ADAPTIVE CONTROL BASED ON NEURAL OBSERVER FOR NONLINEAR SYSTEMS

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Abstract: A neural network (NN) based adaptive output feedback controller is proposed for a class of nonlinear systems. In this control scheme, the adaptive output feedback NN controller is proposed by using an observer to estimate the states of the system. The weights of the neural network can be adjusted in terms of Lyapnuov's stability criterion. The proposed controller can be applied to nonlinear systems without exactly available knowledge of system dynamics.

Keywords: Adaptive control, neural network, nonlinear system, output feedback.

1. INTRODUCTION

It is known that output feedback control for nonlinear systems is a very attractive topic in practical applications such as robot manipulators control, chemical process control and smart actuator applications etc. However, the applications of this approach are quite limited because it relies on the exact knowledge of plant. In the past decades, research on neural network (NN) based output feedback control has become very active. The powerful capability of neural computing makes it possible to be a good candidate for implementing real-time adaptive control for nonlinear dynamical systems. Calise et al (2001) developed a direct adaptive feedback controller of highly uncertain nonlinear systems without states estimation. In their scheme, feedback linearizatoin coupled with an on-line NN, whose weights are updated by a linear combination of measured tracking error, was employed to compensate for dynamical model errors. Kim and Lewis (1999) proposed a robust NN output feedback scheme for the motion control of robot manipulators and a NN observer is used to estimate the joint velocities. Hovakimyan et al (2002) designed an adaptive output feedback controller for non-affine minimum phase nonlinear systems by using a three-layer NN adapted by output of a linear tracking error observer. Ge et al (1999) as well as Seshagir et al (2000) applied the output feedback control schemes to continuous-time nonlinear plants. Jankovic (1997) and Choil et al (2000) developed the output feedback controllers combined with high-gain observer and back-stepping strategy.

In the paper, a neural network based adaptive output feedback controller is developed for a class of SISO nonlinear systems with high-gain observer for the estimation of unavailable states. The overall system is proved to be ultimately bounded and the tracking error converges to a small neighbourhood of origin.

2. PROBLEM STATEMENT

Consider a single-input single-output (SISO) nonlinear system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \cdots\cdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = f(x,u) \\ y = x_1 \end{cases} \tag{1}$$

where $x = [x_1, x_2 \cdots x_n]^T \in R^n$ is the vector of system states, $u, y \in R$ are respectively the system control input and measured output. $f(x,u)$ is a smooth nonlinear function which satisfies $\frac{\partial f(x,u)}{\partial u} \neq 0$.

The control objective is to find a control, u, such that the system output tracks the prescribed trajectory, $y_d(t)$, with an acceptable accuracy. Define desired state and tracking error as

$$x_d = [y_d, \dot{y}_d, \ddot{y}_d \cdots, y_d^{(n-1)}]^T, \tag{2}$$

$$e(t) = x - x_d \tag{3}$$

Assumption 1: $\left\| \left(x_d^T, y_d^{(n)} \right) \right\| \le d$ with a known positive constant d.

If only the output y is measurable and the rest states of the system are not available for feedback, it needs to estimate x_2, x_3, ..., x_n for the implementation of feedback control. The following high-gain observer is introduced to estimate the plant state x.

Lemma 1 : Suppose the function *y(t)* and its first *n* derivative are bounded. Consider the following linear system

$$\begin{cases} \rho \dot{z}_1 = z_2 \\ \rho \dot{z}_2 = z_3 \\ \vdots \\ \rho \dot{z}_{n-1} = z_n \\ \rho \dot{z}_n = -\beta_1 z_n - \beta_2 z_{n-1} - \cdots - \beta_{n-1} z_2 - z_1 + y(t) \end{cases} \tag{4}$$

where the parameters $\beta_1 \cdots \beta_{n-1}$ are chosen so that $s^n + \beta_1 s^{n-1} + \cdots + \beta_{n-1} s + 1$ is Hurwitz polynomial. Then, there exist positive constants $h_i, i = 2, 3 \cdots n$ and T such that for all $t > T$ it leads to

$$\frac{z_{i+1}}{\rho^i} - y^{(i)} = -\rho \psi^{(i+1)}, \quad \left| \frac{z_{i+1}}{\rho^i} - y^{(i)} \right| \le \rho h_{i+1} \quad i = 1, \cdots, n-1 \tag{5}$$

where ρ is a positive constant, $\psi = z_n + \beta_1 z_{n-1} + \cdots + \beta_{n-1} z_1$ and $\left| \psi^{(i)} \right| \le h_i$. $\psi^{(i)}$ denotes the i^{th} derivative of ψ. (Behatsh, 1990)

The estimate $\hat{x}$ of plant state x is defined as

$$\hat{x} = \left[x_1, \frac{z_2}{\rho}, \frac{z_3}{\rho^2}, \cdots, \frac{z_n}{\rho^{n-1}} \right]^T . \tag{6}$$

Also, define

$$e_o = \hat{x} - x, \tag{7}$$

$$e_d = \hat{x} - x_d, \tag{8}$$

From (3), (7) and (8), we can obtain that

$$e_d = e + e_o \tag{9}$$

3. OUTPUT FEEDBACK CONTROL

From (1), the plant can be described by

$$\dot{x}_n = \delta + \tilde{f}(x,u) \tag{10}$$

$$\delta = \hat{f}(y,u) \tag{11}$$

$$u = \hat{f}^{-1}(y, \delta) \tag{12}$$

$$\tilde{f}(x,u) = f(x,u) - \hat{f}(y,u) . \tag{13}$$

and $\hat{f}(y,u)$ is invertible with respect to u and satisfies (Hovakimyan et al, 2002)

1. $\operatorname{sgn} \frac{\partial f}{\partial u} = \operatorname{sgn} \frac{\partial \hat{f}}{\partial u}$; 2. $\left| \frac{\partial \hat{f}}{\partial u} \right| > \frac{1}{2} \left| \frac{\partial f}{\partial u} \right| > 0$.

The pseudo-control signal δ is designed as

$$\delta = -K e_d - \delta_{ad} + \delta_r + y_d^{(n)} \tag{14}$$

where $K = [k_1, k_2 \cdots k_n]$ is a vector of gains chosen as the cefficients of Hurwitz polynomial; δ_{ad} is designed to cancel $\tilde{f}$; δ_r is the term for robust design that will be specified in the sequel.

From (9), (10) can be expressed as

$$\dot{e}_n = -Ke + \tilde{f}(x,u) - \delta_{ad} + \delta_r - K e_o . \tag{15}$$

A three-layer NN, which is consisted of n_2 hidden neurons and n_3 output neurons with linear activation function, can be written by vectors i.e.

$$y = W^T \sigma \left(V^T x_{nn} \right) . \tag{16}$$

where $V = [V_{i,j}]$ $i = 1, 2, \cdots n_1$; $j = 0, 1 \cdots n_2$; and $W = [W_{j,k}]$, $k = 1, \cdots n_3$ are interconnecting weights $\sigma(.)$ are the sigmoidal function which satisfies $\|\sigma(.)\| \le \sqrt{n_2}$.

It is well known that any smooth function can be approximated on a compact set using a three-layer NN with appropriate weights. This implies that function error $\tilde{f} \in C(\Omega)$, with Ω compact subset, can be written as

$$\tilde{f}(x,u) = W^T \sigma \left(V^T x_{nn} \right) + \varepsilon(x_{nn}) \tag{17}$$

where $\varepsilon \left(x_{nn} \right)$ is the NN reconstruction error and $\|\varepsilon(x_{nn})\| \le \varepsilon_N$ with a positive constant ε_N ; W and V are optimal weights that minimize $\varepsilon(x_{nn})$ for all $x_{nn} \in \Omega$.

From the equations shown in (12), (13), (14) and (34) that will be presented later, the network input can be chosen as

$$x_{nn} = \left[1, x^T, y_d^{(n)}, e_d^T, \delta_{ad}, \left\| \hat{Z} \right\|_F \right]^T . \tag{18}$$

Define the following notation for convenient

$$Z = \begin{bmatrix} W & 0 \\ 0 & V \end{bmatrix}, \ \hat{Z} = \begin{bmatrix} \hat{W} & 0 \\ 0 & \hat{V} \end{bmatrix}, \text{ and } \tilde{Z} = \begin{bmatrix} \tilde{W} & 0 \\ 0 & \tilde{V} \end{bmatrix} \tag{19}$$

where $\hat{W}, \hat{V}$ are the estimate of W, V . And $\tilde{W} = W - \hat{W}, \tilde{V} = V - \hat{V}$.

Assumption 2: The optimal weights W and V are bounded by the pre-specified positive values W_P, V_P, Z_P so that $\|W\|_F \le W_P$ and $\|V\|_F \le V_P$, and $\|Z\|_F \le Z_P$, where $\|.\|_F$ represents Frobenius norm.

δ_{ad} is chosen as output of a three-layer network

$$\delta_{ad} = \hat{W}^T \sigma(\hat{V}^T \hat{x}_{nn}) ; \tag{20}$$

and the NN input is chosen as

$$\hat{x}_{nn} = \left[1, \hat{x}^T, y_d^{(n)}, e_d^T, \delta_{ad}, \left\| \hat{Z} \right\|_F \right]^T . \tag{21}$$

From (8) and (20), we can obtain that

$$\|\hat{x}_{nn}\| \le 1 + \|x_d\| + \|e_d\| + \left\| y_d^{(n)} \right\| + \|e_d\| + \left\| \hat{W} \right\|_F \left\| \sigma \left(\hat{V}^T \hat{x} \right) \right\| + \left\| \hat{Z} \right\|_F .$$

Combining with *Assumption 1-2*, one can obtain that $\hat{x}_{nn}$ is bounded by

$$\hat{x}_{nn} \le c_0 + c_1 \|e_d\| + c_2 \left\| \tilde{Z} \right\|_F \tag{22}$$

with the computable positive constants $c_0, c_1, c_2 > 0$. Using (20), (15) can be expressed as

$$\begin{aligned}\dot{e}_n &= -Ke + W^T\sigma(V^T x_{nn}) - \hat{W}^T\sigma(\hat{V}^T\hat{x}_{nn}) \\ &+\delta_r - Ke_o + \varepsilon(x_{nn})\end{aligned} \quad (23)$$

The error dynamics of the closed-loop system can be written as

$$\dot{e} = Ae + b\begin{bmatrix} W^T\sigma(V^T x_{nn}) - \hat{W}^T\sigma(\hat{V}^T\hat{x}_{nn}) \\ +\delta_r - Ke_o + \varepsilon(x_{nn})\end{bmatrix} \quad (24)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ -k_1 & -k_2 & -k_3 & \cdots & -k_n \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

A is an asymptotically stable matrix. Therefore, given any positive definite symmetric matrix Q, there exists a unique positive definite symmetric matrix P such that

$$A^T P + AP = -Q\,. \quad (25)$$

From (5) and (6), we can obtain that, for a known $c_3 > 0$

$$\|e_o\| \le c_3\,. \quad (26)$$

And the following inequality holds

$$\begin{aligned}&\left\|W^T\sigma(V^T x_{nn}) - \hat{W}^T\sigma(\hat{V}^T\hat{x}_{nn}) - Ke_o + \varepsilon(x_{nn})\right\| \\ &\le \|W\|_F\left\|\sigma(V^T x_{nn})\right\| + \left\|\hat{W}\right\|_F\left\|\sigma(\hat{V}^T x_{nn})\right\| \\ &+K\|e_o\| + \|\varepsilon(x_{nn})\| \le c_4 + c_5\left\|\tilde{Z}\right\|_F\end{aligned} \quad (27)$$

where c_4, c_5 are computable positive constants.

The Taylor series expansion of $\sigma\left(V^T\hat{x}_{nn}\right)$ for the given $\hat{x}_{nn}$ can be described by

$$\sigma = \hat{\sigma} + \hat{\sigma}'\tilde{V}^T\hat{x}_{nn} + o\left(\tilde{V}^T\hat{x}_{nn}\right)^2 \quad (28)$$

where $\sigma := \sigma(V^T\hat{x}_{nn}), \hat{\sigma} := \sigma(\hat{V}^T\hat{x}_{nn})$, $\hat{\sigma}' := \sigma'(\hat{V}^T\hat{x}_{nn})$; σ' is the Jacobian matrix of $\sigma(\hat{V}^T\hat{x}_{nn})$.

Together with (22), the higher-order terms $o\left(\tilde{V}^T\hat{x}_{nn}\right)^2$ is bounded by

$$\begin{aligned}&\left\|o\left(\tilde{V}^T\hat{x}_{nn}\right)^2\right\| = \left\|\sigma - \hat{\sigma} - \hat{\sigma}'\tilde{V}^T\hat{x}_{nn}\right\| \\ &\le c_6 + c_7\left\|\tilde{V}\right\|_F + c_8\|e_d\|\left\|\tilde{V}\right\|_F + c_9\left\|\tilde{V}\right\|_F\left\|\tilde{Z}\right\|_F\end{aligned} \quad (29)$$

with some computable and positive constant c_6, c_7, c_8, c_9.

For the stability proof, the following representation is also considered

$$\begin{aligned}&W^T\sigma(V^T x_{nn}) - \hat{W}^T\hat{\sigma} \\ &= \tilde{W}^T\left(\hat{\sigma} - \hat{\sigma}'\tilde{V}^T\hat{x}_{nn}\right) + \hat{W}^T\hat{\sigma}'\tilde{V}^T\hat{x}_{nn} + \omega_1 + \omega_2\end{aligned} \quad (30)$$

$$\omega_1 = W^T\sigma(V^T x_{nn}) - W^T\sigma(V^T\hat{x}_{nn}) \quad (31)$$

$$\omega_2 = \tilde{W}^T\hat{\sigma}'V^T\hat{x}_{nn} + W^T o(\tilde{V}^T\hat{x}_{nn})^2\,. \quad (32)$$

Combining (22), (29), (31) and (32), one can obtain

$$\begin{aligned}&\left\|\omega_1 + \omega_2 - Ke_0 + \varepsilon\right\| \\ &\le l_0 + l_1\left\|\tilde{Z}\right\|_F + l_2\|e_d\|\left\|\tilde{Z}\right\|_F + l_3\left\|\tilde{Z}\right\|_F^2\end{aligned} \quad (33)$$

for some specified positive constant l_0, l_1, l_2, l_3.

Theorem 1: Suppose that assumptions 1 and 2 hold. Take the control law given by (12) and (14) with the term for robust design, thus

$$\begin{aligned}\delta_r &= \begin{cases} -K_\delta\frac{\varsigma}{\|\varsigma\|}\left(\left\|\hat{Z}\right\|_F + Z_p\right)\|e_d\| & \|\varsigma\| \ne 0 \\ 0 & \|\varsigma\| = 0\end{cases}, \\ \varsigma &= e_d^T Pb\end{aligned} \quad (34)$$

and $K_\delta > l_2$ where l_2 the is the constant in (33). Let the weight adaptation laws be provided by

$$\dot{\hat{W}} = F\left(\hat{\sigma} - \hat{\sigma}'\hat{V}^T\hat{x}_{nn}\right)\varsigma - F\left(\kappa_0\|\varsigma\| + \kappa_1\right)\hat{W}$$

$$\dot{\hat{V}} = R\hat{x}_{nn}\varsigma\hat{W}^T\hat{\sigma}' - R\left(\kappa_0\|\varsigma\| + \kappa_1\right)\hat{V}$$

with any constant matrices $F = F^T > 0$, $R = R^T > 0$ and $\kappa_0 > l_3$, $\kappa_1 > \left(c_3 l_5\|Pb\|\right)^2/2$, where P satisfies (25) for the matrix Q with the minimal eigenvalue $\lambda_{\min}(Q) > 1$; l_5 is defined by (39). Then, if the initial errors belong to the compact set Ω_r defined in (49), the NN controller guarantees that all the signals in the closed-loop system are all ultimately bounded.

Proof: Define the Lyapunov function candidate

$$L = e^T Pe + tr\tilde{W}^T F^{-1}\tilde{W} + tr\tilde{V}^T R^{-1}\tilde{V}\,. \quad (35)$$

Considering (24), the derivative of L with respect to t yields

$$\begin{aligned}\dot{L} &= e^T(A^T P + PA)e + 2e^T Pb[W^T\sigma(V^T x_{nn}) \\ &-\hat{W}^T\hat{\sigma} + \delta_r - Ke_o + \varepsilon] + 2tr\tilde{W}^T F^{-1}\dot{\tilde{W}} \\ &+2tr\tilde{V}^T R^{-1}\dot{\tilde{V}}\end{aligned} \quad .(36)$$

Using (9), (25) and (30), (36) can be written as

$$\begin{aligned}&\dot{L} = -e^T Qe \\ &+2e_d^T Pb\begin{bmatrix}\tilde{W}^T\left(\hat{\sigma} - \hat{\sigma}'\tilde{V}^T\hat{x}_{nn}\right) + \hat{W}^T\hat{\sigma}'\tilde{V}^T\hat{x}_{nn} \\ +\omega_1 + \omega_2 + \delta_r - Ke_o + \varepsilon\end{bmatrix}. \\ &-2e_o{}^T Pb\left[W^T\sigma(V^T x_{nn}) - \hat{W}^T\hat{\sigma} + \delta_r - Ke_o + \varepsilon\right] \\ &+2tr\tilde{W}^T F^{-1}\dot{\tilde{W}} + 2tr\tilde{V}^T R^{-1}\dot{\tilde{V}}\end{aligned}$$

Then, we have

$$\begin{aligned}\dot{L} &\le -e^T Qe + 2\varsigma\left(\omega_1 + \omega_2 + \delta_r - Ke_o + \varepsilon\right) \\ &+2\left\|e_o{}^T Pb\right\|\left(c_4 + c_5\left\|\tilde{Z}\right\|_F + \|\delta_r\|\right) \\ &+2\left(\kappa_0\|\varsigma\| + \kappa_1\right)\left(tr\tilde{W}^T\hat{W} + tr\tilde{V}^T\hat{V}\right)\end{aligned} \quad (37)$$

From (33), we have

$$\begin{aligned}\dot{L} &\le -e^T Qe + 2\|\varsigma\|(l_0 + l_1\left\|\tilde{Z}\right\|_F + l_2\|e_d\|\left\|\tilde{Z}\right\|_F \\ &+l_3\left\|\tilde{Z}\right\|_F^2) + 2\varsigma\delta_r + 2\left\|e_o{}^T Pb\right\|(c_4 + c_5\left\|\tilde{Z}\right\|_F \\ &+\|\delta_r\|) + 2\left(\kappa_0\|\varsigma\| + \kappa_1\right)tr\tilde{Z}^T\hat{Z}\end{aligned} \quad .(38)$$

Using (34), δ_r is bounded for $l_4, l_5 > 0$

$$\|\delta_r\| \le l_4 \|e_d\| + l_5 \|e_d\| \|\tilde{Z}\|_F . \quad (39)$$

Combining (34) and (39), (38)becomes

$$\begin{aligned}\dot{L} \le &-e^T Qe + 2\|\varsigma\|[l_0 + l_1 \|\tilde{Z}\|_F - (K_\delta - l_2)(\|\hat{Z}\|_F \\ &+ Z_P)\|e_d\| + l_3 \|\tilde{Z}\|_F^2] + 2\|e_o{}^T Pb\|(c_4 + c_5 \|\tilde{Z}\|_F \\ &+ l_4 \|e_d\| + l_5 \|e_d\|\|\tilde{Z}\|_F) + 2(\kappa_0 \|\varsigma\| + \kappa_1) tr\tilde{Z}^T \hat{Z}\end{aligned} \quad (40)$$

Considering (26) and $K_\delta > l_2$, (40) can be rewritten as

$$\begin{aligned}\dot{L} \le &-\lambda_{\min}(Q)\|e\|^2 + 2\|\varsigma\|\left(l_0 + l_1\|\tilde{Z}\|_F + l_3\|\tilde{Z}\|_F^2\right) \\ &+ 2c_3c_4\|Pb\| + 2c_3c_5\|Pb\|\|\tilde{Z}\|_F + 2c_3l_4\|Pb\|\|e_d\| \\ &+ 2c_3l_5\|Pb\|\|e_d\|\|\tilde{Z}\|_F + 2(\kappa_0\|\varsigma\| + \kappa_1) tr\tilde{Z}^T\hat{Z}\end{aligned} \quad (41)$$

For $tr\tilde{Z}^T\hat{Z} \le Z_P\|\tilde{Z}\|_F - \|\tilde{Z}\|_F^2$ and $\|e_d\| \le \|e\| + c_3$, we obtain

$$\begin{aligned}\dot{L} \le &-[\lambda_{\min}(Q) - 1]\|e\|^2 - \left(\|e\| - c_3l_5\|Pb\|\|\tilde{Z}\|_F\right)^2 \\ &- 2(\kappa_0 - l_3)\|\varsigma\|\left(\|\tilde{Z}\|_F - \frac{l_1 + \kappa_0 Z_P}{2(\kappa_0 - l_3)}\right)^2 + 2\left(\frac{(l_1 + \kappa_0 Z_P)^2}{4(\kappa_0 - l_3)} + l_0\right)\|\varsigma\| \\ &+ 2c_3l_4\|Pb\|\|e\| + 2\left(c_3c_5\|Pb\| + c_3^2 l_5\|Pb\| + \kappa_1 Z_p\right)\|\tilde{Z}\|_F \\ &- \left[2\kappa_1 - (c_3l_5\|Pb\|)^2\right]\|\tilde{Z}\|_F^2 + 2\left(c_3c_4 + c_3^2 l_4\right)\|Pb\|\end{aligned} \quad (42)$$

Combining $\|\varsigma\| \le c_3\|Pb\| + \|Pb\|\|e\|$ with $\kappa_0 > l_3$, (42) becomes

$$\begin{aligned}\dot{L} \le &-[\lambda_{\min}(Q) - 1]\left[\|e\| - \left(\frac{(l_1 + \kappa_0 Z_P)^2}{4(\kappa_0 - l_3)} + l_0 + c_3 l_4\right)\frac{\|Pb\|}{\lambda_{\min}(Q) - 1}\right]^2 \\ &+ \frac{1}{\lambda_{\min}(Q) - 1}\left[\left(\frac{(l_1 + \kappa_0 Z_P)^2}{4(\kappa_0 - l_3)} + l_0 + c_3 l_4\right)\|Pb\|\right]^2 \\ &- \left[2\kappa_1 - (c_3 l_5\|Pb\|)^2\right]\left(\|\tilde{Z}\|_F - \frac{c_3c_5\|Pb\| + c_3^2 l_5\|Pb\| + \kappa_1 Z_p}{2\kappa_1 - (c_3 l_5\|Pb\|)^2}\right)^2 \\ &+ \frac{\left(c_3c_5\|Pb\| + c_3^2 l_5\|Pb\| + \kappa_1 Z_p\right)^2}{2\kappa_1 - (c_3 l_5\|Pb\|)^2} \\ &+ c_3\left(\frac{(l_1 + \kappa_0 Z_P)^2}{2(\kappa_0 - l_3)} + 2l_0\right)\|Pb\| + 2\left(c_3c_4 + c_3^2 l_4\right)\|Pb\|\end{aligned} \quad (43)$$

Define

$$\begin{aligned}\Phi = &\frac{1}{\lambda_{\min}(Q) - 1}\left[\left(\frac{(l_1 + \kappa_0 Z_P)^2}{4(\kappa_0 - l_3)} + l_0 + c_3 l_4\right)\|Pb\|\right]^2 \\ &+ \frac{\left(c_3c_5\|Pb\| + c_3^2 l_5\|Pb\| + \kappa_1 Z_p\right)^2}{2\kappa_1 - (c_3 l_5\|Pb\|)^2} + c_3\left(\frac{(l_1 + \kappa_0 Z_P)^2}{2(\kappa_0 - l_3)} + 2l_0\right)\|Pb\| \\ &+ 2\left(c_3c_4 + c_3^2 l_4\right)\|Pb\|\end{aligned} , \quad (44)$$

$$\mu_0 = \left(\frac{(l_1 + \kappa_0 Z_P)^2}{4(\kappa_0 - l_3)} + l_0 + c_3 l_4\right)\frac{\|Pb\|}{\lambda_{\min}(Q) - 1} , \quad (45)$$

$$\mu_1 = \frac{\left(c_3c_5\|Pb\| + c_3^2 l_5\|Pb\| + \kappa_1 Z_p\right)}{2\kappa_1 - (c_3 l_5\|Pb\|)^2} . \quad (46)$$

Thus, $\dot{L} < 0$ as long as one of the following conditions holds

$$\begin{aligned}&1.\ \|e\| > \mu_0 + \sqrt{\frac{\Phi}{\lambda_{\min}(Q) - 1}} \\ &2.\ \|\tilde{Z}\|_F > \mu_1 + \sqrt{\frac{\Phi}{2\kappa_1 - (c_3 l_5\|Pb\|)^2}}\end{aligned} . \quad (47)$$

The above results are valid provided (17) holds on the compact set $x_{nn} \in \Omega$ for all t>0. From (18), x_{nn} can be considered as a function of $e, \tilde{Z}, x_d, z, y_d^{(n)}, Z$, i.e.

$$x_{nn} = F(e, \tilde{Z}, x_d, z, y_d^n, Z) . \quad (48)$$

According to the assumption 1, 2 and (4), there exists the corresponding compact set

$$\Omega_r = \left\{(e, \tilde{Z}) \middle| \|(e, \tilde{Z})\| \le r\right\} , \quad (49)$$

which leads to $x_{nn} \in \Omega$ when $\forall (e, \tilde{Z}) \in \Omega_r$. Define a compact set

$$\Omega_\alpha = \left\{(e, \tilde{Z}) \middle| \begin{aligned}&\|e\| \le \mu_0 + \sqrt{\frac{\Phi}{\lambda_{\min}(Q) - 1}} \\ &\|\tilde{Z}\|_F \le \mu_1 + \sqrt{\frac{\Phi}{2\kappa_1 - (c_3 l_5\|Pb\|)^2}}\end{aligned}\right\} . \quad (50)$$

From (35), one obtains that

$$\begin{aligned}L_{\max} = &\max_{\Omega_\alpha}(L) = \lambda_{\max}(P)\left(\mu_0 + \sqrt{\frac{\Phi}{\lambda_{\min}(Q) - 1}}\right)^2 \\ &+ \lambda_{\max}(\Gamma)\left(\mu_1 + \sqrt{\frac{\Phi}{2\kappa_1 - (c_3 l_5\|Pb\|)^2}}\right)^2\end{aligned} \quad (51)$$

where $\lambda_{\max}(P), \lambda_{\max}(\Gamma)$ are the maximal eigenvalues of P and Γ, respectively; and $\Gamma = \begin{bmatrix} F^{-1} & 0 \\ 0 & R^{-1} \end{bmatrix}$.

For any $(e, \tilde{Z}) \in \left\{(e, \tilde{Z}) \middle| \|(e, \tilde{Z})\| = r\right\}$, we can obtain that $L \ge \lambda_{\min} r^2$, where $\lambda_{\min}$ is the minimal eigenvalue of P and Γ. The requirement of $\Omega_\beta = \left\{(e, \tilde{Z}) \middle| L \le L_{\max}\right\} \subset \Omega_r$ leads to

$$r \ge \sqrt{L_{\max}/\lambda_{\min}} \quad (52)$$

Then $e, \tilde{Z}$ are semi-globally uniformly ultimately bounded for $\forall\left\{e(0), \tilde{Z}(0)\right\} \in \Omega_r$ as long as Ω is sufficient large such that (52) holds. From the assumption and structure of observer, we can conclude that all the signals of the closed-loop are all semi-globally, uniformly and ultimately bounded.

4. SIMULATION

Consider the following nonlinear plant

$$\begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = (1 + x_1^2 + x_2^2) + u + \sin u + \tanh u^2 \\ y = x_1\end{cases} . \quad (53)$$

The tracking objective is to make the output, *y(t)*, follow a desired output, $y_d(t) = \sin 0.5t + e^{-0.1t}\sin t$. The initial states are set to $x(0) = [0.6,\ 1.6]^T$. Then, the corresponding observer for x_2 is designed as

$$\begin{cases} \rho \dot{z}_1 = z_2 \\ \rho \dot{z}_2 = -\beta_1 z_2 - z_1 + y(t) \end{cases} . \quad (54)$$

A three-layer NN with n_1=8, n_2=15, n_3=1 is utilized. The hidden-layer activation functions are chosen as $\sigma(x) = \frac{1}{1+e^{-x}}$ .The weights $\hat{W}, \hat{V}$ of the neural network are initialized to zeros. The other parameters are respectively chosen as

$F = R = 5I$, $\kappa_0 = \kappa_1 = 0.1$, $K = [1, 2]$, $K_\delta = 1.5$, $Z_P = 5$, $\beta_1 = 2$, $\rho = 0.001$, $z(0) = [0,0]^T$, $P = \begin{bmatrix} 7.5 & 2.5 \\ 2.5 & 2.5 \end{bmatrix}$, $\hat{f}(u, y) = u$.

where I is an identity matrix. In order to avoid the peaking phenomenon, the saturation of the control input $u(t)$ is ± 10 . Fig.1 and Fig. 3 respectively show the tracking results for 50s. The observer error $e_o(t)$ is plotted in Fig. 4. We see that the tracking error and the observer error converge to a small neighborhood of origin. Fig. 5 shows the history of the control input u(t). The norm of weight estimates is also given in Fig.6 to illustrate the boundedness of the NN weight estimates. A PID controller is also designed to make comparison with the NN controller. The integral gain and the proportional gain are respectively chosen as 4.0 and 2.0 which are specified through the off-line optimizing procedure. The control signal is chosen as $u(t)=-5e_2-20\int e_1 dt-20e_1$. The tracking performances are respectively plotted in Fig. 2 and Fig. 3. We can see from simulation results that the performance of the proposed NN controller is better than that of the PID controller.

5. CONCLUSION

A scheme of ouput feedback neural controller is proposed for nonlinear systems with hysteresis. Comparing with previous adaptive controller, the proposed controller can be applicable to more extensively nonlinear systems. In order to handle the case where some of the states are not measurable, a high gain observer to estimate the states of the system. The overall system is proved to be ultimately bounded and the tracking error converges to a small neighborhood of the origin. The theoretical analysis and simulation results show that the proposed controller is rather promising for the control of non-smooth nonlinear systems..

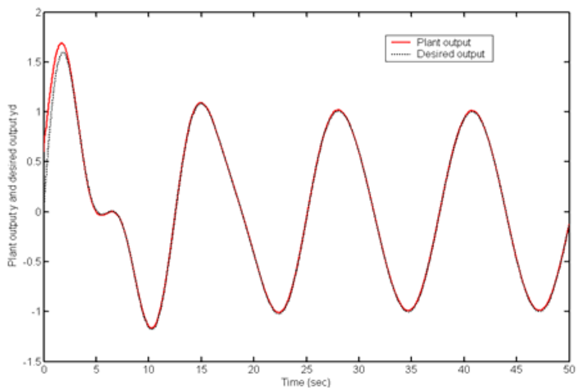

Fig.1 Tracking performance of the NN controller

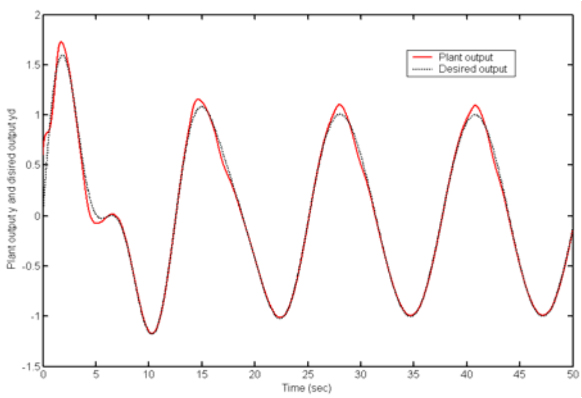

Fig.2 Tracking performance of the PID controller

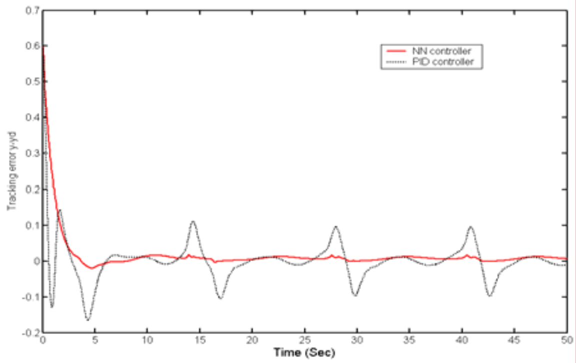

Fig. 3 Tracking error of the NN and PID controller

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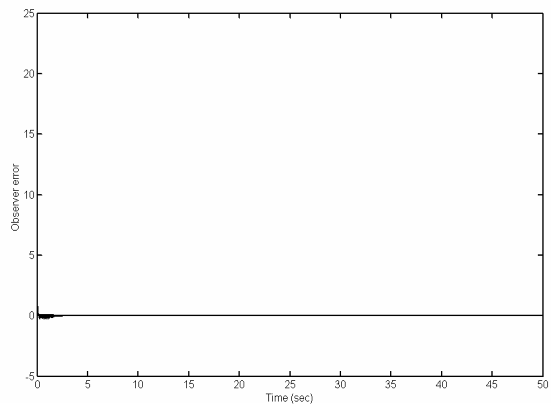

Fig.4 The estimate state error $\hat{x}_2 - x_2$

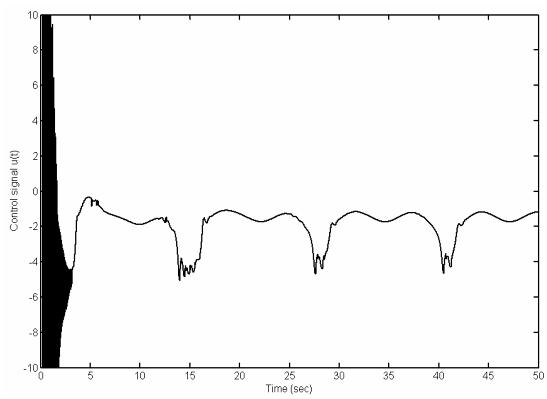

Fig.5 Control input of NN controller

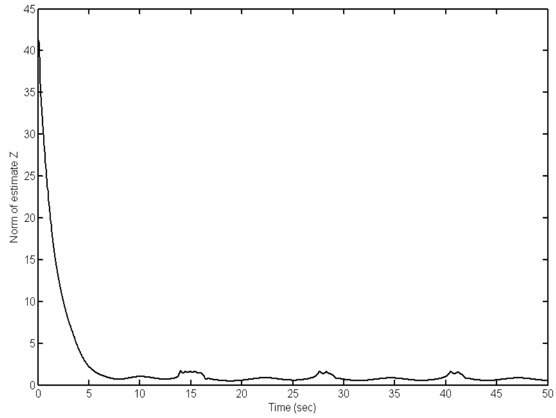

Fig. 6 The Norm of estimate NN weights $\|\hat{Z}\|_F$

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