Robust Decentralized Controller Design for Power Systems Using Convex Optimization Involving LMIs

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Abstract: This paper deals with the application of robust decentralized controller design for power systems using linear matrix inequality (LMI) techniques. In the design, the desired stability of the system is guaranteed while at the same time the tolerable bounds in the uncertainties due to structural changes, nonlinearities and load variations, are maximized. The approach allows the inclusion of additional design constraints such as the size and structure of the gain matrices. The paper also presents a decomposition procedure using the clustering technique of the states, inputs and outputs structure information to compute directly the appropriate diagonal structures of the output gain matrix for practical implementation. The algorithms were implemented on a test system and simulation results for power system stabilizer (PSS) design are presented.

Keywords: Convex optimisation; Decentralized control; Interconnected systems; Robust control.

1. Introduction

The emergence of deregulation and the subsequent restructuring of national electric power systems have necessitated the use of accurate model descriptions and new controller design techniques for guaranteeing end-to-end reliable electric service (Ilic and Zaborszky, 2000; Kundur, 1994). In such environment, in which the number of components of the power system grows significantly, the system tends to be subjected to severe loading conditions and it becomes increasingly difficult to predict the system response to disturbances. With such conditions it is imperative to develop and design new robust control strategies that can guarantee satisfactory system performance against a wide range of disturbances.

This paper presents a robust decentralized controller design for power systems that is formulated in the framework of convex optimization involving LMIs. An interconnection based modeling approach is introduced – an approach which explicitly takes into account the interactions among subsystems and the effects of nonlinearities within each subsystems.

During the last three decades, decentralized controller structure for interconnected power systems which conforms to each subsystem is one of the predominant subjects in this area (Aoki, 1972; Siljak, 1978, 1991). A large number of results concerning robust decentralized stabilization of interconnected power systems in this perspective have been reported (Chapman, et al., 1994; Guo, *et al*., 2000; Jain, *et al*., 1997; Xie, *et al*., 2000). The interesting aspect of these approaches is that of finding bounds on the gains of the local feedback in conjunction with appropriately chosen Lyapunov functions so as to guarantee stability over a wide range of operating points and disturbances. However, the results from these approaches turn out to be conservative and need additional simulations to obtain the optimal gains for the controllers. Another interesting decentralized controller scheme for governor/turbine control, based on the approach outlined in the works

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of Siljak and Stipanovic (2000), was presented in (Siljak, *et al*., 2002). The remarkable feature of this approach is the use of LMIs optimization (Boyd, *et al*., 1994) in addressing the problem of robust stability in the presence of interconnection uncertainties between subsystems.

In line with this perspective, this paper presents a general approach for designing power system controllers, whereby the interactions between subsystems, changes in operating conditions as well as the effects of system nonlinearities can all be taken into account. The application of this approach to a multimachine power system allows a coordinated tuning of controllers that incorporates robustness to changes in the system.

This paper is organized as follows. In Section 2, the robust decentralized controller design problem is formulated in the framework of convex optimization. Then in Section 3, simulation results for PSS structured controllers together with performance indices are given. Finally, the paper is concluded by brief remarks in Section 4. The instrumental theorem in this paper is placed in Appendix-A. In Appendix-B, a brief introduction on nonlinear optimization based parameter tuning of controllers in multimachine systems is presented.

2. ROBUST DECENTRALIZED CONTROL IN CONVEX OPTIMIZATION FRAMEWORK INVOLVING LMIS

2.1 Mathematical Model for Large Scale Systems with Interconnection Terms

Consider a large-scale interconnected system $\mathbf{S}$ composed of N subsystems $\mathbf{S}_i$, $i=1,2,\ldots,N$ described by the following equations:

$$\dot{\mathbf{x}}_i(t)=\mathbf{A}_i\mathbf{x}_i(t)+\mathbf{B}_i\mathbf{u}_i(t)+\sum_{j=1}^{N}[\mathbf{E}_{ij}+\Delta\mathbf{E}_{ij}(t)]\mathbf{x}_j(t)$$
$$\mathbf{y}_i=\mathbf{C}_i\mathbf{x}_i(t) \tag{1}$$

where $\mathbf{x}_i(t)\in\mathbf{R}^{n_i}$ is the state vector, $\mathbf{u}_i(t)\in\mathbf{R}^{m_i}$ is the control variable, $\mathbf{y}_i(t)\in\mathbf{R}^{q_i}$ is the output variable of the subsystem $\mathbf{S}_i$. In general, formulating power system model in the form of (1) involves linearizing the nonlinear system equations for a particular operating condition and then decomposing the corresponding system equations as a sum of two sets of equations. While the former describes the system as a hierarchical interconnection of N subsystems, the latter represents the interactions among the subsystems. The matrices $\mathbf{A}_i$, $\mathbf{B}_i$, $\mathbf{C}_i$ and $\mathbf{E}_{ij}$ are constant matrices of appropriate dimensions conformable to each $\mathbf{S}_i$. Furthermore, the matrix $\mathbf{E}_{ij}$ represents the interconnections and/or interactions among subsystems. The term $\Delta\mathbf{E}_{ij}(\cdot)$ is intentionally included to take into account the effect of any deviation from the given operating condition due to nonlinearities and structural changes in the system.

The interconnection and uncertainty term in (1) can be rewritten in the form of

$$\mathbf{G}_i\mathbf{g}_i(t,\mathbf{x})=\sum_{j=1}^{N}[\mathbf{E}_{ij}+\Delta\mathbf{E}_{ij}]\mathbf{x}_j(t) \tag{2}$$

where $\mathbf{G}_i$ are constant matrices of appropriate dimensions and assume further that the following quadratic constraints hold:

$$\mathbf{g}_i(t,\mathbf{x})^{\mathrm{T}}\mathbf{g}_i(t,\mathbf{x})\le\xi_i^2\mathbf{x}^T\mathbf{H}_i^T\mathbf{H}_i\mathbf{x} \tag{3}$$

where $\xi_i>0$ are parameters related to interconnection uncertainties in the system and $\mathbf{H}_i$ are matrices that reflect the nature of interconnections among subsystems. Moreover, assume that the pairs $(\mathbf{A}_i,\mathbf{B}_i)$ and $(\mathbf{A}_i,\mathbf{C}_i)$ are stabilizable and observable, respectively.

With the assumption of no "overlapping" among $\mathbf{x}_i(t)$, the state variable $\mathbf{x}(t)\in\mathbf{R}^n$ of the overall system is denoted by $\mathbf{x}(t)=[\mathbf{x}_1^T(\cdot),\mathbf{x}_2^T(\cdot),\ldots,\mathbf{x}_N^T(\cdot)]^T$. Thus, the interconnected system $\mathbf{S}$ can then be written in a compact form as

$$\dot{\mathbf{x}}(t)=\mathbf{A}_D\mathbf{x}+\mathbf{B}_D\mathbf{u}+\mathbf{G}_D\mathbf{g}(t,\mathbf{x})$$
$$\mathbf{y}(t)=\mathbf{C}_D\mathbf{x} \tag{4}$$

where $\mathbf{x}\in\mathbf{R}^n$ is the state, $\mathbf{u}\in\mathbf{R}^m$ is the input and $\mathbf{y}\in\mathbf{R}^q$ is the output of the overall system $\mathbf{S}$, and all matrices are constant matrices of appropriate dimensions with $\mathbf{A}_D=\mathrm{diag}\{\mathbf{A}_1,\mathbf{A}_2,\ldots,\mathbf{A}_N\}$, $\mathbf{B}_D=\mathrm{diag}\{\mathbf{B}_1,\mathbf{B}_2,\ldots,\mathbf{B}_N\}$, $\mathbf{C}_D=\mathrm{diag}\{\mathbf{C}_1,\mathbf{C}_2,\ldots,\mathbf{C}_N\}$ and $\mathbf{G}_D=\mathrm{diag}\{\mathbf{G}_1,\mathbf{G}_2,\ldots,\mathbf{G}_N\}$.

The interconnection and uncertainty function $\mathbf{g}=[\mathbf{g}_1^T,\mathbf{g}_2^T,\ldots,\mathbf{g}_N^T]^T$ is bounded as

$$\mathbf{g}^T(t,\mathbf{x})\mathbf{g}(t,\mathbf{x})\le\mathbf{x}^T[\sum_{i=1}^{N}\xi_i^2\mathbf{H}_i^T\mathbf{H}_i]\mathbf{x} \tag{5}$$

In the following, the feedback control law must satisfy decentralized information structure constraints conformable to the subsystems, so that each subsystem is controlled using only its locally available information (Aoki, 1972; Siljak, 1991). This critical requirement implies that the i^{th}-subsystem is controlled by local control law

$$\mathbf{u}_i(\mathbf{x}_i)=\mathbf{K}_i\mathbf{x}_i \tag{6}$$

where $\mathbf{K}_i$ is an $m_i\times n_i$ constant matrix. Thus, the control law for the overall system will be

$$\mathbf{u}(\mathbf{x})=\mathbf{K}_D\mathbf{x} \tag{7}$$

where $\mathbf{K}_D = \text{diag}\{\mathbf{K}_1, \mathbf{K}_2, ..., \mathbf{K}_N\}$ is a constant $m \times n$ matrix with diagonal blocks compatible with those of $\mathbf{A}_D$ and $\mathbf{B}_D$.

The instrumental theorem presented in Appendix-A is used in establishing the robust stability of the closed-loop interconnected system

$$\dot{\mathbf{x}}(t) = [\mathbf{A}_D + \mathbf{B}_D \mathbf{K}_D]\mathbf{x} + \mathbf{G}_D \mathbf{g}(t, \mathbf{x}) \tag{8}$$

via a decentralized robust control strategy (6) under the constraints (5) on the function $\mathbf{g}(t, \mathbf{x})$.
In most practical cases, it is not possible to get all the state variables $\mathbf{x}_i$ of the system. However if only linear combinations called output variables $\mathbf{y}_i$ are used as feedback signals for the system then the decentralized output feedback strategies will have the following form

$$\mathbf{u}_i = \mathbf{F}_i \mathbf{y}_i \equiv \mathbf{F}_i \mathbf{C}_i \mathbf{x} \tag{9}$$

where $\mathbf{F}_i$ is an $m_i \times q_i$ constant output controller matrix. One way of achieving similar effect with (9) as with the local state controllers (6) is to require $\mathbf{B}_i \mathbf{K}_i \equiv \mathbf{B}_i \mathbf{F}_i \mathbf{C}_i$ so that the closed-loop in (8) will be unaltered. By defining the structure of the matrices $\mathbf{Y}_D$ and $\mathbf{L}_D$ in Theorem 1 of the Appendix A as

$$\mathbf{Y}_D \equiv \mathbf{Y}_o + \mathbf{U}\mathbf{Y}_C \mathbf{U}^T \tag{10}$$

where $\mathbf{U} = \text{diag}\{\mathbf{U}_1, \mathbf{U}_2, ..., \mathbf{U}_N\}$ is a fixed user defined block diagonal matrix with blocks $\mathbf{U}_i$ of dimension $n_i \times q_i$. Moreover, $\mathbf{Y}_O = \text{diag}\{\mathbf{Y}_O^{(1)}, \mathbf{Y}_O^{(2)}, ..., \mathbf{Y}_O^{(N)}\}$ and $\mathbf{Y}_C = \text{diag}\{\mathbf{Y}_C^{(1)}, \mathbf{Y}_C^{(2)}, ..., \mathbf{Y}_C^{(N)}\}$ are symmetric block diagonal matrices with blocks $\mathbf{Y}_O^{(i)}$ and $\mathbf{Y}_C^{(i)}$ of dimensions $n_i \times n_i$ and $q_i \times q_i$, respectively.

Similarly, by defining the structure of the matrix $\mathbf{L}_D$ as

$$\mathbf{L}_D \equiv \mathbf{L}_C \mathbf{U}^T \tag{11}$$

where $\mathbf{L}_C = \text{diag}\{\mathbf{L}_C^{(1)}, \mathbf{L}_C^{(2)}, ..., \mathbf{L}_C^{(N)}\}$ is a block diagonal matrix with blocks of $\mathbf{L}_C^{(i)}$ dimension $m_i \times q_i$. With this, the matrix $\mathbf{K}_D$ can be computed by using matrix inversion lemma as:

$$\mathbf{K}_D = \mathbf{L}_C \mathbf{U}^T \mathbf{Y}_O^{-1}[\mathbf{I} - \mathbf{U}(\mathbf{Y}_C^{-1} + \mathbf{U}^T \mathbf{Y}_O^{-1}\mathbf{U})^{-1}]\mathbf{U}^T \mathbf{Y}_O^{-1} \tag{12}$$

Using relation (12) together with the general form of (9) for *output-decentralized system* and moreover by requiring $\mathbf{U}^T \mathbf{Y}_O^{-1} = \mathbf{C}_D$ as an additional constraint within the LMI optimization framework, it is possible to compute directly the matrix $\mathbf{F}_D$ as

$$\mathbf{F}_D = \mathbf{L}_C \mathbf{U}^T \mathbf{Y}_O^{-1}[\mathbf{I} - \mathbf{U}(\mathbf{Y}_C^{-1} + \mathbf{U}^T \mathbf{Y}_O^{-1}\mathbf{U})^{-1}] \tag{13}$$

2.2 Robust decentralized controller design formulation

Using Theorem 1 together with (10) and (11) of the preceding subsection, the decentralized controller design problem for the interconnected system, with uncertainties vector $\boldsymbol{\xi}$, has been reduced to that of finding symmetric block diagonal matrices $\mathbf{Y}_O$, $\mathbf{Y}_C$ and a block diagonal matrix $\mathbf{L}_C$. The basic idea, motivated by the work of Siljak and Stipanovic (2000) on maximizing the class of perturbations that can be tolerated by the closed loop system, is as follows. By applying repeatedly the Schur complement to (A.1) (see Appendix A) and using the structure of matrices (10) and (11), the robust decentralized controller design can be formulated as convex optimization involving the LMIs. This formulation guarantees the solvability of $\mathbf{Y}_O$, $\mathbf{Y}_C$ and $\mathbf{L}_C$ by maximizing at the same time the interconnection uncertainty bounds, and consequently, solving the robust output decentralized controller for the interconnected system. To make the problem more practical, the sum of $1/\xi_i^2$ which is related to the uncertainties in the system can be minimized while at the same time ensuring a prescribed upper uncertainty bound on the individual interconnection terms. Furthermore, by limiting the norm of the individual gains of the controller the optimization problem can be formulated as

$$\text{Min} \quad \sum_{i=1}^{N}(\gamma_i + K_{Y_i} + K_{L_i})$$
$$\text{Subject to} \quad \mathbf{Y}_D > \mathbf{0} \text{ and}$$
$$\begin{bmatrix} \mathbf{Q}_D & \mathbf{G}_D & \mathbf{Y}_D\mathbf{H}_1^T & \cdots & \mathbf{Y}_D\mathbf{H}_N^T \\ \mathbf{G}_D^T & -\mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{H}_1\mathbf{Y}_D & \mathbf{0} & \gamma_1\mathbf{I} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_N\mathbf{Y}_D & \mathbf{0} & \mathbf{0} & \cdots & \gamma_N\mathbf{I} \end{bmatrix} < \mathbf{0} \tag{14}$$

and

$$\gamma_i - \frac{1}{\bar{\xi}_i^2} < 0; \quad \begin{bmatrix} -K_{L_i}\mathbf{I} & \mathbf{L}_D^{(i)T} \\ \mathbf{L}_D^{(i)} & -\mathbf{I} \end{bmatrix} < \mathbf{0}; \quad \begin{bmatrix} \mathbf{Y}_D^{(i)} & \mathbf{I} \\ \mathbf{I} & K_{Y_i}\mathbf{I} \end{bmatrix} > \mathbf{0} \tag{15}$$

where $\mathbf{Q}_D = \mathbf{A}_D\mathbf{Y}_D + \mathbf{Y}_D\mathbf{A}_D^T + \mathbf{B}_D\mathbf{L}_D + \mathbf{L}_D^T\mathbf{B}_D^T$, $\gamma_i = 1/\xi_i^2$; and $\mathbf{Y}_D$ and $\mathbf{L}_D$ are given according to (10) and (11), respectively. Moreover, K_{Y_i} and K_{L_i} are constraints on the magnitudes of decentralized gains, satisfying

$$\mathbf{L}_D^{(i)T}\mathbf{L}_D^{(i)} < K_{L_i}\mathbf{I}; \qquad \mathbf{Y}_D^{(i)^{-1}} < K_{Y_i}\mathbf{I} \tag{16}$$

Based on (14) and (15), the algorithm for determining the robust output decentralized controller and the associated class of perturbations that can be tolerated by the interconnected system is given as follows:

Algorithm:

Step 1: Select the degree of exponential stability $\alpha > 0$ (see Remark 1 in Appendix A).

Step 2: Check the feasibility of the convex optimization problem (14) subject to (15). If it is infeasible, then go to *Step 1* and modify the required degree of exponential stability, else proceed to the next step.

Step 3: Solve the optimization problem in (14) and (15) so as to determine $\mathbf{Y}_O$, $\mathbf{Y}_C$ and $\mathbf{L}_C$ while at the same time maximizing the degree of interconnection bounds.

Step 4: Determine the decentralized output feedback control matrix $\mathbf{F}_D$ using (13).

3. CASE STUDY

The robust decentralized controller design approach presented in the previous section of this paper is now applied on a test system. This system, which is shown in Fig. 1, has been specifically designed to study the fundamental behaviour of large interconnected power systems including inter-area oscillations in power systems (Klein, *et al*., 1991). The system has four machines and each machine is equipped with IEEE standard exciter and governor controllers. In the design, speed signals from each generator are used for robust decentralized PSS control through the excitation systems. Fig. 2 shows the PSS block structure for the i^{th} - machine including the values for T_{iw}, T_{i1} and T_{i2} that are used in the design. While the washout filter and a first order phase-lead block parameters are chosen according to conventional PSS design methods, the gain K_i were estimated based on the technique described in the previous section. After augmenting the washout filter and phase-lead block in the system, the design problem is formulated as a convex optimization problem involving LMIs so as to determine the optimal gain K_i for each controller. Moreover, issues such as upper bounds on the gains of the controllers that guarantee prescribed uncertainty bounds and robust stability are included in the formulation while designing the optimal gain K_i for each PSS.

To demonstrate the advantages along with the robustness of the proposed method, comparisons were made with simulation results obtained from a nonlinear based optimization (NBO) for tuning power system controllers. A short description of the NBO technique can be found in Appendix B.

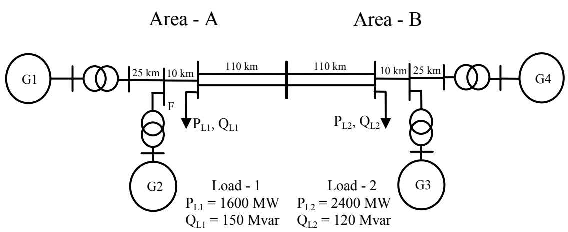

Fig. 1. One-line diagram of four machine two area system.

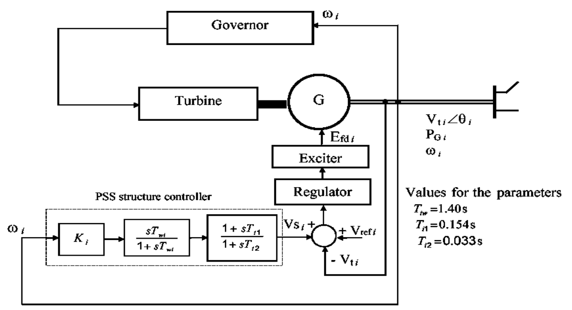

Fig. 2. General structure of the i^{th} - generator together with the PSS structured controller in the multimachine power system.

A three-phase fault with different fault durations was applied at different locations to verify the performance of the proposed LMI based approach as well as that of NBO approach. The NBO design approach was carried out for a fault duration of 150 ms applied to the bus near to generator G2 in Area-A for the base loading condition of $[P_{L1} = 1600 \text{ MW}, Q_{L1} = 150 \text{ Mvar}]$ and $[P_{L2} = 2400 \text{ MW}, Q_{L2} = 120 \text{ Mvar}]$. For the LMI based controller design, the system was linearized for the same operating condition and the corresponding system equations were decomposed as a sum of two sets of equations. While the former describes the system as a hierarchical interconnection of N subsystems, the latter represents the interactions among the subsystems. The damping characteristics of the system with and without PSS controllers for both approaches are shown in Fig.3. The computed PSS gains are also given in Table 1. The PSSs designed through NBO approach were focused on minimizing the quadratic deviation of generator power following a short circuit. Therefore, it is obvious that these PSSs provide slightly better damping for the considered operating condition. It is also observed that the damping achieved from the LMI controllers is quite good and acceptable.

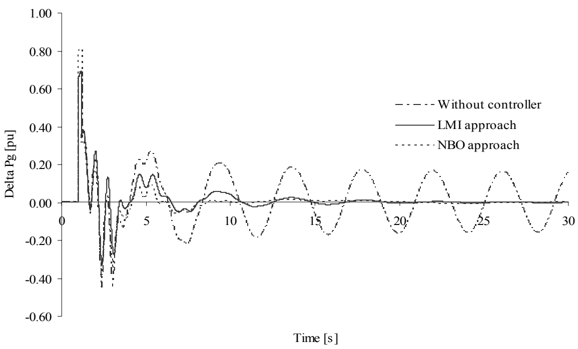

Fig. 3. Transient responses of Generator G2 to a short circuit at node F in Area A

To further assess the effectiveness of the proposed approach regarding robustness, the transient performance indices were computed for different loading conditions at node 1 $[P_{L1}, Q_{L1}]$ and node 2 $[P_{L2}, Q_{L2}]$ while keeping constant total load in the system. The transient performance indices for generator powers P_{Gi}, generator terminal voltages V_{ti} and excitation voltages E_{fdi} following a short circuit of 150 ms duration at node F in Area-A are computed using the following equations, respectively.

$$I^{\mathrm{PG}} = \sum_{i=1}^{N} \int_{t0}^{tf} \left| P_{gi}(t) - P_{gi}^{0} \right| dt \qquad (17a)$$

$$I^{\mathrm{Vt}} = \sum_{i=1}^{N} \int_{t0}^{tf} \left| V_{ti}(t) - V_{ti}^{0} \right| dt \qquad (17b)$$

$$I^{\mathrm{Efd}} = \sum_{i=1}^{N} \int_{t0}^{tf} \left| E_{\mathrm{fd}i}(t) - E_{\mathrm{fd}i}^{0} \right| dt \qquad (17c)$$

These transient performance indices, which are used as a qualitative measure of the post-disturbance behaviour of the system for any fault and/or sudden load changes, are then normalized to the transient performance indices of the base loading condition (without PSSs) for which the design has been carried-out for both approaches. The normalized transient performance indices and the ratios of the normalized transient performance indices are shown in Fig. 4 for the proposed and the NBO approaches.

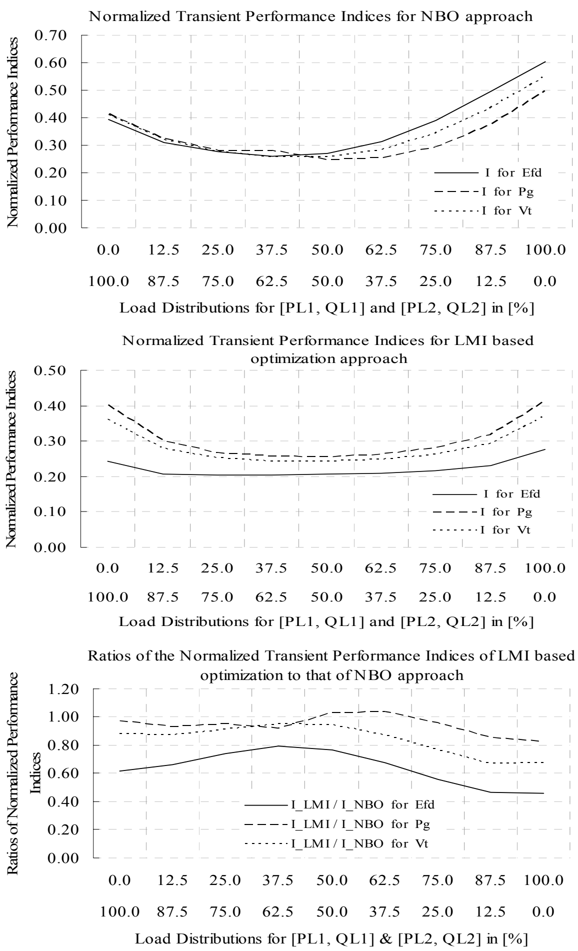

Fig. 4. Plot of the normalized transient performances for LMI with degree of stability α = 0.15 and NBO approaches.

It can be seen from these results that, for I^{PG} performance index, the NBO approach performs better near or in the vicinity of the base loading condition, i.e. $[P_{L1} = 1600\ \mathrm{MW},\ Q_{L1} = 150\,\mathrm{Mvar}]$ and $[P_{L2} = 2400\ \mathrm{MW},\ Q_{L2} = 120\ \mathrm{Mvar}]$ corresponding to $[40\%,\ 60\%]$ in Fig. 4. This is due to the fact that the NBO approach aimed only to improve the damping behaviour of the system by minimizing the transient responses of the generator output powers. However, as the loading conditions vary over wide ranges, the robustness of the controllers designed by the convex optimization involving LMI outperform that of NBO approach as can be seen from the results shown in Fig. 4. Though the approach proposed in this paper seems computationally more involved and needs a relatively long computation time for large problems, its performances in all cases are adequate as compared with the NBO approach for PSSs tuning.

Table 1 Gains computed for LMI approach with degree of stability α = 0.15 and NBO approach.

Approaches	Gains	Gains upper bounds
LMI	$K_1 = 0.6544$ $K_2 = 4.8921$ $K_3 = 4.3837$ $K_4 = 0.2458$	$K_{Li} < 4$ $K_{Yi} < 5$ (i = 1, 2, 3, 4) $\lVert K_i \rVert^2 < K_{Li} K_{Yi}^2 = 100$
NBO	$K_1 = 9.15$ $K_2 = 0.0$ $K_3 = 0.0$ $K_4 = 0.0$	$\lVert K_i \rVert^2 < \lVert K_{\max i} \rVert^2 = 100$

Moreover, the approach has the merit of incorporating additional design constraints such as the size and the structure of the gain matrices, the degree of exponential stability and delays. While the NBO approach is straightforward for designing controllers for even large sized problems without encountering any computational difficulties, due to the nature of the problem the optimal solutions depend on the initial values of the controller parameters, fault duration and fault locations used for determining the solution of the problem.

4. CONCLUSION

A framework for a robust decentralized PSS controllers design for power systems that explicitly takes into account the interactions between subsystems, changes in operating conditions and effects of system nonlinearities has been investigated. The applicability of the approach has been demonstrated through designing robust PSS in a four-machine test system. This design problem formulated as a convex optimization problem involving LMIs, which is computationally efficient and guarantees the existence of solution in polynomial time. Moreover the approach is flexible enough to allow the inclusion of additional design parameters such as the size and structure of the gain matrices, the degree of exponential stability and delays so as to maximize the class of perturbations that can be tolerated by the interconnected system. An additional benefit of this approach, besides computational issues, is the fact that all the controllers are linear and use minimum local feedback information. Thus their implementation is straightforward and cost effective.

APPENDIX-A

Theorem 1: The interconnected system (1) is robustly stabilized by the decentralized control strategy of (6) with uncertainty degree vector $\boldsymbol{\xi}=[\xi_1,\xi_2,...,\xi_N]^T$ if and only if there exist a symmetric positive definite matrix $\mathbf{Y}_D=\text{diag}\{\mathbf{Y}_D^{(1)},\mathbf{Y}_D^{(2)},...,\mathbf{Y}_D^{(N)}\}$ with blocks $\mathbf{Y}_D^{(i)}$ of dimensions $n_i\times n_i$ and a block diagonal matrix $\mathbf{L}_D=\text{diag}\{\mathbf{L}_D^{(1)},\mathbf{L}_D^{(2)},...,\mathbf{Y}_D^{(N)}\}$ with blocks $\mathbf{L}_D^{(i)}$ of dimensions $m_i\times n_i$ satisfying the following matrix inequality:

$$\mathbf{Y}_D\mathbf{A}_D^T+\mathbf{A}_D\mathbf{Y}_D+\mathbf{L}_D^T\mathbf{B}_D^T+\mathbf{B}_D\mathbf{L}_D + \sum_{i=1}^{N}\xi_i^2\mathbf{Y}_D\mathbf{H}_i^T\mathbf{H}_i\mathbf{Y}_D+\mathbf{G}_D\mathbf{G}_D^T<\mathbf{0} \quad \text{(A.1)}$$

Moreover, the controller matrix $\mathbf{K}_D$ is computed as

$$\mathbf{K}_D\equiv\mathbf{L}_D\mathbf{Y}_D^{-1} \quad \text{(A.2)}$$

Proof: This theorem can be proved via a quadratic Lyapunov function of the form $\mathbf{V}(\mathbf{x})=\mathbf{x}^T\mathbf{P}_D\mathbf{x}$ that ensures the negative definiteness of the derivative of $\mathbf{V}(\mathbf{x})$, i.e. $d\mathbf{V}(\mathbf{x})/dt<0$, under the constraint of (5) for all trajectories of (8). Then with change of variables $\mathbf{Y}_D=\tau\mathbf{P}_D^{-1}$ and $\mathbf{L}_D=\mathbf{K}_D\mathbf{Y}_D$, the expression in (A.1) can be obtained.

Remark 1: In proving the above theorem, it is possible to establish an α - degree of stability for the system by finding a quadratic Lyapunov function that proving $d\mathbf{V}(\mathbf{x})/dt=-2\alpha\mathbf{V}(\mathbf{x})$ for some positive α.

APPENDIX-B

The objective of the nonlinear based optimization (NBO) for tuning power system controllers is to force the system to have a better damping behaviour and as well as a post-disturbance stable operating point within short time (Cai and Erlich, 2003). The formulation of NBO problem can be expressed within the generalized cost function of the form as:

$$\min_{\boldsymbol{\lambda}}\sum_{i=1}^{N}\{\alpha_i\int_{t_0}^{t_f}P_{gi}^2\,dt+\beta_i\int_{t_0}^{t_f}V_{ti}^2dt+\int_{t_0}^{t_f}E_{fdi}^2dt\} \quad \text{(B.1)}$$

where

α_i and β_i are weighting factors for P_{gi} and V_{ti}, respectively.

P_{gi} - the generator real power for the i^{th} - generator.

V_{ti} - the generator terminal voltage for the i^{th} - generator.

E_{efi} - the exciter field voltage for the i^{th} - generator.

$\boldsymbol{\lambda}$ - the gains for PSS structured controllers.

Solving problem (B.1) can be carried out using gradient-based methods from IMSL mathematic library routines (IMSL Math/Library, 1997). However, due to the nonlinear nature of the problem, the solutions of the optimization problem depend on the initial values of the controller parameters, fault locations and fault durations in the system.

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