PMID OBSERVER DESIGN FOR UNKNOWN INPUT GENERALIZED DYNAMICAL SYSTEMS

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Abstract: A new design approach of observer is proposed for generalized dynamical systems with unknown input disturbances. By introducing proportional gain, derivative gain and multiple-integral gains, the estimation error dynamics can be guaranteed to be internally proper stable and the disturbance can be de-coupled successfully. The system state and the input disturbance can be asymptotically estimated simultaneously. The proposed observer is then extended to the bilinear generalized system case, bilinear time-delay generalized system case, and is also applied to estimate both system states and output noises for normal system case. Finally, an illustrative example is included.

Keywords: Estimation, generalized systems, multiple-integral observer, PID design, unknown input disturbances.

1. INTRODUCTION

Consider the following linear time-invariant generalized dynamical system

$$\begin{cases} E\dot{x} = Ax + Bu + Nd \\ y = Cx \end{cases} \tag{1}$$

where $x \in \mathbf{R}^n$ is the generalized state vector, $u \in \mathbf{R}^m$ and $y \in \mathbf{R}^p$ are the control input and measured output vectors, respectively, and $d \in \mathbf{R}^k$ is the unknown input disturbance vector. The matrix E may be singular, i.e., $\operatorname{rank}(E) < n$.

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For ensuring the unique solvability, the pair $(E, \quad A)$ is assumed to be regular, that is, $\det(sE - A) \neq 0$, $\exists s \in \mathbf{C}$ ($\mathbf{C}$ denotes the set of all complex numbers).

For $E = I$, the system (1) reduces to a normal system directly, which means that generalized dynamical system is a class of more general systems. Generalized dynamical system arises from a convenient and natural modeling process, and many applications can be found in economic systems, electrical circuit, chemical process and robotic systems etc (Dai, 1989). During the past several years, some results have been reported about the observer design for generalized dynamical systems with unknown input disturbances either by using proportional (P) gain (Darouach, 1996), proportional and integral (PI) gains (Busawon et al, 2001; Koenig et al, 2002), or the parameterization technique (Gao, 1999; Gao 2005). Recently, multiple-integral approach was proposed by Jiang et al (2002) for the observer design of normal systems, which can de-couple a large class of input disturbances. The multiple-integral technique and singular observer approach were synthesized in the reference (Gao et al, 2004) and a PI multiple-integral observer was proposed for singular systems with both input disturbances and output disturbances. Multiple-integral technique was also utilized to investigate robust state estimation for single-output normal systems with both measurement errors and unmodeled dynamics by Ibrir (2004).

In this paper, a new unknown input observer approach is presented. For avoiding the undesired impulsive behavior in observer design of generalized dynamical systems, derivative (D) gain is introduced. Multiple-integral technique is used simultaneously for attenuating or de-coupling a large class of disturbances. As a result, a new type of observers, that is PID multiple-integral observer or called PMID observer, is proposed in this study. The presented PMID observer is causal and the estimated error dynamics can be guaranteed to be internally proper (regular and impulse-free) and stable.

2. DESIGN PROCEDURES

Without loss of generality, the input disturbance $d(t)$ is assumed to have the following polynomial function form (Jiang et al, 2000; Gao et al, 2004):

$$d = A_0 + A_1 t + A_2 t^2 + \cdots A_{q-1} t^{q-1} \qquad (2)$$

where $A_i \ (i = 0, 1, 2, \ldots, q-1)$ are unknown constant matrices. It is clear that the q^{th} derivative of the disturbance d equals to zero, i.e., $d^{(q)} = 0$. Now we construct a PMID observer as follows:

$$\begin{cases} E\dot{\hat{x}} = A\hat{x} + Bu + L_P(y - C\hat{x}) + L_D(\dot{y} - C\dot{\hat{x}}) + N\hat{f}_q \\ \dot{\hat{f}}_q = L_I^q(y - C\hat{x}) + \hat{f}_{q-1} \\ \dot{\hat{f}}_{q-1} = L_I^{q-1}(y - C\hat{x}) + \hat{f}_{q-2} \\ \vdots \\ \dot{\hat{f}}_2 = L_I^2(y - C\hat{x}) + \hat{f}_1 \\ \dot{\hat{f}}_1 = L_I^1(y - C\hat{x}) \end{cases} \qquad (3)$$

where $\hat{x} \in \mathbf{R}^n$ is the estimated vector of the generalized state vector x ; $\hat{f}_i \in \mathbf{R}^k \ (i = 1, 2, \ldots, q)$ is an estimation of the $(q-i)^{th}$ derivative of the disturbance d ; L_P , L_D and $L_I^i \ (i = 1, 2, \ldots, q)$ are the proportional, derivative, and integral gains with appropriated dimensions, respectively.

Let $e_s = x - \hat{x}$ and $e_i = \hat{f}_i - d^{(q-i)} \quad (i = 1, 2, \ldots, q)$. Then from (1) and (3), one has

$$(\overline{E} + \overline{L}_D \overline{C})\dot{\overline{e}} = (\overline{A} - \overline{L}\,\overline{C})\overline{e} \qquad (4)$$

where

$$\overline{e} = \left[(e_s)^T, \ (e_1)^T, \ (e_2)^T, \ \cdots \ (e_q)^T\right]^T \in \mathbf{R}^{n+kq},$$

$$\overline{L} = \left[(L_P)^T, \ (L_I^1)^T, \ (L_I^2)^T, \ \cdots \ (L_I^q)^T\right]^T \in \mathbf{R}^{(n+kq)\times p},$$

$$\overline{L}_D = \left[(L_D)^T, \ 0, \ \cdots \ 0, \ 0\right]^T \in \mathbf{R}^{(n+kq)\times p},$$

$$\overline{C} = \left[C, \ 0, \ \cdots \ 0, \ 0\right] \in \mathbf{R}^{p\times(n+kq)},$$

$$\overline{E} = \begin{bmatrix} E & 0 & \cdots & 0 & 0 \\ 0 & I_k & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_k & 0 \\ 0 & 0 & \cdots & 0 & I_k \end{bmatrix} \in \mathbf{R}^{(n+kq)\times(n+kq)},$$

$$\overline{A} = \begin{bmatrix} A & 0 & \cdots & 0 & N \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & I_k & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_k & 0 \end{bmatrix} \in \mathbf{R}^{(n+kq)\times(n+kq)}, \qquad (5)$$

I_k denotes an $k \times k$ identity matrix, $(\overline{L})^T$ represents the transpose of the matrix $\overline{L}$. There exist gains $\overline{L}_D$ and $\overline{L}$ such that $(\overline{E} + \overline{L}_D \overline{C})$ is non-singular and the pair $(\overline{E} + \overline{L}_D \overline{C}, \ \overline{A} - \overline{L}\,\overline{C})$ is stable provided that the triple $(\overline{E}, \ \overline{A}, \ \overline{C})$ is completely observable in terms of the previous work by Gao (2003). Moreover, the completely observability means that

$$\mathrm{rank}\left[(\overline{E})^T, \ (\overline{C})^T\right]^T = n + kq \qquad (6)$$

and

$$\mathrm{rank}\left[(s\overline{E} - \overline{A})^T, \ (\overline{C})^T\right]^T = n + kq, \ \forall s \in \mathbf{C}. \qquad (7)$$

According to (5), the conditions (6) and (7) hold if and only if

$$\mathrm{rank}\left[(E)^T, \ (C)^T\right]^T = n, \qquad (8)$$

$$\mathrm{rank}\left[(sE - A)^T, \ (C)^T\right]^T = n, \ s \neq 0, s \in \mathbf{C}, \qquad (9)$$

and

$$\operatorname{rank}\begin{pmatrix} A & N \\ C & 0 \end{pmatrix} = n + k \,. \tag{10}$$

To summarize, we have the following theorem.

Theorem 1. If the plant (1) is completely observable and the equation (10) is satisfied, the PMID observer in the form (3) can be designed such that the estimation error dynamics in (4) is regular, impulse-free and asymptotically stable. □

Remark 1. Under the assumption (6) or (8), the inverse of $(\overline{E}+\overline{L}_D\overline{C})$ exists for almost all $\overline{L}_D \in \mathbf{R}^{(n+kq)\times p}$. Thus, the derivative gain $\overline{L}_D$ can be chosen easily such that $(\overline{E}+\overline{L}_D\overline{C})$ is non-singular. After that, one can select the gain $\overline{L}$ to assign the poles of the matrix $(\overline{E}+\overline{L}_D\overline{C})^{-1}(\overline{A}-\overline{L}\,\overline{C})$ arbitrary under the controllability condition (7) or (9)-(10). For ensuring the internal properness and stability of the estimation error dynamics, the completely observability condition for $(\overline{E},\ \overline{A},\ \overline{C})$ can be replaced by the completely detectability condition, i.e.,

$$\operatorname{rank}\left[(\overline{E})^T,\quad (\overline{C})^T\right]^T = n + kq \tag{11}$$

and

$$\operatorname{rank}\left[(s\overline{E}-\overline{A})^T,\quad (\overline{C})^T\right]^T = n + kq\,,\ \forall s \in \mathbf{C}_+\,, \tag{12}$$

where $\mathbf{C}_+$ denotes the closed right-half complex plane. Using $\overline{E}$, $\overline{A}$ and $\overline{C}$ defined in (5), Eqs. (11) and (12) are equivalent to

$$\operatorname{rank}\left[(E)^T,\quad (C)^T\right]^T = n\,, \tag{13}$$

and

$$\operatorname{rank}\begin{pmatrix} sE-A & -N \\ 0 & sI_k \\ \hline C & 0 \end{pmatrix} = n + k\,,\ \forall s \in \mathbf{C}_+\,. \tag{14}$$

□

Let

$$z = (E + L_D C)\hat{x} - L_D y\,, \tag{15}$$

then

$$\hat{x} = (E + L_D C)^{-1} z + (E + L_D C)^{-1} L_D y\,. \tag{16}$$

Substituting (16) into (3), one has the following modified PMID observer

$$\begin{cases} \dot{z} = (A - L_P C)(E + L_D C)^{-1} z + Bu \\ \quad + [L_P + (A - L_P C)(E + L_D C)^{-1} L_D] y + N\hat{f}_q \\ \hat{x} = (E + L_D C)^{-1} z + (E + L_D C)^{-1} L_D y \\ \dot{\hat{f}}_q = L_I^q (y - C\hat{x}) + \hat{f}_{q-1} \\ \dot{\hat{f}}_{q-1} = L_I^{q-1} (y - C\hat{x}) + \hat{f}_{q-2} \\ \vdots \\ \dot{\hat{f}}_2 = L_I^2 (y - C\hat{x}) + \hat{f}_1 \\ \dot{\hat{f}}_1 = L_I^1 (y - C\hat{x})\,. \end{cases} \tag{17}$$

Thus, one can present the following theorem:

Theorem 2. Under the assumptions (8)-(10) or (13)-(14), the modified PMID observer in the form of (17) can be designed such that the estimation error dynamics in (4) is regular, impulse-free and asymptotically stable. □

Remark 2. Notice that there are not the derivative terms of the output y in the observer (17). Therefore, this modified observer (17) is preferable in control applications.

Remark 3. When $d^{(q)} \neq 0$, but $d^{(q)}$ is bounded, the error equation becomes

$$(\overline{E}+\overline{L}_D\overline{C})\dot{\overline{e}} = (\overline{A}-\overline{L}\,\overline{C})\overline{e} - \overline{N}\,d^{(q)} \tag{18}$$

where $\overline{N} = (0 \quad I_k \quad 0 \quad \cdots \quad 0)^T$. From the above equation, the bounded $d^{(q)}$ is only affected by the known matrix $\overline{N}$. Therefore, one can choose a derivative gain $\overline{L}_D$ such that (18) is normalized, and then select a reasonable high gain $\overline{L}$ such that the stabilizing term prevails over the perturbed term $\overline{N}d^{(q)}$. Consequently, the proposed PMID observer has a good robust tracking performance against any input disturbances approximated by (2).

Remark 4. For a normal system both with input disturbances and output disturbances

$$\begin{cases} \dot{x} = Ax + Bu + Nd \\ y = Cx + d \end{cases} \tag{19}$$

where $(C,\ A)$ is observable pair (or detectable pair) $\operatorname{rank}\begin{bmatrix} A & N \\ C & I \end{bmatrix} = n + k$.

We can consider the following augmented generalized dynamical plant

$$\begin{cases} \widetilde{E}\dot{\widetilde{x}} = \widetilde{A}\widetilde{x} + \widetilde{B}u + \widetilde{N}d \\ y = \widetilde{C}x \end{cases} \tag{20}$$

where

$$\widetilde{x} = \begin{pmatrix} x \\ d \end{pmatrix} \in \mathbf{R}^{n+k}, \; \widetilde{E} = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix},$$

$$\widetilde{A} = \begin{pmatrix} A & 0 \\ 0 & -I \end{pmatrix}, \; \widetilde{B} = \begin{pmatrix} B \\ 0 \end{pmatrix},$$

$$\widetilde{N} = \begin{pmatrix} N \\ I \end{pmatrix} \text{ and } \widetilde{C} = \begin{pmatrix} C & I \end{pmatrix}.$$

Notice that

$$\operatorname{rank}\left[(\widetilde{E})^T, \quad (\widetilde{C})^T\right]^T$$

$$= \operatorname{rank}\begin{bmatrix} I_n & 0 \\ 0 & 0 \\ \hdashline C & I_k \end{bmatrix} = n + k \;, \tag{21}$$

and

$$\operatorname{rank}\begin{pmatrix} s\widetilde{E} - \widetilde{A} & -\widetilde{N} \\ 0 & sI_k \\ \hdashline \widetilde{C} & 0 \end{pmatrix}$$

$$= \operatorname{rank}\begin{bmatrix} sI - A & -N \\ 0 & sI \\ \hdashline C & I \end{bmatrix} + k$$

$$= \begin{cases} \operatorname{rank}\begin{bmatrix} A & N \\ C & I \end{bmatrix} + k \\ \operatorname{rank}\begin{bmatrix} sI - A \\ C \end{bmatrix} + 2k, \; s \neq 0, \; s \in \mathbf{C}\,(\text{or } \mathbf{C}_+) \end{cases}$$

$$= n + 2k, \forall s \in \mathbf{C}\,(\text{or } \mathbf{C}_+)\,. \tag{22}$$

$\begin{pmatrix} \overline{E} & \overline{A} & \overline{C} \end{pmatrix}$ is thus completely observable (or detectable) where $\overline{E}$, $\overline{A}$ and $\overline{C}$ are defined in (5) only with the sub-matrices E, A, C and N being replaced by $\widetilde{E}$, $\widetilde{A}$, $\widetilde{C}$ and $\widetilde{N}$, respectively. Therefore, for the plant (20), we can use the present technique to construct PMID observer in the form of (3) or (17) (provided that E, A, B, C and N are replaced by $\widetilde{E}$, $\widetilde{A}$, $\widetilde{B}$, $\widetilde{C}$ and $\widetilde{N}$, respectively). Thus, the system state and the disturbance of (19) can be estimated asymptotically at the same time. Consequently, this kind of observers can be utilized to estimate the measurement output noise for a normal system.

Remark 5. For a generalized bilinear dynamical system with input disturbances

$$\begin{cases} E\dot{x} = Ax + \sum_{i=1}^{m} D_i x u_i + Bu + Nd \\ y = Cx \end{cases} \tag{23}$$

where u_i, $(i = 1, 2, \ldots, m)$, is the component of the control input vector u and the assumptions (8)-(10) or (13)-(14) hold. Suppose $\operatorname{rank}\begin{pmatrix} C \\ D_i \end{pmatrix} = \operatorname{rank}(C)$ and choose $G_i = D_i C^+$ $(i = 1, 2, \ldots, m)$ with C^+ being the generalized inverse of C, we can construct the PMID observer as follows:

$$\begin{cases} E\dot{\hat{x}} = A\hat{x} + \sum_{i=1}^{m} G_i y u_i + Bu + L_P(y - C\hat{x}) \\ \qquad + L_D(\dot{y} - C\dot{\hat{x}}) + N\hat{f}_q \\ \dot{\hat{f}}_q = L_I^q(y - C\hat{x}) + \hat{f}_{q-1} \\ \dot{\hat{f}}_{q-1} = L_I^{q-1}(y - C\hat{x}) + \hat{f}_{q-2} \\ \vdots \\ \dot{\hat{f}}_2 = L_I^2(y - C\hat{x}) + \hat{f}_1 \\ \dot{\hat{f}}_1 = L_I^1(y - C\hat{x}) \end{cases} \tag{24}$$

or

$$\begin{cases} \dot{z} = (A - L_P C)(E + L_D C)^{-1} z \\ \qquad + [L_P + (A - L_P C)(E + L_D C)^{-1} L_D \\ \qquad + \sum_{i=1}^{m} u_i G_i] y + Bu + N\hat{f}_q \\ \hat{x} = (E + L_D C)^{-1} z + (E + L_D C)^{-1} L_D y \\ \dot{\hat{f}}_q = L_I^q(y - C\hat{x}) + \hat{f}_{q-1} \\ \dot{\hat{f}}_{q-1} = L_I^{q-1}(y - C\hat{x}) + \hat{f}_{q-2} \\ \vdots \\ \dot{\hat{f}}_2 = L_I^2(y - C\hat{x}) + \hat{f}_1 \\ \dot{\hat{f}}_1 = L_I^1(y - C\hat{x})\,. \end{cases} \tag{25}$$

In this case, the estimation error dynamic is governed by (4) or (18). Thus, the observer (24) or (25) has a good robust tracking performance against any input disturbances approximated by (2).

Remark 6. For a generalized time-delay bilinear dynamical system with input disturbances

$$\begin{cases} E\dot{x} = Ax + \sum_{i=1}^{m} D_i x u_i + \sum_{i=1}^{l} A_{di} x(t-\tau_i) + Bu + Nd \\ y = Cx \end{cases} \tag{26}$$

where u_i, $(i = 1, 2, \dots, m)$, is the component of the control input vector u; τ_i, $(i = 1, 2, \dots, l)$, is a constant delay duration; the assumptions (8)-(10) or (13)-(14) hold.

Suppose $\operatorname{rank}\begin{pmatrix} C \\ D_i \end{pmatrix} = \operatorname{rank}\begin{pmatrix} C \\ A_{di} \end{pmatrix} = \operatorname{rank}(C)$ and choose $G_i = D_i C^+$ $(i = 1, 2, \dots, m)$, and $H_{di} = A_{di} C^+$ $(i = 1, 2, \dots, l)$, one has the following PMID observer:

$$\begin{cases} E\dot{\hat{x}} = A\hat{x} + \sum_{i=1}^{m} G_i y u_i + Bu \\ \quad \sum_{i=1}^{l} H_{di} y(t-\tau_i) + L_P(y - C\hat{x}) \\ \quad + L_D(\dot{y} - C\dot{\hat{x}}) + N\hat{f}_q \\ \dot{\hat{f}}_q = L_I^q (y - C\hat{x}) + \hat{f}_{q-1} \\ \dot{\hat{f}}_{q-1} = L_I^{q-1} (y - C\hat{x}) + \hat{f}_{q-2} \\ \vdots \\ \dot{\hat{f}}_2 = L_I^2 (y - C\hat{x}) + \hat{f}_1 \\ \dot{\hat{f}}_1 = L_I^1 (y - C\hat{x}) \end{cases} \tag{27}$$

or

$$\begin{cases} \dot{z} = (A - L_P C)(E + L_D C)^{-1} z \\ \quad + [L_P + (A - L_P C)(E + L_D C)^{-1} L_D \\ \quad + \sum_{i=1}^{m} u_i G_i] y + \sum_{i=1}^{l} H_{di} y(t-\tau_i) \\ \quad + Bu + N\hat{f}_q \\ \hat{x} = (E + L_D C)^{-1} z + (E + L_D C)^{-1} L_D y \\ \dot{\hat{f}}_q = L_I^q (y - C\hat{x}) + \hat{f}_{q-1} \\ \dot{\hat{f}}_{q-1} = L_I^{q-1} (y - C\hat{x}) + \hat{f}_{q-2} \\ \vdots \\ \dot{\hat{f}}_2 = L_I^2 (y - C\hat{x}) + \hat{f}_1 \\ \dot{\hat{f}}_1 = L_I^1 (y - C\hat{x}) \ . \end{cases} \tag{28}$$

In this case, the estimation error dynamic is also governed by (4) or (18). Thus, the observer (27) or (28) has a good robust tracking performance against any input disturbances approximated by (2).

3. ILLUSTRATIVE EXAMPLE

Consider the plant in the form (1), where

$$E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ A = \begin{bmatrix} 2 & -1 \\ -1 & -2 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \ N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \ u = \begin{bmatrix} 2\sin(t) \\ 3\cos(t) \end{bmatrix},$$

$$d = \begin{bmatrix} 0.05t + 1.2 \\ 0.02\sin(2t) + 1 \end{bmatrix}. \tag{29}$$

Observe that

$$\operatorname{rank}\begin{pmatrix} E \\ C \end{pmatrix} = \operatorname{rank}\begin{pmatrix} 0 & 1 \\ 0 & 0 \\ \hdashline 2 & 1 \\ 0 & 1 \end{pmatrix} = 2 = n,$$

$$\operatorname{rank}\begin{pmatrix} sE - A \\ C \end{pmatrix} = \operatorname{rank}\begin{pmatrix} -2 & s+1 \\ 1 & 2 \\ \hdashline 2 & 1 \\ 0 & 1 \end{pmatrix} = 2 = n \ , \ \forall s \in \mathbf{C} \ ,$$

and

$$\operatorname{rank}\begin{pmatrix} A & N \\ C & 0 \end{pmatrix} = \operatorname{rank}\left(\begin{array}{cc:cc} 2 & -1 & 1 & 0 \\ -1 & -2 & 0 & 1 \\ \hdashline 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right) = 4 = n + k \ .$$

That is, the conditions (8)-(10) are satisfied. Moreover, it is noted that $d^{(2)} \approx 0$, i.e., $q = 2$. Thus, we can design a PMID observer (3) with $q = 2$. We choose

$$L_D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$L_P = \begin{bmatrix} 10.4160 & -4.8489 \\ -0.3720 & 5.2119 \end{bmatrix},$$

$$L_I^1 = \begin{bmatrix} 33.4207 & -89.4206 \\ 4.4925 & -4.8389 \end{bmatrix},$$

$$L_I^2 = \begin{bmatrix} 30.3378 & -50.6435 \\ 1.8729 & 8.9709 \end{bmatrix}$$

such that the poles of (4) are $-1+i$, $-1-i$, -2, -3, -4, -5.

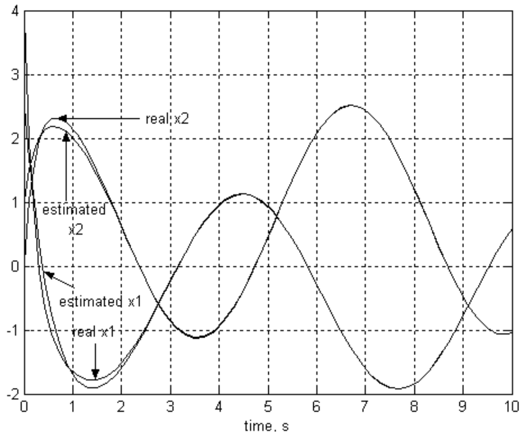

Fig. 1. The states and their estimates

The observer can be obtained as follows:

$$\begin{cases} \dot{z} = \begin{bmatrix} -9.4160 & 12.2649 \\ -0.1280 & -6.5840 \end{bmatrix} z + \begin{bmatrix} 1.0000 & 7.4160 \\ -0.5000 & -1.3720 \end{bmatrix} y \\ \quad + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} u + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \hat{f}_2 \\ \hat{x} = \begin{bmatrix} 0.5000 & -1.0000 \\ 0 & 1.0000 \end{bmatrix} z + \begin{bmatrix} 0.5000 & -1.0000 \\ 0 & 1.0000 \end{bmatrix} y \\ \dot{\hat{f}}_1 = \begin{bmatrix} 33.4207 & -89.4206 \\ 4.4925 & -4.8389 \end{bmatrix} (y - C\hat{x}) \\ \dot{\hat{f}}_2 = \begin{bmatrix} 30.3378 & -50.6435 \\ 1.8729 & 8.9709 \end{bmatrix} (y - C\hat{x}) + \hat{f}_1 \ . \end{cases} \tag{30}$$

Figures 1 and 2 respectively show the tracking performance of the state and disturbance. It can be seen that the tracking performance is satisfactory.

4. CONCLUSION

A new type of observer, that is, PID multiple-integral observer or called PMID observer, is presented for generalized dynamical systems, which allows us to asymptotically estimate the system state and de-couple unknown input disturbances successfully. The estimates of the system sate and input disturbance can be obtained simultaneously. The bilinear PMID observer has also been designed. The proposed PMID observer can be applied to estimate the output noise or the sensor fault for normal systems. This design will be useful in some control topics such as observer-based control and fault detection and fault tolerant control etc.

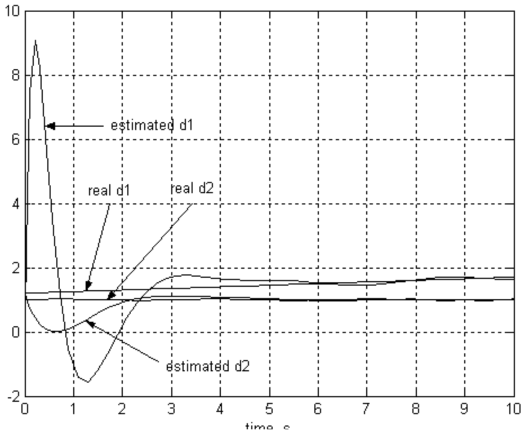

Fig.2 The disturbances and their estimates

REFERENCES

Budawon, K. and P. Kabore (2001). Disturbance attenuation using proportional integral observer. *International Journal of Control*, **74**, 618-627.

Dai, L (1989). *Singular control systems*. Springer-Verlag, New York.

Darouach, M., M. Zasadzinski and M. Hayar (1996). Reduced-order observer design for descriptor systems with unknown inputs. *IEEE Transactions on Automatic Control*, **41**, 1068-1072.

Gao, Z. (1999). Parameterization of linear function observers for descriptor systems. *IFAC World Control Congress*, Beijing, **D**, 1-6.

Gao, Z. (2005). PD observer parameterization design for descriptor systems. Journal of The Franklin Institute, in press.

Gao Z. and D. W. C. Ho (2004). Proportional multiple-integral observer design for descriptor systems with measurement output disturbances. *IEE Proc.-Control Theory & Applications*, **151**, 279-288.

Ibrir S. (2004). Robust state estimation with q-Integral observers. *Proceeding of the American Control Conference,* Boston, Massachusetts.

Koenig, D. and S. Mammar (2002). Design of proportional-integral observer for unknown input descriptor systems. *IEEE Transactions on Automatic Control*, **47**, 2057-2062.

Jiang G., S. Wang and W. Song (2000). Design of observer with integrators for linear system with unknown input disturbances. *Electronics Letters*, **36**, 1168-1169.