

AN APPLICATION OF THE VIRTUAL REFERENCE FEEDBACK TUNING METHOD TO A MULTIVARIABLE PROCESS CONTROL

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Abstract: This paper provides an application of the virtual reference feedback tuning (VRFT) method to a process control. The VRFT method is a direct method which determines parameters of a feedback controller using a set of input and output data of a plant. Originally, the VRFT method is presented for designing a single-input single-output control system. In this paper, the VRFT method is applied for a design of a multi-input multi-output control system. Experimental results show availability of the VRFT method. *Copyright ©2005 IFAC*

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1. INTRODUCTION

This paper describes an application of the virtual reference feedback tuning (Campi *et al.*, 2002; Guardabassi and Savaresi, 2000) method for the design of a multi-input multi-output (MIMO) control system which regulates a gas flow rate and a gas pressure.

In an industrial plant there are many controllers which have fixed structures, for example a PID controller. The parameters of such a controller are usually adjusted by a heuristic method. It is often difficult to tune control parameters heuristically for an MIMO system. There are several methods to design a multivariable control system. They are described in a standard textbook (Skogestad and Postlethwaite, 1996). Although those methods are useful and powerful, they require efforts and knowledges to make a plant model and to use a computer tool. It is preferable that tuning of control parameters is economical in practice.

There are some approaches which obtains control parameters directly using data generated by a

plant. The VRFT is a direct method which is able to obtain parameters of the feedback controller from input and output data of the plant. Originally, the VRFT method is proposed for a single input and single output (SISO) system. Simulation results of the VRFT method are shown in references (Campi *et al.*, 2002; Guardabassi and Savaresi, 2000; Guardabassi and Savaresi, 2001). In the reference (Previdi *et al.*, 2004), an experimental application using VRFT is shown.

In this paper the VRFT method is applied for an MIMO process. Experimental tests are carried out and test results show availability.

2. VIRTUAL REFERENCE FEEDBACK TUNING

2.1 Idea of the VRFT method

In this section, the virtual reference feedback tuning is briefly explained. Although the VRFT method is originally proposed for an SISO system,

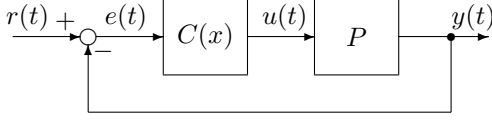


Fig. 1. feedback loop

it can be applied for an MIMO plant in almost same way.

It is assumed that the plant is described by a linear discrete-time system $P(z^{-1})$ and has same number of inputs and outputs, the controller $C(z^{-1}, x)$ has a fixed structure and has parameters $x = (x_1, \dots, x_n)^T$, and a set of inputs and outputs (I/O) data $\{u(t), y(t), t = 1, \dots, N\}$ of the plant $P(z^{-1})$ is available.

In the followings, writes $|v|_2 = \sqrt{v^T v}$ for a vector v and $\text{diag}[T_1, \dots, T_n]$ for a diagonal matrix with elements T_1, \dots, T_n . z^{-1} will be omitted for the purpose to shorten the notation.

The idea of the VRFT method is explained as follows.

- (1) Consider a feedback control system in Fig.1. A reference model T is assigned as a desired closed loop system.
- (2) Define a reference $\bar{r}(t)$ to satisfy $y(t) = T\bar{r}(t)$.
- (3) The data $y(t)$ can be considered as the output signal of the feedback control system when the closed loop transfer function is T and the reference signal is $r(t) = \bar{r}(t)$. Also the data $u(t)$ can be considered as the plant input signal of the closed loop because $y(t) = Pu(t)$.
- (4) If the controller is $C(x)$ and the reference signal is $\bar{r}(t)$, then the control signal will be $C(x)(\bar{r}(t) - y(t))$.
- (5) Therefore, when $u(t)$ and $C(x)(\bar{r}(t) - y(t))$ is near by some measure, the closed loop system with the controller $C(x)$ is near the reference model T .
- (6) Then, the idea is to find such a controller $C(x)$ that makes $u(t)$ and $C(x)(\bar{r}(t) - y(t))$ near. The problem can be formulated by minimizing the following index.

$$J(x) = \sum_{t=1}^N |u(t) - C(x)(T^{-1} - I)y(t)|_2^2 \quad (1)$$

The index (1) can be evaluated because $u(t)$ and $y(t)$ are the prepared data and T is specified by a designer.

This minimization problem is regarded as a system identification problem to identify the parameters of the controller $C(x)$ by using data $u(t)$ and $(T^{-1} - I)y(t)$.

2.2 Filtering

The reference model T specifies the closed loop response and is defined by a designer. When the elements of T are assigned by the dynamic model with slow response, T^{-1} has a large gain. On this case, the noise signal included in the data may causes to make a bad estimation of the control parameters by minimizing the equation (1). In order to reduce the influence of noise, a filter should be applied. When the filter L is applied, the equation (1) becomes as

$$J(x) = \sum_{t=1}^N |Lu(t) - C(x)(T^{-1} - I)Ly(t)|_2^2. \quad (2)$$

In the MIMO case, it is required that the structure of the filter L is satisfied $PL = LP$ so that the minimization of the index (2) means to minimize the index (1). Assume that C_o is a desired controller, P^{-1} and C_o^{-1} exists and $PL = LP$. As $T = (I + PC_o)^{-1}PC_o$ and $y(t) = Pu(t)$, the equation (2) becomes,

$$\begin{aligned} J(x) &= \sum_{t=1}^N |Lu(t) - C(x)(PC_o)^{-1}LPu(t)|_2^2 \\ &= \sum_{t=1}^N |(C_o - C(x))C_o^{-1}Lu(t)|_2^2. \end{aligned}$$

This equation means that $C_o - C(x)$ is near when $J(x)$ is small.

As the plant model P is unknown, L should be diagonal to ensure the condition $PL = LP$.

In the reference (Campi *et al.*, 2002) the filter $L = T(1 - T)/U$ for an SISO system, where U is a square root of the spectrum of $u(t)$, is applied in order to strengthen the model matching. This kind of filter can not be applied in the MIMO case, because it does not have a required structure.

There are simple ways which satisfy the condition $PL = LP$ and avoid the calculation of T^{-1} .

- (1) select $T = \text{diag}[T_1, T_2, \dots, T_m]$.
- (2) calculate the least common multiple M of T_1, T_2, \dots, T_m .
- (3) define $H = \text{diag}[M/T_1, M/T_2, \dots, M/T_m]$.
- (4) set $L = TH$.
- When T is diagonal and has same elements, H is I and the filter L becomes T .

In the followings, the reference model T is selected as diagonal and the next cost function is considered.

$$J(x) = \sum_{t=1}^N |THu(t) - C(x)(I - T)Hy(t)|_2^2 \quad (3)$$

where $H = I$ or $H = \text{diag}[M/T_1, M/T_2, \dots, M/T_m]$.

2.3 Algorithm

If the parameters of the controller is described linearly, the equation (3) can be written as,

$$J(x) = \sum_{t=1}^N |THu(t) - K(t)x|_2^2 \quad (4)$$

where

$$K(t)x = C(x)(I - T)Hy(t). \quad (5)$$

The problem that minimize the equation (4) can be solved by the least square method.

In this case, an algorithm for the VRFT method is summarized as,

- (1) prepare a set of I/O data $\{u(t), y(t), t = 1, \dots, N\}$.
- (2) define a reference model T .
- (3) calculate $K(t)$ such that $K(t)x = C(x)(I - T)Hy(t)$.
- (4) set a vector U and a matrix E such that

$$U = \begin{bmatrix} THu(1) \\ THu(2) \\ \vdots \\ THu(N) \end{bmatrix}, \quad E = \begin{bmatrix} K(1) \\ K(2) \\ \vdots \\ K(N) \end{bmatrix}$$

- (5) calculate the parameter vector

$$x = (E^T E)^{-1} E^T U.$$

This is a simple algorithm. As minimizing the equation (1)(or (2),(3)) is a kind of a system identification problem to identify the transfer function $C(x)$, there may be a more proper algorithm.

3. EXPERIMENTAL TEST

3.1 Test facility

In this section experimental results are presented. A schematic diagram of the test facility is shown in Fig.2. This facility has a circulation loop of fluid in which the evaporator heats liquid and generates gas, the condenser condenses gas and the pump feeds liquid to the evaporator. A gas flow rate and a gas pressure are regulated by modulating two control valves. The gas flow rate y_2 is measured at the upstream of the control valve CV_2 . The gas pressure y_1 is measured at the outlet header of the evaporator. The controller output u_1, u_2 are the position demand of the each control valve.

A conceptual model of the test plant is shown in Fig.3. The features of the test plant are summarized as,

- the plant is interactive and open loop stable.
- the speed of the control valves are limited.
- the plant have dead times caused by delay of the actuator and signal conversions.

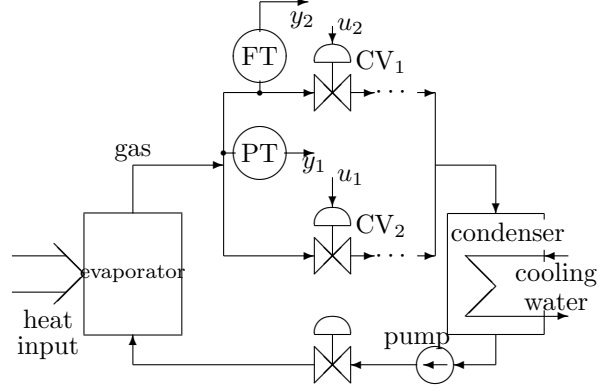


Fig. 2. Experimental facility

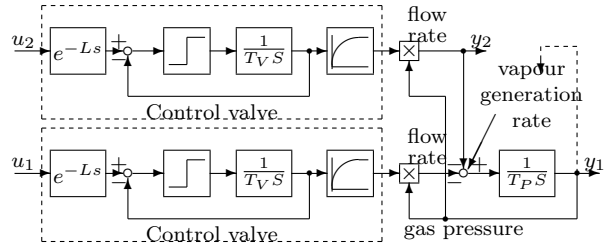


Fig. 3. Plant model

Practically, it is usual that two PI controllers are installed in spite of interactions.

3.2 Controller design

3.2.1. Controller and parameters The structure of the multivariable controller to be designed is defined as follows,

$$C(x) = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} + \frac{1}{1 - z^{-1}} \begin{bmatrix} x_5 & x_6 \\ x_7 & x_8 \end{bmatrix} + (1 - z^{-1}) \begin{bmatrix} x_9 & x_{10} \\ x_{11} & x_{12} \end{bmatrix}. \quad (6)$$

$K(t)$ and x in the equation (5) can be written as,

$$K(t) = \begin{bmatrix} P_1 y_1(t) & P_2 y_2(t) & 0 & 0 \\ 0 & 0 & P_1 y_1(t) & P_2 y_2(t) \end{bmatrix} \times \begin{bmatrix} I_4 & \frac{1}{1 - z^{-1}} I_4 & (1 - z^{-1}) I_4 \end{bmatrix} \quad (7)$$

$$x = (x_1, x_2, \dots, x_{12})^T \quad (8)$$

where $\text{diag}[P_1, P_2] = (I - T)H$ and I_4 is the 4×4 identity matrix.

A controller which consists of two SISO PID controllers is compared to a multivariable controller.

$$C(x) = \begin{bmatrix} x_1 & 0 \\ 0 & x_4 \end{bmatrix} + \frac{1}{1 - z^{-1}} \begin{bmatrix} x_5 & 0 \\ 0 & x_8 \end{bmatrix} + (1 - z^{-1}) \begin{bmatrix} x_9 & 0 \\ 0 & x_{12} \end{bmatrix} \quad (9)$$

In this case, $K(t)$ and x are

$$K(t) = \begin{bmatrix} P_1 y_1(t) & 0 \\ 0 & P_2 y_2(t) \end{bmatrix} \times \begin{bmatrix} I_2 & \frac{1}{1-z^{-1}} I_2 (1-z^{-1}) I_2 \end{bmatrix} \quad (10)$$

$$x = (x_1, x_4, x_5, x_8, x_9, x_{12})^T \quad (11)$$

where I_2 is the 2×2 identity matrix.

3.2.2. Data and reference model In order to evaluate the VRFT method, four reference models and three plant data are used to calculate the control parameters.

The plant data are obtained by closed loop and open loop tests.

data 1 closed-loop test: PI gain $P=1, I=0.05$

data 2 closed-loop test: PI gain $P=0.5, I=0.02$

data 3 open loop test: step change u_1 and u_2 .

The closed loop tests are carried by using two SISO PI controllers that have same PI gains and providing the step in the reference signal. Figure 4(a)(b)(c) show responses of the plant inputs $u(t)$ and outputs $y(t)$. In each figure, upper graphs show inputs $u_1(t), u_2(t)$ and lower graphs show outputs $y_1(t), y_2(t)$. In the lower graphs, the solid lines are outputs and the dotted lines are the reference signals.

The diagonal elements used in the reference model T are the discrete time transfer function obtained by converting the following continuous time transfer functions.

model 1 (T_1) $e^{-1.6s} / (s^2 + 1.4s + 1)$

model 2 (T_2) $e^{-1.6s} / (2.25s^2 + 2.4s + 1)$

model 3 (T_3) $e^{-1.6s} / (4s^2 + 3.2s + 1)$

model 4 (T_4) $e^{-1.6s} / (16s^2 + 6.4s + 1)$.

The zero-order-hold sampling with the sampling time 0.2 second which is same as the sampling time of the controller, converts each of these transfer functions to the discrete time transfer function. Each model has a dead time $e^{-1.6s}$. As the dead time can not be changed by the feedback controller, the value of the dead time is chosen same as the plant's dead time which is decided by using the input and output data.

Figure 5 shows the step responses of the element of these reference model.

3.3 Experimental results

3.3.1. Result 1 The first results compare the MIMO design and the SISO design and show the effects of the data. Table 1 shows the parameters calculated by the VRFT method. MIMO stands

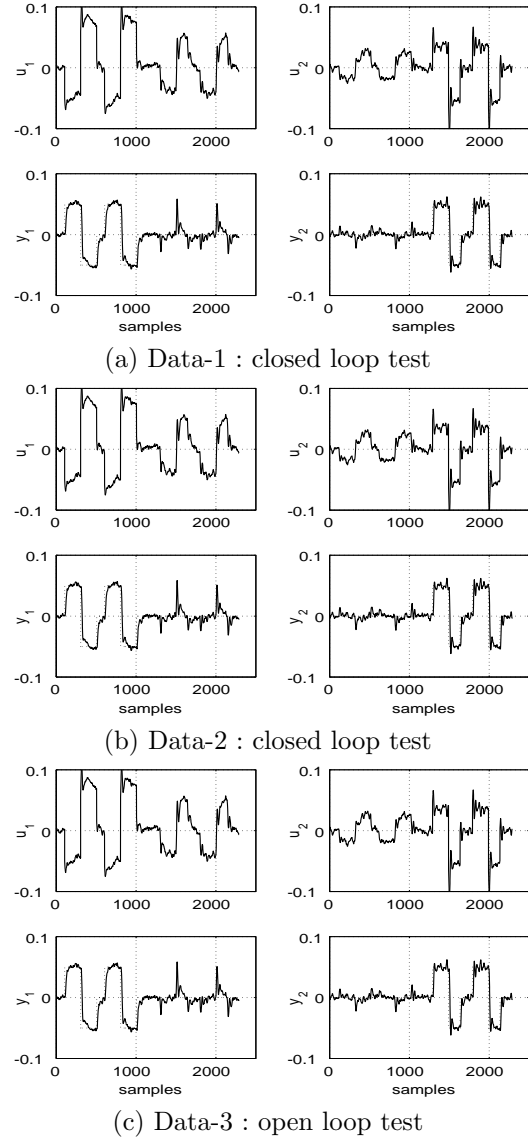


Fig. 4. Input and output data

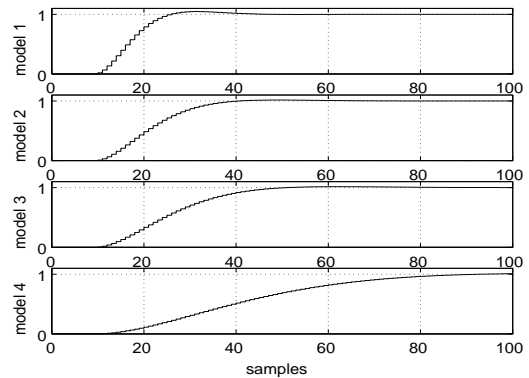


Fig. 5. Step responses of the reference models

for the controller using the equation (6) and SISO stands for the controller using the equation (9). MIMO design shows the parameters using data 1, data 2 and data 3. SISO design shows the parameters using data 3. The reference model is $T = \text{diag}[T_2, T_2]$ and the filter is $H = I$ for all cases.

Table 1. control parameters design by the VRFT method

controller	MIMO			SISO
parameter	data1	data2	data3	data3
x_1	-0.775	-0.995	-0.604	-0.366
x_2	-0.260	-0.370	-0.279	0
x_3	-0.057	-0.159	-0.104	0
x_4	0.406	0.181	0.326	0.337
x_5	-0.062	-0.064	-0.068	-0.046
x_6	-0.045	-0.047	-0.042	0
x_7	-0.020	-0.022	-0.023	0
x_8	0.041	0.040	0.044	0.053
x_9	-0.194	0.601	0.376	0.145
x_{10}	-1.801	-1.132	0.067	0
x_{11}	-0.308	-0.145	-0.481	0
x_{12}	-0.266	0.616	0.147	0.347
result	Fig.6	Fig.7	Fig.8	Fig.9

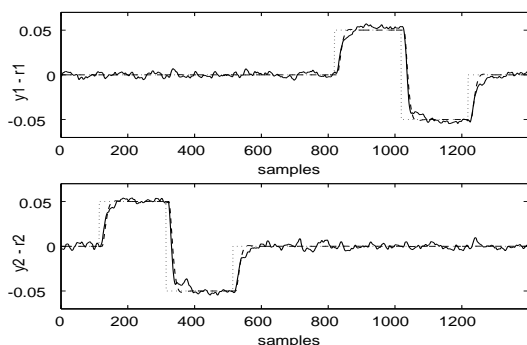


Fig. 6. Closed loop response : designed by using data - 1

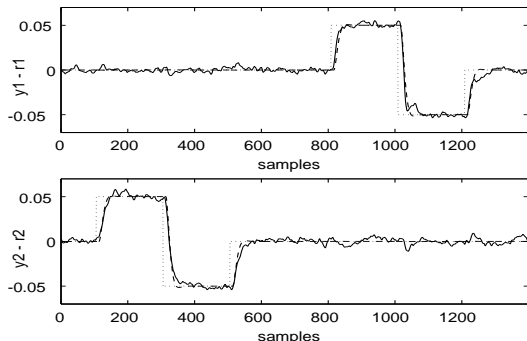


Fig. 7. Closed loop response : designed by using data - 2

The experimental results which are the responses to the step in the reference signal are shown in Fig.6 to Fig.9.

The upper graph in each figure shows the plant output y_1 by a solid line and its reference signal by a dotted line and the lower graph shows the output y_2 by a solid line and its reference signal by a dotted line. The dashed lines that are the required responses calculated by using the reference model and the reference signal are added to both graphs to be compared to the test results.

In all cases of the MIMO design, the responses of the plant output are almost same as the required responses. Figure 9 shows the interaction of the SISO design.

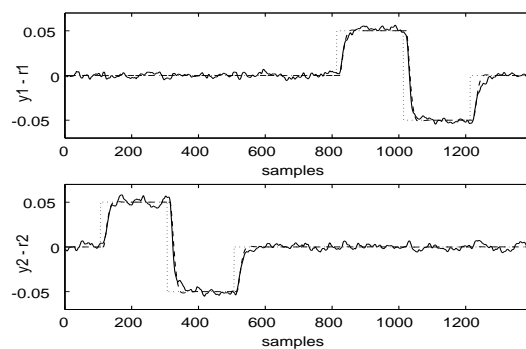


Fig. 8. Closed loop response : designed by using data - 3

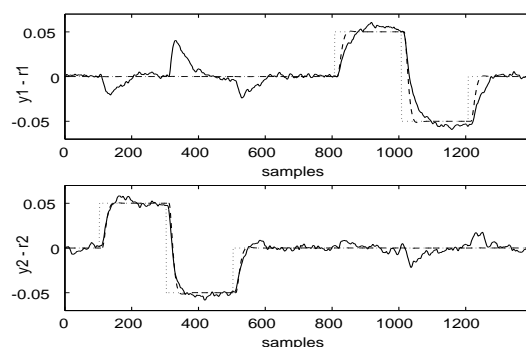


Fig. 9. Closed loop response : controlled by 2-SISO controllers

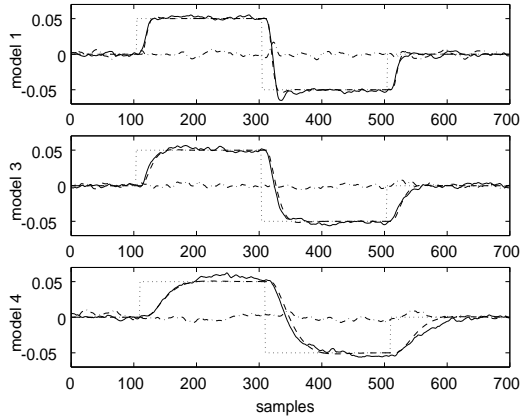
3.3.2. Result 2 The next results displayed in Fig.10 show the responses to the step in the reference using the controllers designed by the different reference models and the same plant data. The reference models are selected as $T = \text{diag}[T_1, T_1]$, $T = \text{diag}[T_3, T_3]$ and $T = \text{diag}[T_4, T_4]$. The plant data 3 and the filter $H = I$ is used for all cases.

In Fig.10 (a) and (b), the upper graph display the results using the model $T = \text{diag}[T_1, T_1]$, the middle graph show the results using the model $T = \text{diag}[T_3, T_3]$ and the lower graph show the results using the model $T = \text{diag}[T_4, T_4]$.

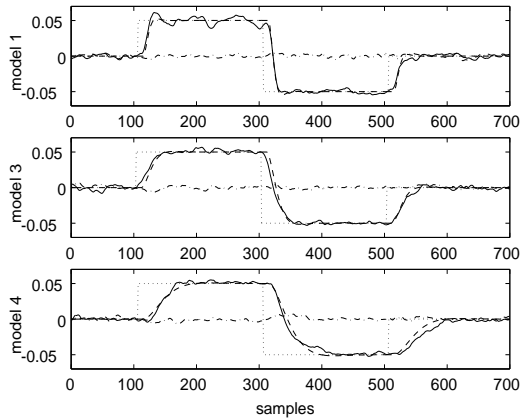
Figure 10 (a) show the response to the step in the reference r_1 . In each graph the solid line is the plant output y_1 , the dotted line is the reference signal r_1 , the dash-dotted line is the plant output y_2 . The dashed lines that are the required responses calculated by using the reference model and the reference signal are added to both graphs to be compare to the test results.

Figure 10 (b) show the response to the step in the reference r_2 . Each line shows the response same as Fig.10 (a) by exchanging the signal number 1 and 2.

These results show that the responses of the closed loop with the controller designed by the VRFT method are close to the responses of the reference models.



(a) r_1 step change



(b) r_2 step change

Fig. 10. Closed loop response : model matching

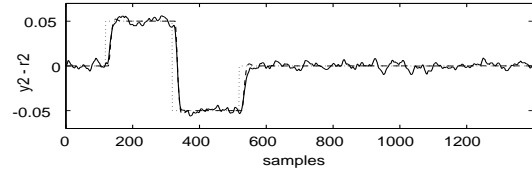
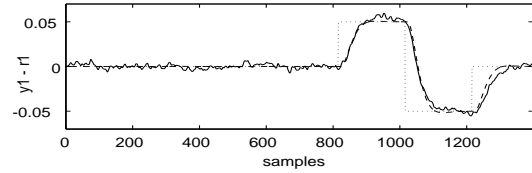
3.3.3. Result 3 Figure 11 compare the closed loop response designed by using the reference model $T = \text{diag}[T_4, T_1]$ and $H = \text{diag}[T_1, T_4]$ with $T = \text{diag}[T_4, T_1]$ and $H = I$. The graph format is same as Fig. 6. Figure 11(a) shows the controller designed with different models works well. Figure 11(b) shows the degraded response by using the inappropriate filter.

4. CONCLUSION

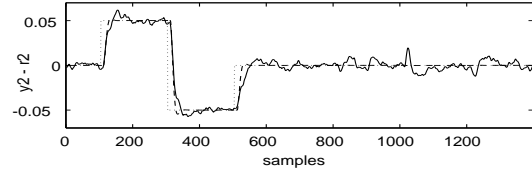
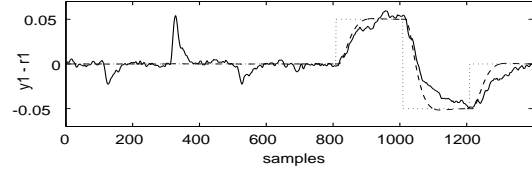
This paper provides an application of the virtual reference feedback tuning (VRFT) method for an MIMO process. The VRFT method is applied for the design of a multivariable controller for a test plant. Experimental results show availability of the VRFT method.

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(a) $T = \text{diag}[T_4, T_1], H = \text{diag}[T_1, T_4]$



(b) $T = \text{diag}[T_4, T_1], H = I$

Fig. 11. Closed loop response designed with model 1 and model 4

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