DESIGN OF REFERENCE GOVERNOR FOR LINEAR SYSTEMS WITH INPUT CONSTRAINTS

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Abstract: This paper presents a design method of the reference governor for a stabilizable linear system with input constraints. When the input has a constraint, the input saturation might happen and thus, the controller cannot work correctly. Furthermore, the output might diverge in the case of an open loop unstable system. To remedy this problem, the reference governor is proposed. The set-point control problem with control constraints is considered. In the proposed method, the behaviors of the states and input are predicted and the reference is modified so that the predicted input never exceeds the saturation limits and the modified one is closest to the given set-point command among the values which are achievable without the input saturation. The convergence of the newly generated reference trajectory is proved. Simulation results are included to verify the performance of the proposed method.

Keywords: reference governor, input constraint, input saturation, set-point regulation

1. INTRODUCTION

Every physical actuator is subject to saturation due to its upper and lower limits. When we apply a control scheme designed without considering this actuator saturation, the performance cannot be guaranteed or the system can become unstable because the controller cannot work as expected. A well-known example of performance degradation is an integrator windup phenomenon in a PID controller (Franklin *et al.*, 1994; Åström and Hägglund, 1995). Whenever control saturation happens, the integrator will keep integrating unnecessarily, resulting in substantial overshoot. The remedy for this is to stop the integral control law from integrating, because the input to the plant does not change at all when saturated. This is referred to as an *anti-windup* scheme. Another solution is the reference modification. By modifying the reference, we can prevent the control input from entering the saturation region. This is a *reference governor* approach. In the case of the unstable open loop plants, changing the control signal can make the closed loop system unstable because the stabilization by the controller cannot be guaranteed. The reference governor is an auxiliary system to the controller, which is already designed to stabilize the system and track the reference in the absence of the input constraints. Therefore, the reference governor approach is effective especially when the open loop plant is unstable.

Many researches have been carried out in the field of feedback control systems with input constraints. Pertinent results and relevant references are suggested in (Sussmann *et al.*, 1994). One of the research directions is anti-windup schemes (Kothare *et al.*, 1994; Hanus *et al.*, 1987; Åström and Rundqwist, 1989; Zheng *et al.*, 1994; Hu and Rangaiah, 2000). Some studies address the adaptive control method to solve an actuator saturation problem (Tao and Kokotović, 1996; Tao and F. F. Lewis, 2001; Hu and Lin, 2001; Kapila and K. M. Grigoriadis, 2002). Recently, the reference governor approach has been much studied, mostly based on the predictive control. Moving horizon optimal control (Mayne and Michalska, 1990; Yang and Polak, 1993) and model predictive control (Mosca, 1995) have proved to be appropriate for dealing with such a problem. Gilbert *et al.* (Gilbert *et al.*, 1995; Gilbert and Kolmanovsky, 1995) suggested discrete-time reference governors. Bemporad *et al.* (Bemporad *et al.*, 1997; Bemporad and Mosca, 1998; Bemporad, 1998) developed a class of discrete-time reference governors based on conceptual tools of predictive control. Also, Gilbert and Kolmanovsky proposed the reference governor for continuous-time nonlinear systems based on families of Lyapunov functions (Gilbert and Kolmanovsky, 1999), which is further extended to the method based on sublevel sets of equilibria-parameterized functions (Gilbert and Kolmanovsky, 2001).

In this paper, we pursue the approach originated by Kapasouris *et al.* (Kapasouris *et al.*, 1988; Kapasouris *et al.*, 1990) and consider a set-point control problem associated with linear systems with input constraints. The main idea is to modify the reference depending on the possibility of present and future violation of the input constraint. We simplify the structure of the reference governor without an integrator. While the approach by Kapasouris can be only applied to the case of saturation in transient state, our approach can also handle the case of saturation in steady-state. Also, we reduce the computational complexity. Using the proposed method, the stability of the closed loop system is maintained, especially for open loop unstable plant.

We first describe the linear plant and the stabilizing state feedback controller in Section 2. In Section 3, we propose the reference governor to modify the reference trajectory to maintain the closed loop stability. The performance of the proposed reference governor for such a system is proved. Moreover, we introduce the time horizon in calculating the time infinity norm to reduce the computational complexity, which is possible because all states will be settled in stable linear systems. In Section 4, Simulation results are included to demonstrate the performance. Section 5 contains conclusions.

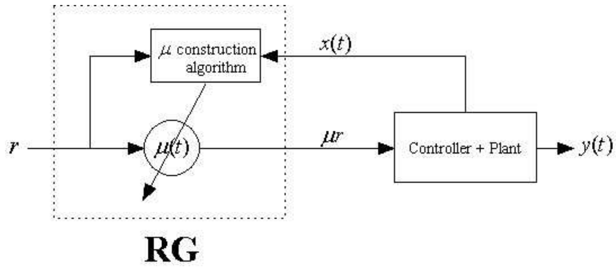


Fig. 1. The overall structure of the proposed reference governor.

2. STATEMENT OF THE PROBLEM

Consider the following single-input single-output (SISO) linear time invariant system with n-dimensional state $x \in \Re^n$,

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Cx, \end{aligned} \tag{1}$$

where $A \in \Re^{n\times n}$, $B \in \Re^{n\times 1}$, $C \in \Re^{1\times n}$. It is desired to pick a state-feedback control u so that the output y follows a given set-point command r and the overall system is stabilized. The state feedback control

$$u = Kx + r, \tag{2}$$

where $K \in \Re^{1\times n}$ is a stabilizing feedback gain vector, can stabilize the system under Assumption 1.

Assumption 1. $A + BK$ is Hurwitz.

The closed loop system including the plant and the controller can be written as:

$$\begin{aligned} \dot{x} &= (A + BK)x + Br, \\ y &= Cx. \end{aligned} \tag{3}$$

3. REFERENCE GOVERNOR DESIGN AND ANALYSIS

In this section, a design method of reference governor for a closed loop system (3) with an actuator saturation is proposed and the properties of the proposed reference governor are discussed.

3.1 Overall Structure

Assume a control input has saturation limits u_{max} and u_{min}. Define an saturation operator as

$$sat(u) = \begin{cases} u_{max} & \text{if} \quad u \geq u_{max} \\ u & \text{if} \quad u_{min} \leq u \leq u_{max} \\ u_{min} & \text{if} \quad u \leq u_{min} \end{cases}. \tag{4}$$

Because the controller is designed without considering the actuator saturation, the controller can guarantee its performance only in the unsaturated region. Therefore, it is desirable to prevent a control input from entering the saturated region.

The goal of the reference governor is to find a modified reference closest to the given reference r with keeping a control input staying in the unsaturated region and thus making the saturation never happen. The overall structure of the proposed reference governor is shown in Figure 1. A time-varying parameter μ is introduced, through which the reference modification is achieved.

3.2 Construction of Reference Governor

If we consider u as an output in the closed-loop system and also apply the proposed reference governor to this system, the overall system equation including the reference governor becomes

$$\begin{aligned} \dot{x} &= A_c x + B\mu r, \\ u &= Kx + \mu r, \end{aligned} \tag{5}$$

where $A_c = A + BK$.

The basic idea is to find μ closest to 1 such that $u(t)$ satisfies the input constraints for all $t \in [0,\ \infty)$. However, it is impossible to get $u(t)$ after the current time t^o (i.e., $t > t^o$). Hence, we have to use predicted signals instead of actual signals. Let $\hat{x}(\tau)$ and $\hat{u}(\tau)$, $\tau = t - t^o$, denote the predicted state and the predicted input signal for $t > t^o$ in the reference governor. Then, the predicted system equation in the reference governor can be written as:

$$\begin{aligned} \dot{\hat{x}}(\tau) &= A_c \hat{x}(\tau) + B\mu r, \\ \hat{u}(\tau) &= K\hat{x}(\tau) + \mu r. \end{aligned} \tag{6}$$

The purpose of the reference governor is to pick μ closest to 1 so that the predicted control input $\hat{u}$ never saturates for given r in (6). If we denote a saturation limit as L with $u_{max} = -u_{min} = L$, the avoidance of the input saturation and convergence of the output to the value closest to the given r among the values which are achievable without the input saturation is shown in the following theorem under the Assumption 2.

Assumption 2. Initial state $x(0)$ satisfies the input constraints, i.e, there exists μ such that

$$|Ke^{A_c\tau}x(0) + K\left(e^{A_c\tau} - I\right)A_c^{-1}B\mu r + \mu r| \le L, \quad \forall \tau \in [0,\ \infty), \tag{7}$$

and is denoted by $\mu(0)$.

Theorem 3. If we choose the largest $\mu \in [\mu(0),\ 1]$ which satisfies (8) when $\mu(0) \le 1$ or the smallest $\mu \in [1,\ \mu(0)]$ which satisfies (8) when $\mu(0) \ge 1$, for $t \ge t^o$,

$$|Ke^{A_c\tau}x(t^o) + K\left(e^{A_c\tau} - I\right)A_c^{-1}B\mu r + \mu r| \le L, \quad \forall \tau \in [0,\ \infty), \tag{8}$$

the actual input can satisfy the given input constraints for $t \ge t^o$, i.e.,

$$|u(t)| \le L, \quad \forall t \in [t^o,\ \infty), \tag{9}$$

and μ converges to some constant μ^*. Then, $\mu^* r$ is the closest value to r among the values which are achievable within the given saturation limits. If the given reference r is attainable without violating the input constraints, $\mu^* = 1$.

Proof: The solution of (6) with $\hat{x}(0) = x(t^o)$ can be obtained as follows:

$$\begin{aligned} \hat{u}(\tau) &= Ke^{A_c\tau}\hat{x}(0) + K\int_0^\tau e^{A_c(\tau-\xi)}B\mu r d\xi + \mu r \\ &= Ke^{A_c\tau}\hat{x}(0) + Ke^{A_c\tau}\int_0^\tau e^{-A_c\xi}d\xi B\mu r + \mu r \\ &= Ke^{A_c\tau}\hat{x}(0) + K\left(e^{A_c\tau} - I\right)A_c^{-1}B\mu r + \mu r \\ &= Ke^{A_c\tau}x(t^o) + K\left(e^{A_c\tau} - I\right)A_c^{-1}B\mu r + \mu r. \end{aligned}$$

Then, (8) implies

$$|\hat{u}(\tau)| \le L, \quad \forall \tau \in [0,\ \infty). \tag{10}$$

If we substitute τ with $t - t^o$, then (10) becomes

$$|\hat{u}(t - t^o)| \le L, \quad \forall t \in [t^o,\ \infty). \tag{11}$$

Also, the solution of (5) on $t \ge t^o$ is:

$$\begin{aligned} u(t) &= Ke^{A_c(t-t^o)}x(t^o) \\ &\quad + K\left(e^{A_c(t-t^o)} - I\right)A_c^{-1}B\mu r + \mu r \qquad (12) \\ &= \hat{u}(t - t^o). \qquad (13) \end{aligned}$$

Therefore,

$$|u(t)| \le L, \quad \forall t \in [t^o,\ \infty). \tag{14}$$

Denote μ which satisfies (8) as admissible μ. From the Assumption 2, there exists admissible $\mu(0)$. Assume $\mu(0)$ is less than 1. If there is no admissible $\mu(t)$ larger than $\mu(0)$ for all $t \in [0,\ \infty)$, $\mu^* = \mu(0)$. Since only the $\mu(t)$ which is larger than μ^* is meaningful, if there is an admissible $\mu(t)$ which is larger than μ^*, $\mu^* = \mu(t)$. Also, the output will reach $\mu^* r$. Therefore, μ^* will increase until there is no admissible $\mu(t)$ which is larger than μ^*. Therefore, μ^* is monotone increasing and it is also bounded, thus it converges. If there exists an admissible $\mu(t) \geq 1$, which means the given reference r is attainable without violating the input constraints, μ^* is set to one, i.e., $\mu^* = 1$. When $\mu(0)$ is larger than 1, the convergence of μ^* can be proved in a similar way. ■

Remark 3.1. The maximum and minimum values of $\mu(t)$ can be directly calculated from (8) using the input saturation limit values. Define $\alpha(\tau) = Ke^{A_c\tau}$ and $\beta(\tau) = K\left(e^{A_c\tau} - I\right)A_c^{-1}B + 1$. Then (8) means that

$$|\alpha(\tau)x(t^o) + \beta(\tau)\mu r| \leq L \quad \forall\tau \in [0,\ \infty).$$

Therefore,

$$\mu_{max} = \min\left\{\min_{\beta(\tau)>0} \frac{L - \alpha(\tau)x(t^o)}{\beta(\tau)r},\ \min_{\beta(\tau)<0} \frac{-L - \alpha(\tau)x(t^o)}{\beta(\tau)r}\right\},$$
$$\mu_{min} = \max\left\{\max_{\beta(\tau)>0} \frac{-L - \alpha(\tau)x(t^o)}{\beta(\tau)r},\ \max_{\beta(\tau)<0} \frac{L - \alpha(\tau)x(t^o)}{\beta(\tau)r}\right\},$$
$$\forall\tau \in [0,\ \infty).$$

Since any μ between μ_{max} and μ_{min} satisfies the input constraints, μ^* should be selected as $\min\{\mu_{max}, 1\}$.

Remark 3.2. From the practical point of view, $\hat{u}(\tau)$ cannot be calculated for all $\tau \in [0,\ \infty)$. In practice, the time interval is restricted to $[0,\ T]$. T should be large enough to get an information for the maximum and minimum values of $|u(t)|$. There is no general rule for picking the time interval T. However, we can think of a guideline in linear systems, for example, the settling time can be one candidate. Simulation results in the next subsection provide the insight about how to get the value of T.

4. SIMULATION RESULTS

The performance of the proposed reference governor is now illustrated through simulations. The set-point regulation problem for an unstable linear time invariant system is considered. The plant is given by

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -2x_1 + 3x_2 + u, \\ y &= x_1. \end{aligned} \tag{15}$$

The plant has unstable poles at $s = 1$ and $s = 2$. To stabilize this plant, the state feedback controller is designed as

$$u = -6x_2 + 2r, \tag{16}$$

where r is a desired set-point. Then, the closed loop system poles are at $s = -1$ and $s = -2$, and y will approach to r asymptotically.

An initial condition is given by $x(0) = [0\ 0]$. Figure 2 shows the regulation performance of the given state feedback controller without an actuator saturation when $r = 1$; Figure 2(a) shows reference and output trajectories and Figure 2(b) shows an input trajectory. As shown in Figure 2(b), input varies between -1 and 2. Assume the actuator works only between -0.5 and 2, i.e.,

$$sat(u) = \begin{cases} 2 & \text{if } u \geq 2 \\ -0.5 & \text{if } u \leq -0.5 \\ u & \text{if } -0.5 \leq u \leq 2 \end{cases}. \tag{17}$$

Then, saturation happens at $u = -0.5$, which causes the instability of the closed loop system since the open loop plant is unstable. Figure 3(a) and 3(b) show the response of the output and the input, respectively. To solve this problem, the proposed reference governor is applied with $T = 5$. The predicted system equation in the reference governor can be represented as follows:

$$\begin{aligned} \dot{\hat{x}}(\tau) &= A_c\hat{x}(\tau) + B\mu r, \\ \hat{u}(\tau) &= K\hat{x}(\tau) + \mu r, \end{aligned} \tag{18}$$

where

$$A_c = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \tag{19}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{20}$$

$$K = \begin{bmatrix} 0 & -6 \end{bmatrix}. \tag{21}$$

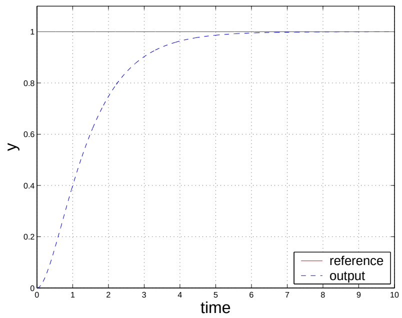
The solution of (18) can be obtained as follows:

$$\begin{aligned} \hat{u}(\tau) = &\left(12e^{-\tau} - 12e^{-2\tau}\right)\hat{x}_1(0) \\ &+ \left(6e^{-\tau} - 12e^{-2\tau}\right)\hat{x}_2(0) \\ &+ \left(-12e^{-\tau} + 12e^{-2\tau} + 2\right)\mu r. \end{aligned} \tag{22}$$

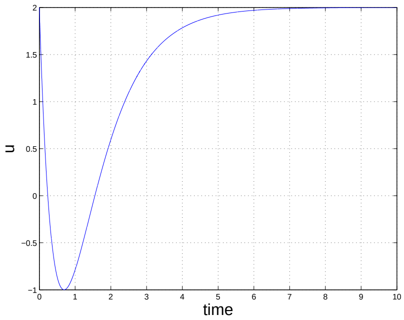
Then, μ can be calculated by checking the maximum and minimum value of $\hat{u}(\tau)$ and restricting both the maximum value is less than u_{max} and the minimum value is larger than u_{min} on $0 \leq \tau \leq 5$. Figure 4(a) depicts modified reference and output trajectories and Figure 4(b) depicts an input trajectory. The changes of parameter μ and maximum and minimum value of the input $\hat{u}$ on $\tau \in [0, 5]$ are described in Figure 4(c) and 4(d). The saturation does not happen and the stability of the system is maintained. Moreover, the output trajectory approaches to the modified set-point asymptotically. From these results, we can see that the proposed reference governor shows the satisfactory performance.

5. CONCLUSIONS

In this paper, we have proposed the design method of the reference governor for a linear plant with

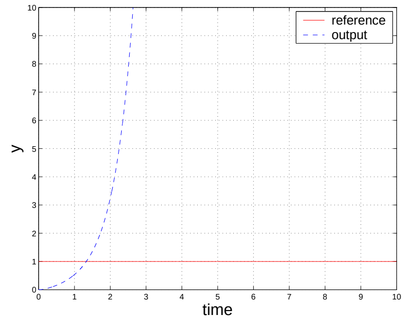


(a) The output trajectory. The solid line indicates the reference and the dashed line indicates the output.

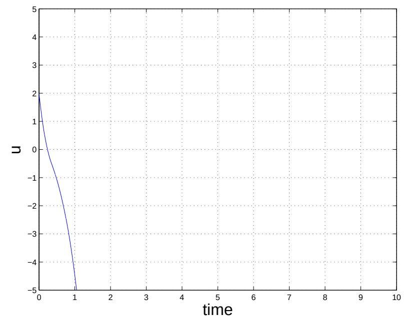


(b) The input trajectory.

Fig. 2. The regulation performance in the case of no input saturation when $x(0) = [0\ 0]$.



(a) The output trajectory. The solid line indicates the reference and the dashed line indicates the output.



(b) The input trajectory.

Fig. 3. The regulation performance in the case of the input saturation without the reference governor when $x(0) = [0\ 0]$.

saturating actuators and a stabilizing controller. Since the plant is open loop unstable, actuator saturation should be avoided to make the stabilizing controller work correctly. The contributions of the proposed reference governor can be summarized as follows:

- By introducing parameter μ, reference is modified depending on the variation of μ.
- As μ goes to 1, the reference governor generates the given reference r. Hence, the performance for unsaturated input can be recovered.
- Whenever the control input reaches the saturation bound, μ is changed such that the control input stays in the unsaturated region.
- The modified reference converges to some constant value achievable with the input which satisfies the constraints, thus avoiding the saturation phenomenon.

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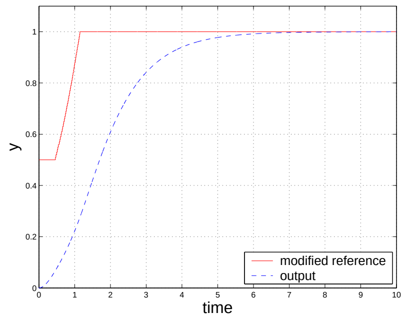
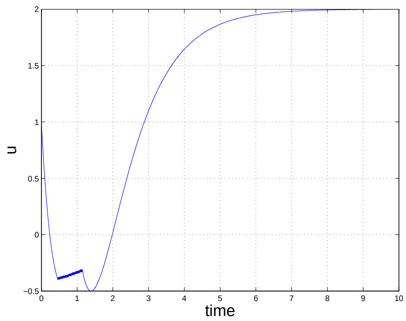
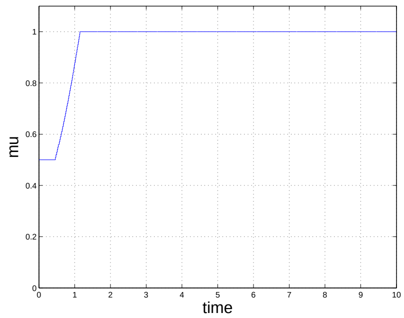


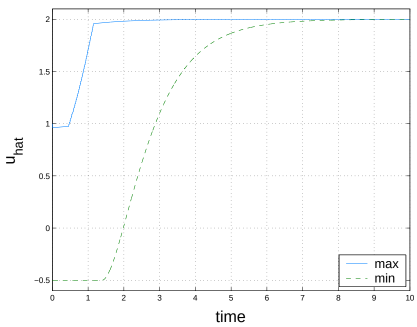
(a) The output trajectory. The solid line indicates the modified reference and the dashed line indicates the output.



(b) The input trajectory.



(c) The parameter μ trajectory in the reference governor



(d) Maximum and minimum value of $\hat{u}$ in the reference governor

Fig. 4. The regulation performance in the case of the input saturation with the reference governor when $x(0) = [0\ 0]$.

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