RECURSIVE IDENTIFICATION FOR HAMMERSTEIN AND WIENER SYSTEMS WITH PIECE-WISE LINEAR MEMORYLESS BLOCK

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Abstract: The paper deals with identification of Hammerstein and Wiener systems with nonlinearity being a discontinuous piece-wise linear function. Recursive estimation algorithms are given to estimate six unknown parameters contained in the nonlinearity and all coefficients of the linear subsystem. The estimates converge to the corresponding true values with probability one. Numerical examples are given to verify the theoretical assertions. *Copyright© 2005 IFAC.*

Keywords: Wiener system, Hammerstein system, identification, strong consistency, kernel function.

1. INTRODUCTION

The nonlinear system consisting of the combination of a nonlinear static block and a linear subsystem is an important model in engineering, biology, communication and in other fields (see, for example, (Celka et al, 2001; Greblicki, 1997; Westwick and Kearney, 1992), among others). The system is called the Hammerstein or Wiener system in accordance with the order of the subsystems: it is called Hammerstein if the nonlinearity is followed by the linear subsystem, and Wiener if the nonlinearity is after the linear subsystem.

For recent years a great research attention has been paid to the identification issue of Wiener and Hammerstein systems. For describing the nonlinearity there are parametric (Al-Duwaish and Krim, 1997; Bei, 2003; Celka et al, 2001; Chen, 2004b; Emerson et al, 1992; Stoica and Söderström, 1982; Vörös (2001, 2003); Wigren, 1994) and nonparametric (Bai, 2002; Chen, 2004a; Greblicki, 2002; Greblicki and Pawlak, 1989; Kalafatis et al, 1989; Lang, 1997; Ljung, 1987; Pawlak, 1991) approaches. In the nonparametric approach, the unknown nonlinear function may be identified with the help of approximation by smooth functions, for example, the nonlinear function is expanded to a series of orthogonal functions in (Bai, 2002; Pawlak, 1991). But values of the nonlinear function may also be directly estimated by using kernel functions (Chen, 2004a; Greblicki, 2002; Greblicki and Pawlak, 1989; Kalafatis et al, 1997). In the parametric approach the nonlinear function is often expressed as a linear combination of known smooth functions, and the identification problem is reduced to estimating coefficients of the linear combination. However, practical nonlinearities may not be smooth, for example, the piece-wise linear functions with preloads and dead zones (Al-Duwaish and Karim, 1997; Chen, 2004b; Vörös (2001, 2003)) are common in engineering and biology.

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There are various methods and algorithms developed for identifying Wiener and Hammerstein systems, but most of them are nonrecursive, and only a few of them are proved to be strongly consistent (Chen (2004a,b)). There are even not too many papers (Bai, 2002; Greblicki, 2002; Greblicki and Pawlak, 1989; Kalafatis et al, 1997) concerned with convergence in probability. Here we are interested in recursive estimation and in convergence of estimates with probability one.

To be precise, we consider the one-dimensional noise-free Wiener and Hammerstein systems as shown in Figures 1 and 2:

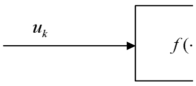

Fig. 1 Hammerstein System

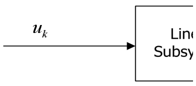

Fig. 2 Wiener System

The nonlinearity of the system is characterized by a static piece-wise linear function

$$f(v) = \begin{cases} c^+(v-d^+)+b^+, & v>d^+ \\ 0, & -d^- \le v \le d^+ \\ c^-(v+d^-)-b^-, & v<-d^-, c^+\ge 0, c^-\ge 0. \end{cases} \quad (1)$$

As concerns the linear subsystem, the ARMA model

$$A(z)y_k = B(z)v_{k-1} \quad (2)$$

with

$$A(z) = 1 + a_1 z + \cdots + a_p z^p, \quad (3)$$
$$B(z) = 1 + b_1 z + \cdots + b_q z^q, \quad zy_k = y_{k-1} \quad (4)$$

is considered for Hammerstein systems, and the MA model

$$v_k = C(z)u_k, \quad (5)$$

with

$$C(z) = 1 + c_1 z + \cdots + c_r z^r \quad (6)$$

is considered for Wiener systems.

The problem is to recursively estimate six parameters c^+, d^+, b^+, c^-, d^-, b^- ($h^+ = c^+d^+ - b^+$, and $h^- = c^-d^- - b^-$) contained in $f(\cdot)$ and all coefficients of the linear subsystems, i.e.,

$$\theta^T \triangleq [-a_1, \ldots, -a_p, b_1, \ldots, b_q] \quad (7)$$

for the Hammerstein system, and

$$\vartheta^T \triangleq [c_1, \ldots, c_r] \quad (8)$$

for the Wiener system.

The identification algorithms for Hammerstein and Wiener systems are given in Sections 2 and 3, respectively. Two numerical examples are demonstrated in Section 4, and a few concluding remarks are given in the last section.

2. IDENTIFICATION OF HAMMERSTEIN SYSTEMS

For identifying the system we need the following conditions H1-H4.

H1. $A(z)$ is stable, i.e., all roots of $A(z)$ are outside the closed unit disk.

H2. $A(z)$ and $B(z)$ are coprime, $|a_p| + |b_q| \neq 0$, and $q > p$.

H3. $B^{-1}(z) - \frac{1}{2}$ is SPR, i.e., $B^{-1}(e^{i\lambda}) + B^{-1}(e^{-i\lambda}) > 1$, $\forall \lambda \in [0, 2\pi]$.

H4. The upper bound D for d^+ and d^- is available:

$$0 \le d^+ < D, \quad \text{and} \quad 0 \le d^- < D.$$

As the system input we take $\{u_k\}$ to be a sequence of mutually independent and identically distributed (iid) random variables with uniform distribution over $[-2D, 2D]$.

Estimation algorithm for θ

Let $\overline{y}_k^0$ be the empirically centralized system output:

$$\overline{y}_k^0 \triangleq y_k - \overline{y}_k, \quad (9)$$

where $\overline{y}_k$ is the sample average recursively calculated according to

$$\overline{y}_k = \Big(1 - \frac{1}{k}\Big)\overline{y}_{k-1} + \frac{1}{k} y_k. \quad (10)$$

Let

$$\begin{aligned} \phi_k^T &= [\overline{y}_k^0, \ldots, \overline{y}_{k-p+1}^0, \hat{v}_{k-1}^0, \ldots, \hat{v}_{k-q}^0], \\ \hat{v}_{k-1}^0 &= \overline{y}_k^0 - \theta_k^T \phi_{k-1}, \end{aligned} \quad (11)$$

and the estimate θ_k for θ be recursively computed according to the ELS algorithm (Chen and Guo, 1991; Ljung, 1987)

$$\theta_{k+1} = \theta_k + a_k P_k \phi_k (\overline{y}_{k+1}^0 - \phi_k^T \theta_k), \quad (12)$$
$$P_{k+1} = P_k - a_k P_k \phi_k \phi_k^T P_k, \quad a_k = (1 + \phi_k^T P_k \phi_k)^{-1} \quad (13)$$

with arbitrary θ_0 and $P_0 > 0$.

Remark 1. If $Ev_k = 0$, for example, $d^+ = d^-$, $c^+ = c^-$, $b^+ = b^-$, then $Ey_k = 0 \quad \forall k \ge 1$. In this case (9) and (10) are not needed and y_k can be directly used in (11), (12) replacing $\overline{y}_k^0$

Estimation Algorithms for c^+, h^+, c^- and h^-

Let us write θ_k given by (12) in the component form:

$$\theta_k^T = [-a_{1k}, \dots, -a_{pk}, b_{1k}, \dots, b_{qk}].$$

Define

$$B_k \triangleq \begin{pmatrix} -b_{1k} & 1 & \cdots & 0 \\ \vdots & & & \vdots \\ \vdots & & & \\ \vdots & & & 1 \\ -b_{qk} & 0 & & 0 \end{pmatrix}, \qquad A_k \triangleq \begin{pmatrix} 1 \\ a_{1k}, \\ \vdots \\ a_{qk} \end{pmatrix}, \tag{14}$$

where $a_{ik} = 0$ for $i > p$ (by A2, $q > p$).

Recursively define $\widehat{x}_k$ with an arbitrary initial $\widehat{x}_0$:

$$\widehat{x}_{k+1} = B_{k+1}\widehat{x}_k + A_{k+1}y_{k+1}. \tag{15}$$

Set

$$\hat{v}_{k-1} \triangleq [1, 0, \dots, 0]\widehat{x}_k, \tag{16}$$

which will serve as the estimate for v_{k-1}, the output of the nonlinear block.

Denote

$$z_{k+1}^+ \triangleq \hat{v}_k I_{[u_k \geq D]}. \tag{17}$$

Let

$$\mu^+ \triangleq [c^+, h^+]^T, \quad \phi_k^+ \triangleq [u_k, -1]^T I_{[u_k \geq D]}. \tag{18}$$

By notice of that $f(u) = c^+u - h^+$ for $u \geq D$ or $v_k = \mu^{+T}\phi_k^+$ for $u_k \geq D$, it is natural to estimate μ^+ by the least squares (LS) algorithm:

$$\mu_k^+ = \mu_{k-1}^+ + a_k^+ P_k^+ \phi_k^+ (z_{k+1}^+ - \phi_k^{+T}\mu_{k-1}^+) \tag{19}$$

$$P_{k+1}^+ = P_k^+ - a_k^+ P_k^+ \phi_k^+ \phi_k^{+T} P_k^+, \tag{20}$$

$$a_k^+ = (1 + \phi_k^{+T} P_k^+ \phi_k^+)^{-1}.$$

The estimation for $\mu^- \triangleq [c^-, h^-]^T$ is carried out in a similar way. Defining

$$z_{k+1}^- \triangleq \hat{v}_k I_{[u_k \leq -D]}, \qquad \phi_k^- \triangleq [u_k \quad 1]^T I_{[u_k \leq -D]}, \tag{21}$$

we estimate μ^- by the recursive LS algorithm:

$$\mu_k^- = \mu_{k-1}^- + a_k^- P_k^- \phi_k^- (z_{k+1}^- - \phi_k^{-T}\mu_{k-1}^-), \tag{22}$$

$$P_{k+1}^- = P_k^- - a_k^- P_k^- \phi_k^- \phi_k^{-T} P_k^-, \tag{23}$$

$$a_k^- = (1 + \phi_k^{-T} P_k^- \phi_k^-)^{-1}.$$

Estimates for d^+, b^+, d^-, and b^-

Set

$$\xi_0^+ = 0, \tag{24}$$

and recursively define

$$\xi_k^+ = \frac{k-1}{k}\xi_{k-1}^+ + \frac{\hat{v}_k}{k} I_{[u_k \geq 0]}, \tag{25}$$

where $\hat{v}_k$ is given by (16). In order to avoid the possible division by zero we modify c_k^+ as follows:

$$\overline{c}_k^+ \triangleq \begin{cases} c_k^+, & \text{if} \quad |c_k^+| \geq \frac{1}{k} \\ \text{sign}(c_k^+)\frac{1}{k}, & \text{if} \quad |c_k^+| < \frac{1}{k}. \end{cases} \tag{26}$$

Then d^+ and b^+ are estimated by d_k^+ and b_k^+, respectively, where

$$d_k^+ = \frac{1}{\overline{c}_k^+}[h_k^+ - \text{sign}(h_k^+) \cdot \left(h_k^{+2} + 4\overline{c}_k^+ D(\overline{c}_k^+ D - h_k^+ - 2\xi_k^+)\right)^{\frac{1}{2}}], \tag{27}$$

and

$$b_k^+ \triangleq c_k^+ d_k^+ - h_k^+. \tag{28}$$

Similarly, set

$$\xi_0^- = 0, \tag{29}$$

and define

$$\xi_k^- = \frac{k-1}{k}\xi_{k-1}^- + \frac{\hat{v}_k}{k} I_{[u_k \leq 0]}. \tag{30}$$

After modifying c_k^- to $\overline{c}_k^-$:

$$\overline{c}_k^- \triangleq \begin{cases} c_k^-, & \text{if} \quad |c_k^-| \geq \frac{1}{k} \\ (\text{sign}c_k^-)\frac{1}{k}, & \text{if} \quad |c_k^-| < \frac{1}{k}, \end{cases} \tag{31}$$

d^- and b^- are respectively estimated by

$$d_k^- = \frac{1}{\overline{c}_k^-}\Big[h_k^- - \text{sign}(h_k^-) \cdot \Big((h_k^-)^2 + 4\overline{c}_k^- D(\overline{c}_k^- D - h_k^- + 2\xi_k^-)\Big)^{\frac{1}{2}}\Big], \tag{32}$$

and

$$b_k^- \triangleq c_k^- d_k^- - h_k^-. \tag{33}$$

Theorem 1. Assume conditions H1–H4 hold. Then i) θ_k given by (9)–(13) is strongly consistent:

$$\theta_k \xrightarrow[k\to\infty]{} \theta \qquad a.s.,$$

ii) μ_k^+ and μ_k^- respectively given by (19), (20) and (22), (23) incorporating with (14)–(17) and (21) are strongly consistent:

$$\mu_k^+ \xrightarrow[k\to\infty]{} [c^+, h^+]^T \text{ a.s.,} \quad \mu_k^- \xrightarrow[k\to\infty]{} [c^-, h^-]^T \text{a.s. ,}$$

iii) d_k^+, b_k^+, d_k^-, and b_k^- are also strongly consistent:

$$d_k^+ \xrightarrow[k\to\infty]{} d^+ \text{ a.s.,} \quad b_k^+ \xrightarrow[k\to\infty]{} b^+ \text{ a.s.,}$$

$$d_k^- \xrightarrow[k\to\infty]{} d^- \text{ a.s.,} \quad \text{and} \quad b_k^- \xrightarrow[k\to\infty]{} b^- \text{ a.s.}$$

Instead of detailed proof we just point out that Ev_k may not be zero mean, and when estimating θ we should consider

$$A(z)(y_k - Ey_k) = B(z)w_k^0$$

with $w_k^0 \triangleq v_{k-1} - Ev_{k-1}$ to replace (2). However, Ey_k is unknown, and we have to approximate

$y_k - Ey_k$ by $\overline{y}_k^0$, for which the following model takes place:

$$A(z)\overline{y}_k^0 = B(z)w_k^0 + \epsilon_k, \tag{34}$$

where

$$\epsilon_k \triangleq -A(z)(\overline{y}_k - Ey_k). \tag{35}$$

The main effort of the proof is devoted to proving strong consistency of the ELS for (34), an ARMA model with errors.

3. IDENTIFICATION OF WIENER SYSTEMS

We now consider the Wiener system defined by (1),(5), and (6). Denoting by

$$\varphi_{k-1}^T = [u_{k-1}, \dots, u_{k-r}], \tag{36}$$

we rewrite (5) as

$$v_k = u_k + \vartheta^T \varphi_{k-1}, \tag{37}$$

where ϑ is given by (8).

The coefficients to be estimated are c^+, d^+, b^+, c^-, d^-, b^- for the nonlinear block, and ϑ for the linear subsystem.

Let us take a sequence of iid Gaussian random variables $u_k \in \mathcal{N}(0,1)$ to serve as the system input. Then the output of the linear subsystem $v_k = u_k + \vartheta^T \varphi_{k-1}$ is Gaussian stationary and ergodic (Loève, 1977-1978), $v_k \in \mathcal{N}(0, \sigma_v^2)$. It is clear that

$$\sigma_v^2 = 1 + \|\vartheta\|^2. \tag{38}$$

For convenience of writing let us denote

$$\alpha^+ = \frac{d^+}{\sigma_v}, \quad \alpha^- = \frac{d^-}{\sigma_v}, \quad \beta^+ = c^+\sigma_v, \quad \beta^- = c^-\sigma_v. \tag{39}$$

Estimates for $\alpha^+, \beta^+, h^+, b^+$ and $\alpha^-, \beta^-, h^-, b^-$

Recursively define

$$p_k^+ = (1 - \frac{1}{k})p_{k-1}^+ + \frac{1}{k}I_{[y_k>0]}, \tag{40}$$

$$p_k^- = (1 - \frac{1}{k})p_{k-1}^- + \frac{1}{k}I_{[y_k<0]}, \tag{41}$$

with arbitrary initial values p_0^+, p_0^-, and then derive α_k^+ and α_k^-, the estimates for α^+ and α^-, according to the table of $\Phi(x) \triangleq \frac{1}{\sqrt{2\pi}}\int_{-\infty}^x e^{-\frac{t^2}{2}}dt$:

$$p_k^+ = 1 - \Phi(\alpha_k^+), \quad p_k^- = \Phi(-\alpha_k^-). \tag{42}$$

For estimating β^+, h^+, β^- and h^- we recursively calculate

$$\bar{y}_k^+ = (1 - \frac{1}{k})\bar{y}_{k-1}^+ + \frac{1}{k}y_k I_{[y_k>0]} \tag{43}$$

$$\bar{y}_k^{2+} = (1 - \frac{1}{k})\bar{y}_{k-1}^{2+} + \frac{1}{k}y_k^2 I_{[y_k>0]} \tag{44}$$

$$\bar{y}_k^- = (1 - \frac{1}{k})\bar{y}_{k-1}^- + \frac{1}{k}y_k I_{[y_k<0]} \tag{45}$$

$$\bar{y}_k^{2-} = (1 - \frac{1}{k})\bar{y}_{k-1}^{2-} + \frac{1}{k}y_k^2 I_{[y_k<0]} \tag{46}$$

with arbitrary initial values, and obtain estimates β_k^+ and h_k^+, by solving the following second order algebraic equations

$$\bar{y}_k^+ = \frac{\beta_k^+}{\sqrt{2\pi}}e^{-\frac{(\alpha_k^+)^2}{2}} - h_k^+ p_k^+ \tag{47}$$

$$\begin{aligned}\bar{y}_k^{2+} =& (\beta_k^+)^2\Big(\frac{\alpha_k^+ e^{-\frac{(\alpha_k^+)^2}{2}}}{\sqrt{2\pi}} + p_k^+\Big) \\ &- \frac{2}{\sqrt{2\pi}}\beta_k^+ h_k^+ e^{-\frac{(\alpha_k^+)^2}{2}} + (h_k^+)^2 p_k^+,\end{aligned} \tag{48}$$

where α_k^+, and p_k^+ are given by (40) and (42).

Similarly, β_k^- and h_k^- are derived from the following algebraic equations:

$$\bar{y}_k^- = -\frac{\beta_k^-}{\sqrt{2\pi}}e^{-\frac{(\alpha_k^-)^2}{2}} + h_k^- p_k^- \tag{49}$$

$$\begin{aligned}\bar{y}_k^{2-} =& (\beta_k^-)^2\Big(\frac{\alpha_k^- e^{-\frac{(\alpha_k^-)^2}{2}}}{\sqrt{2\pi}} + p_k^-\Big) \\ &- \frac{2}{\sqrt{2\pi}}\beta_k^- h_k^- e^{-\frac{(\alpha_k^-)^2}{2}} + (h_k^-)^2 p_k^-.\end{aligned} \tag{50}$$

It is worth noting that (47), (48)(or (49), (50)) can easily be solved with respect to β_k^+ and h_k^+ (or β_k^- and h_k^-). For this it suffices to replace h_k^+ in (48) with

$$h_k^+ = \frac{1}{p_k^+}\Big(\frac{\beta_k^+}{\sqrt{2\pi}}e^{-\frac{(\alpha_k^+)^2}{2}} - \bar{y}_k^+\Big) \tag{51}$$

derived from (47). As a result, (48) becomes a second order algebraic equation with unknown β_k^+ and its solution is

$$\beta_k^+ = \left(\frac{\bar{y}_k^{2+} - (\bar{y}_k^+)^2\frac{1}{p_k^+}}{\alpha_k^+\gamma_k^+ + p_k^+ - \frac{(\gamma_k^+)^2}{p_k^+}}\right)^{1/2}, \tag{52}$$

where $\gamma_k^+ \triangleq \frac{1}{\sqrt{2\pi}}e^{-\frac{(\alpha_k^+)^2}{2}}$.

Similarly, we have

$$h_k^- = \frac{1}{p_k^-}(\beta_k^-\gamma_k^- + \bar{y}_k^-) \tag{53}$$

and

$$\beta_k^- = \Big(\frac{\bar{y}_k^{2-} - \frac{1}{p_k^-}(\bar{y}_k^-)^2}{-\alpha_k^-\gamma_k^- + p_k^- - \frac{(\gamma_k^-)^2}{p_k^-}}\Big)^{1/2}, \tag{54}$$

where $\gamma_k^- \triangleq \frac{1}{\sqrt{2\pi}}e^{-\frac{(\alpha_k^-)^2}{2}}$.

Set

$$b_k^+ \triangleq \alpha_k^+\beta_k^+ - h_k^+, \quad b_k^- = \alpha_k^-\beta_k^- - h_k^-. \tag{55}$$

Estimates for $\|\vartheta\|, c^+, d^+$ and c^-, d^-

We apply the kernel function approach used in (Greblicki and Pawlak, 1989; Greblicki, 2002) and also in (Chen, 2004b).

Define the kernel function

$$w_k \triangleq k^{-2\epsilon} e^{-k^{4\epsilon} u_k^2}, \quad \epsilon \in (0, \frac{1}{4}) \tag{56}$$

where $u_k \in \mathcal{N}(0,1)$ is the system input.

Let us recursively compute

$$\mu_k = (1 - \frac{1}{k})\mu_{k-1} + \frac{1}{k} w_k y_k I_{[y_k > 0]}, \tag{57}$$

where w_k is given by (56) and y_k is the system output, and find the root of the following algebraic equations with respect to x:

$$\mu_k + \frac{h_k^+}{\sqrt{2}}(1 - \Phi(\alpha_k^+ x)) - \frac{\beta_k^+}{2\sqrt{\pi}x} e^{-\frac{(\alpha_k^+)^2 x^2}{2}} = 0. \tag{58}$$

It can be shown that the solution to (58) uniquely exists for all sufficiently large k. Denote by x_k the solution of (58), and the estimate $\|\vartheta\|_k$ for $\|\vartheta\|$ is defined by

$$\|\vartheta\|_k = \frac{1}{\sqrt{x_k^2 - 1}}. \tag{59}$$

Define

$$c_k^+ \triangleq \frac{\beta_k^+}{(1 + \|\vartheta\|_k^2)^{1/2}}, \quad c_k^- \triangleq \frac{\beta_k^-}{(1 + \|\vartheta\|_k^2)^{1/2}} \tag{60}$$

$$d_k^+ \triangleq \alpha_k^+ (1 + \|\vartheta\|_k^2)^{1/2}, \quad d_k^- \triangleq \alpha_k^- (1 + \|\vartheta\|_k^2)^{1/2}. \tag{61}$$

Estimate for ϑ

The nonlinearity $f(\cdot)$ has been estimated, it remains to estimate $C(z)$ in the linear subsystem.

Define

$$\widehat{v}_k \triangleq \begin{cases} \frac{1}{\bar{c}_k^+}(h_k^+ + y_k), & \text{if } y_k > 0 \\ 0, & \text{if } y_k = 0 \\ \frac{1}{\bar{c}_k^-}(y_k - h_k^-), & \text{if } y_k < 0 \end{cases} \tag{62}$$

where

$$\bar{c}_k^+ = c_k^+ \vee \frac{1}{k}, \quad \bar{c}_k^- = c_k^- \vee \frac{1}{k}.$$

Further, define

$$\bar{\varphi}_{k-1} \triangleq \varphi_{k-1} I_{[y_k \neq 0]}, \tag{63}$$

$$z_k \triangleq (\widehat{v}_k - u_k) I_{[y_k \neq 0]} = (\vartheta^T \varphi_{k-1} + \epsilon_k) I_{[y_k \neq 0]}. \tag{64}$$

The unknown ϑ is estimated by the least squares algorithm:

$$\vartheta_{k+1} = \vartheta_k + a_k P_k \bar{\varphi}_k (z_{k+1} - \vartheta_k^T \bar{\varphi}_k) \tag{65}$$

$$P_{k+1} = P_k - a_k P_k \bar{\varphi}_k \bar{\varphi}_k^T P_k, \tag{66}$$

$$a_k = (1 + \bar{\varphi}_k^T P_k \bar{\varphi}_k)^{-1}$$

with arbitrary ϑ_0 and $P_0 > 0$.

Theorem 2. For the Wiener system described by Figure 2 with nonlinearity and linear system given by (1), and (5) and (6), respectively, if the system input $\{u_k\}$ is iid and $u_k \in \mathcal{N}(0,1)$, then α_k^+, α_k^- given by (40)-(42), $h_k^+, h_k^-, \beta_k^+, \beta_k^-$ given by (43)-(54), $c_k^+, c_k^-, d_k^+, d_k^-$ given by (60)-(61), and ϑ_k defined by (62)-(66) are strongly consistent.

The central part of the proof is to show

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} w_k y_k I_{[y_k > 0]} = \frac{-h^+}{\sqrt{2}} \Big(1 - \Phi(\frac{d^+}{\|\vartheta\|})\Big) + \frac{c^+ \|\vartheta\|}{2\sqrt{\pi}} e^{-\frac{1}{2}(\frac{d^+}{\|\vartheta\|})^2} \quad \text{a.s.} \tag{67}$$

4. NUMERICAL EXAMPLES

We now give numerical examples to demonstrate the strong consistency of the algorithms proposed in Section 3.

Matlab is used to generate the iid sequences $\{u_k\}$ for both Hammerstein and Wiener systems, and to carry out all computation. In all figures the solid lines represent the true values and the dotted lines are their estimates.

For the Hammerstein system let the parameters of $f(\cdot)$ and the ARMA subsystem take the following values:

$$a_1 = 1.5,\ a_2 = 0.6,\ b_1 = 0.15,\ b_2 = -0.3,\ b_3 = 0.45$$
$$d^+ = 1, \quad c^+ = 0.7, \quad b^+ = 1.6$$
$$d^- = 1.2, \quad c^- = 0.6, \quad b^- = 1.7, \quad \text{and} \quad D = 1.5.$$

It is clear that the polynomials

$$A(z) = 1 + 1.5z + 0.6z^2$$
$$B(z) = 1 + 0.15z - 0.3z^2 + 0.45z^3$$

meet all requirements listed in A1-A3.

Fig. 3 demonstrates the estimates for a_1 and a_2, while Fig. 4 for b_1, b_2, and b_3. Fig. 5 and Fig 6 give estimates for c^+, b^+, d^+ and c^-, b^-, d^-, respectively. It is seen that all estimates converge to their true values.

Fig. 3.

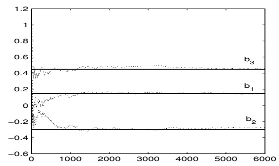

Fig. 4.

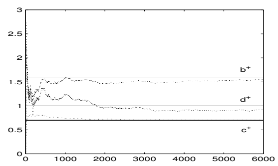

Fig. 5.

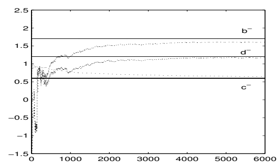

Fig. 6.

For the Wiener system let the parameters of $f(v)$ and the coefficients in (6) be as follows

$$d^+ = 0.65, \quad d^- = 0.86, \quad c^+ = 0.8,$$
$$c^- = 0.6, \quad b^+ = 1.15, \quad b^- = 1.36$$
$$r = 2, \quad c_1 = 1.2, \quad \text{and} \quad c_2 = 0.9.$$

The parameter ϵ in the kernel function (56) should be small, because otherwise w_k would tend to zero too fast so that the new data would be negligible. Here we take $\epsilon = \frac{1}{11000}$.

The simulation results are shown in Figs 7, 8, 9, and 10, from which we see that all estimates asymptotically converge to the the true values, fluctuating at beginning .

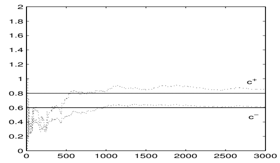

Fig. 7.

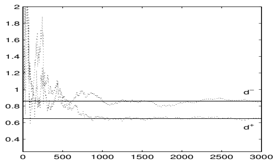

Fig. 8.

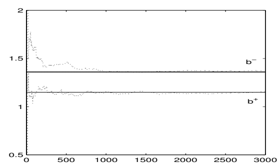

Fig. 9.

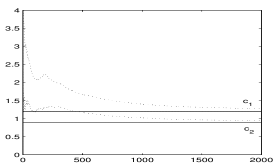

Fig. 10.

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