on the simultaneous stabilization of two systems with a single compensator. Thus, there is likely to be some compromise between achieving the robustness to microactuator failure and achieving the highest possible performance when the microactuator is intact.

The fact that microactuator failure is an extreme form of saturation suggests that a controller that is robust to total failure of the microactuator is likely to be robust to microactuator saturation also. However, saturation robustness of the controller resulting from the Modified PQ Method is not known a priori. Instead, the Circle Criterion provides a straightforward way to check for such robustness. Future work will attempt to eliminate the need for this check.

If robustness to saturation of the microactuator, rather than its failure, is the concern of the design,

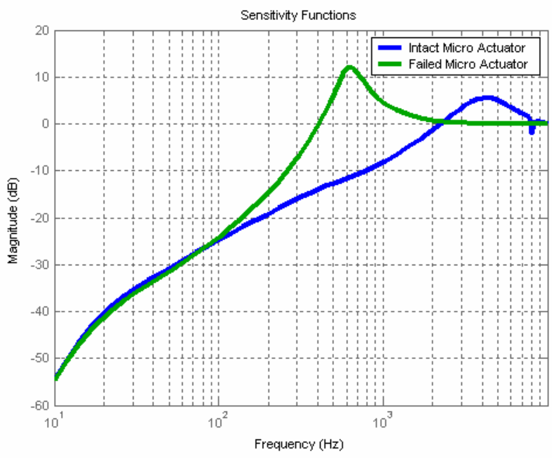

Figure 16. Sensitivity functions for lower F_{single}, K_1 phase margin.

then the phase margin of in the failure mode can be low, because only recovery from saturation is needed, rather than good performance in saturation. The simultaneous stabilization problem can also change to the stabilization of the "failure" mode where the gain at the input to the microactuator decreases to a positive value less than one, rather than to zero for complete failure. The check for stability in saturation correspondingly relaxes.

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